INTERNATIONAL LIBRARY OF TECHNOLOGY

A SERIES OF TEXTBOOKS FOR PERSONS ENGAGED IN THE ENGINEERING PROFESSIONS AND TRADES OR FOR THOSE WHO DESIRE INFORMATION CONCERNING THEM. FULLY ILLUSTRATED AND CONTAINING NUMEROUS PRACTICAL EXAMPLES AND THEIR SOLUTIONS

STRENGTH OF MATERIALS
APPLIED MECHANICS
MACHINE DESIGN

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PREFACE

The International Library of Technology is the outgrowth of a large and increasing demand that has arisen for the Reference Libraries of the International Correspondence Schools on the part of those who are not students of the Schools. As the volumes composing this Library are all printed from the same plates used in printing the Reference Libraries above mentioned, a few words are necessary regarding the scope and purpose of the instruction imparted to the students of—and the class of students taught by—these Schools, in order to afford a clear understanding of their salient and unique features.

The only requirement for admission to any of the courses offered by the International Correspondence Schools, is that the applicant shall be able to read the English language and to write it sufficiently well to make his written answers to the questions asked him intelligible. Each course is complete in itself, and no textbooks are required other than those prepared by the Schools for the particular course selected. The students themselves are from every class, trade, and profession and from every country; they are, almost without exception, busily engaged in some vocation, and can spare but little time for study, and that usually outside of their regular working hours. The information desired is such as can be immediately applied in practice, so that the student may be enabled to exchange his present vocation for a more congenial one, or to rise to a higher level in the one he now pursues. Furthermore, he wishes to obtain a good working

knowledge of the subjects treated in the shortest time and in the most direct manner possible.

In meeting these requirements, we have produced a set of books that in many respects, and particularly in the general plan followed, are absolutely unique. In the majority of subjects treated the knowledge of mathematics required is limited to the simplest principles of arithmetic and mensuration, and in no case is any greater knowledge of mathematics needed than the simplest elementary principles of algebra, geometry, and trigonometry, with a thorough, practical acquaintance with the use of the logarithmic table. To effect this result, derivations of rules and formulas are omitted, but thorough and complete instructions are given regarding how, when, and under what circumstances any particular rule, formula, or process should be applied; and whenever possible one or more examples, such as would be likely to arise in actual practice—together with their solutions—are given to illustrate and explain its application.

In preparing these textbooks, it has been our constant endeavor to view the matter from the student's standpoint, and to try and anticipate everything that would cause him trouble. The utmost pains have been taken to avoid and correct any and all ambiguous expressions—both those due to faulty rhetoric and those due to insufficiency of statement or explanation. As the best way to make a statement, explanation, or description clear, is to give a picture or a diagram in connection with it, illustrations have been used almost without limit. The illustrations have in all cases been adapted to the requirements of the text, and projections and sections or outline, partially shaded, or full-shaded perspectives, have been used, according to which will best produce the desired results. Half-tones have been used rather sparingly, except in those cases where the general effect is desired rather than the actual details.

It is obvious that books prepared along the lines mentioned must not only be clear and concise beyond anything heretofore attempted, but they must also possess unequaled value for reference purposes. They not only give the

maximum of information in a minimum space, but this information is so ingeniously arranged and correlated, and the indexes are so full and complete, that it can at once be made available to the reader. The numerous examples and explanatory remarks, together with the absence of long demonstrations aad abstruse mathematical calculations, are of great assistance in helping one to select the proper formula, method, or process and in teaching him how and when it should be used.

The subjects comprised in this volume are as follows: Strength of Materials, Applied Mechanics, and Machine Design. None of the subjects is treated exhaustively, the treatment being limited to the ordinary needs of the draftsman or young machine designer. On account of the clear and concise manner in which the different principles and methods of applying them are presented, the work should prove of great value to any one desiring to use it for reference purposes. Particular attention is called to the practical manner in which the different subjects are arranged and presented. The paper on Strength of Materials is based to a certain extent on Professor Mansfield Merriman's well-known work, "Mechanics of Materials"; extended tables giving values of tests for different materials have been omitted, the reader being advised to make these tests himself, or else to specify the requirements that the manufacturer should meet. In place of extended tables, a fair average value for the different results to be obtained from tests are given and large factors of safety used, the result being a greater harmony in treating the subject and the securing of uniformity in the subjects that follow.

As mentioned above, this volume is printed from the plates used in printing the Reference Libraries of the International Correspondence Schools. On account of the omission of certain papers, the material contained in which is given in better form elsewhere, there are several breaks in the continuity of the page numbers, formula numbers, article numbers, etc. This, however, does not impair the value of the volume, as the index has been reprinted and made to conform to the present arrangement.

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NOTICE.

There are a number of references in Machine Design to a treatise on the Steam Engine which is not included in this Library. Such references are given below in the form of equivalent references to Vol. 8 of the Library, with the exception of a short treatment of fly-wheels, which is given entire at the end of Machine Design. Any references not noted below and which cannot be found in this volume are unimportant.

Art. **1161,** see Art. **14,** § 25, Vol. 8.
Art. **1222,** see Art. **14,** § 25, Vol. 8.
Art. **1226,** see Art. **6,** § 23, Vol. 8.
Art. **1238,** see Art. **23,** § 23, Vol. 8.
Art. **1270,** see Art. **48,** § 25, Vol. 8.
Art. **1306,** see Tables I, II, III, § 30, Vol. 8.
Art. **1327,** see end of volume.
Art. **1328,** see end of volume.

STRENGTH OF MATERIALS.

MATERIALS USED FOR CONSTRUCTION.

1331. The principal materials used in engineering construction are *timber*, *brick*, *stone*, *cast iron*, *wrought iron*, and *steel.* Table 23 gives their average weights per cubic foot.

TABLE 23.

Material.	Average Weight. Pounds per Cubic Foot.	Approximate Weight of Piece 1" Square and 1' Long in Lb.
Timber	40	.278
Brick	125	
Stone	160	
Cast Iron	450	3.125
Wrought Iron	480	3.333⅓
Steel	490	3.403

CAST IRON.

1332. Cast iron is a combination of pure iron with from 2% to 6% of carbon.

Pig iron is the result of the first smelting, and is obtained directly from the ore. Pig iron is rarely, if ever, used for anything except to be remelted and made into cast iron or wrought iron.

1333. Cast iron is of two kinds, *white cast iron* and *gray cast iron.* The first is a chemical compound of iron with from 2% to 6% of carbon, nearly all the carbon being chemically combined with the iron. The second, or gray cast iron, contains a part of the carbon in chemical combination, and the rest in the state of graphite mechanically mixed with the iron. When a piece of gray cast iron is

broken, a large number of black specks are seen on the broken ends; these specks are pure carbon in the form of graphite.

1334. White cast iron contains hardly any free carbon. It is of two kinds, granular and crystalline. Both are very hard and brittle, and are only used for conversion into wrought iron or steel.

1335. Gray cast iron is divided into three classes, known as Nos. 1, 2, and 3.

No. 1 contains the largest amount of carbon in mechanical mixture, which makes it soft and fusible, though not as strong as Nos. 2 and 3. It is very suitable for making castings where precision in form is required, as, owing to its fluidity, it fills the mold well; but it is not suitable for castings requiring strength.

No. 2 is most suitable for use in constructions, as it is stronger than No. 1, and not so soft.

No. 3 contains the smallest amount of carbon in the graphitic (uncombined) form, and is, in consequence, harder and more brittle. It is fit only for the massive and heavy parts of machinery.

1336. Cast iron has certain advantages and disadvantages as a material for engineering construction. It is easy to give it any desired form. A pattern of the piece desired is made; a mold is made in the sand, the pattern is removed, and the melted iron poured in. Cast iron resists oxidation (rust) better than either wrought iron or steel. Its compressive (crushing) strength is very high, but its tensile (stretching) strength is comparatively low. It cannot be riveted, or welded by forging. It is brittle, breaking off without giving much warning, and stretching but little before giving away. It is liable to have hidden and small surface defects and air bubbles, and this makes its strength uncertain.

Another serious drawback in the use of cast iron is its liability to initial stresses from inequality in cooling after having been poured into the molds. Thus, if one part of

the casting is very thin and another very thick, the thin part cools first, and, in cooling, contracts; the thick part, cooling afterwards, causes stresses in the thin part, which may be sufficient to break it, or, if not, there may be so great a stress in the thin part that a small additional force will break it.

Cast iron is not well adapted to a tensile stress, nor to resist shocks. It is used for columns and posts in buildings, on account of its high compressive strength. In machinery, it is used in all those parts where weight, mass, or form is of more importance than strength, as in frames and bed plates of machines, and for hangers, pulleys, gear wheels, etc. It is also used for water mains where the pressure to be resisted is not too great.

1337. Malleable cast iron is made by heating the casting in an annealing oven, in powdered hematite ore. It can be hammered into any desired shape when cold, but is very brittle when hot.

WROUGHT IRON.

1338. Wrought iron is the product resulting from the reduction of the carbon in cast iron. It is obtained by melting white cast iron, and passing an oxidizing flame over it. When the carbon is burned out, the mass of iron is left in a pasty condition, full of holes. It is then taken out, and hammered or rolled in order to unite it into one mass. The result of this first process is not suitable for use in any construction of importance, and has to be reheated and rerolled a number of times, in order to make it homogeneous, and to remove flaws from within the iron.

At best, therefore, wrought iron is a series of welds; if a piece is broken, the separate layers of which it is composed can be seen plainly. It cannot be melted and run into molds, as can be done with cast iron; but it can be easily welded by forging; that is, two pieces of wrought iron can be united by raising them to the proper temperature and hammering them together. Wrought iron is much more capable of bearing a tensile or transverse stress than cast

iron; it is tougher, stretches more, and gives more warning before fracture.

It withstands shocks far better than cast iron. Two pieces may be punched or drilled and riveted together. The entire process weakens the iron, and cast iron would not withstand it. It has also to withstand flanging and the stresses due to changes of temperature.

Wrought iron cannot be hardened like steel by heating and then dipping in water, but may be *case-hardened* by rubbing the surface with potassium cyanide or potassium ferrocyanide while at a cherry-red heat and then dipping in water. The cyanide causes the iron to be carbonized to a slight depth; that is, through a depth of about $\frac{1}{32}$ of an inch the iron is converted into steel which can be hardened. Cast iron may be hardened in the same way.

The quality of wrought iron varies considerably, and the terms by which it is known in the market refer to the amount of working which the iron has received. We thus have common bar iron, best iron, double best, and triple best. These terms are only rough indications of quality. When wrought iron is rolled cold under great pressure, it has a smooth polished surface, and its strength is greatly increased.

When the word iron is used alone, wrought iron is meant.

STEEL.

1339. Steel is a chemical compound of iron and carbon; that is, it contains no carbon in a free state, as cast iron does. Its tensile strength is greater than that of wrought iron, and its compressive strength greater than that of cast iron. It is by far the strongest material used in the mechanic arts. Its strength varies greatly with its purity and the amount of carbon it contains. According to the amount of carbon in it, steel is divided into high grade, medium grade, and low grade, the high grades having the most carbon. Steel, unlike wrought iron, is fusible; unlike cast iron, it can be forged; and, with the exception of the higher grades,

it can be welded by heating and hammering, although care must be exercised in so doing.

1340. The special characteristic of steel (the very lowest grades excepted) is that, when it is raised to a cherry red heat and suddenly cooled, it becomes brittle and exceedingly hard, and that by subsequent heating and slow cooling, the hardness may be reduced to any desired degree down to the point of least hardness that steel possessing that amount of carbon can have. The first process is called *hardening*, and the second *tempering*.

If the surface of a piece of steel that has been hardened is polished slightly so as to remove the dark scale or soot which covers it, and is then reheated, it will be found that, as the temperature increases, a series of colors appear. These colors are always the same for the same temperature, and, if the steel is suddenly cooled when one of the colors appears, it acquires a degree of hardness which is always the same for the same color and for the same quality of steel.

In Table 24 are given the colors with the corresponding temperatures that occur in tempering different kinds of tools.

TABLE 24.

Tools.	Color.	Temperature, Fahr.
Lancets	Pale yellow	430°
Razors and scalpels	Pale straw	450°
Penknives, chisels for cast iron, and screw taps	Yellow	470°
Scissors and chisels for wrought iron	Brown	490°
For carpenters' tools in general	Red	510°
Fine watch springs and table knives	Purple	530°
Swords and lock springs	Blue, bright	550°
Daggers, fine saws and needles	Blue, full	560°
Common saws and springs	Blue, dark	600°

Steel that has been hardened may be softened by heating it to a hardening temperature and then cooling it very slowly; this process is called **annealing.**

1341. Steel is made in one of the three following ways:

1. By adding carbon to wrought iron.
2. By removing carbon from cast iron.
3. By melting together cast and wrought iron in suitable proportions.

Several processes, varying with the quality of the product desired and the use for which it is intended, are used in making steel. The processes in general use are the following:

(*a*) **Cementation,** in which bars of very pure wrought iron are heated to a high temperature in contact with carbon. The product, known as **blister steel,** is used for cutlery, tools, etc.

(*b*) **Crucible steel,** also called **cast steel,** is made by melting pure wrought iron in a crucible with enough charcoal and cast iron to introduce the required amount of carbon. It is used for making springs, cutlery, tools, etc.

(*c*) **Bessemer steel** is made by decarbonizing cast iron by forcing a powerful blast of air through a melted mass of the iron. This removes the greater part of its carbon. A small quantity of very pure cast iron, rich in carbon, is then added, bringing up the percentage of carbon to the required amount.

(*d*) **Open-hearth steel** is made by fusing a charge consisting of the suitable proportions of cast iron with wrought iron scrap, or with Bessemer steel scrap.

1342. Bessemer and open-hearth steel contain more impurities than blister and crucible steel do; but they are much cheaper, and are just as suitable for many purposes. It is only in consequence of the introduction of these two cheap varieties that steel can be extensively used, as blister and crucible steels would, in the majority of cases, be too expensive.

Bessemer and open-hearth steels contain from .05% to 1½% of carbon. The proportion of carbon in the best kinds of tool and cutlery steels is as follows:

Razor steel,	1½%.	Very difficult to forge, and easily burnt.
Saw file,	1⅜%.	Bears heat not above cherry redness.
Tool steel,	1¼%.	Ordinary cutting tools. Welds with difficulty.
" "	1⅛%.	For mandrils and heavy cutting tools.
" "	1%.	For chisels, gravers, etc.

At the Imperial Works at Neuberg, Austria, the following percentages of carbon are present in the different grades of Bessemer steel:

1.58% to 1.38%.	Cannot be welded, and is rarely used.
1.38% " 1.12%.	Great care must be used in working.
1.12% " .88%.	Welds easily; used for bits, chisels, etc.
.88% " .62%.	Used for cutting tools, files, etc.
.62% " .38%.	Mild steel; for tires, etc.
.38% " .15%.	Tempers slightly; for boiler plates and axles.
.15% " .05%.	Does not temper; steel for pieces of machinery.

1343. It will be noticed from what precedes that the hardness of steel depends upon the amount of carbon it contains.

Some kinds of crucible cast steel can be hardened by heating to a low red heat and then allowing them to cool slowly in the air without dipping in water. They are called self-hardening steels, the best known being Mushet's special tool steel. This contains about 2% carbon with 7% to 12% tungsten in alloy with the iron. The same property is characteristic of Hadfield's manganese steel, which contains between .8% and 1.2% of carbon and 7% to 20% of manganese.

As a rule, when a piece of steel is broken across the grain, the finer the grain and the whiter and cleaner the fracture the more carbon it contains.

STRESSES AND STRAINS.

1344. The molecules of a solid or rigid body being held together by the force of cohesion, this force must be overcome to a greater or less degree in order to change the form and size of the body, or to break it into parts. The internal resistance which a body offers to any force tending to overcome the force of cohesion is called a **stress.** If a weight of 1,000 pounds is held in suspension by a rod, there will be a stress of 1,000 pounds in the rod. In this country and England, stresses are measured in pounds or tons; in nearly all other civilized countries, in kilograms. Whenever a body is subjected to a stress, the total stress induced by the acting force at any section of the body is the same as the total stress at any other section.

1345. The **unit stress** (called also the **intensity** of stress) is the stress per unit of area; or, it is the total stress divided by the area of the cross-section. In the above illustration, if the area had been 4 sq. in. the unit stress would have been $\frac{1,000}{4} = 250$ lb. per sq. in. Had the area been $\frac{1}{2}$ sq. in., the unit stress would have been $\frac{1,000}{.5} = 2,000$ lb. per sq. in.

Let $P =$ the total stress in pounds;
$A =$ area of cross-section in square inches;
$S =$ unit stress in pounds per square inch.

Then, $$S = \frac{P}{A}, \text{ or } P = A\,S. \qquad \textbf{(108.)}$$

That is, *the total stress equals the area of the section, multiplied by the unit stress.*

When a body is stretched, shortened, or in any way deformed through the action of a force, the amount of deformation is called a **strain.** Thus, if the rod before mentioned had been elongated $\frac{1}{10}''$ by the load of 1,000 pounds, the strain would have been $\frac{1}{10}''$. Within certain limits, to be given hereafter, strains are proportional to the stresses producing them.

1346. The **unit strain** is the strain per unit of length or of area, but is usually taken per unit of length and called the *elongation* per unit of length. In this paper, the unit of length will be considered as one inch. The unit strain, then, equals the total strain divided by the length of the body in inches.

Let l = length of body in inches;
e = elongation in inches;
s = unit strain.

Then, $s = \frac{e}{l}$, or $e = ls$. **(109.)**

1347. Whenever a force, no matter how small, acts upon a body, it produces a stress and a corresponding strain.

According to the manner in which forces act upon a body, the stresses are divided into the following classes:

1. *Tension*, which produces a tensile or pulling stress.
2. *Compression*, which produces a compressive or crushing stress.
3. *Shear*, which produces a shearing or cutting stress.
4. *Torsion*, which produces a torsional or twisting stress.
5. *Flexure*, which produces a transverse or bending stress.

TENSION.

1348. When two forces act upon a body in opposite directions (away from each other) the body is said to be in tension. The two forces tend to elongate the body and thus produce a tensile stress and strain. A weight supported by a rope affords a good example. The weight acts downwards, and the reaction of the support to which the upper end of the rope is fastened acts upwards; the result being that the rope is stretched more or less, and a tensile stress is produced in it. Another familiar example is the connecting rod of a steam engine on the return stroke. The cross-head then exerts a pull on one end of the rod, which is resisted by the crank-pin on the other.

EXAMPLE.—An iron rod, 2 inches in diameter, sustains a load of 90,000 pounds; what is the unit stress?

SOLUTION.—Using formula **108**,

$$P = A\,S, \text{ or } S = \frac{P}{A} = \frac{90{,}000}{2^2 \times .7854} = 28{,}647.82 \text{ lb. per sq. in.} \quad \text{Ans.}$$

EXPERIMENTAL LAWS.

1349. The following laws have been established by experiment:

1. *When a body is subjected to a small stress, a small strain is produced, and when the stress is removed the body springs back to its original shape. This leads to the conclusion that, for small stresses, bodies are perfectly elastic.*

2. *Within certain limits, the change of shape (strain) is directly proportional to the applied force.*

3. *When the stress is sufficiently great, a strain is produced which is partly permanent; that is, the body does not spring back entirely to its original form when the stress is removed. This lasting part of the strain is called a* **set,** *and in such cases the strain is not proportional to the stress.*

4. *Under a still greater stress, the strain rapidly increases, and the body is finally ruptured or broken.*

5. *A force acting suddenly, as a shock, causes greater injury than a force gradually applied.*

According to the first law, the body will resume its original form when the force is removed, provided the stress is not too great. This property is called *elasticity*. According to the second law, the strain is proportional to the stress within certain limits. Thus, if a pull of 1,000 pounds elongates a body .1″, a pull of 2,000 pounds would elongate it .2″. This is true up to a certain limit, beyond which the body will not resume its original form upon the removal of the stress, but will be permanently strained more or less, according to the amount of stress. The stress at the point where the set begins is called the **elastic limit.** All strains produced by stresses within (less than) the elastic limit are directly proportional to the stresses.

1350. Stresses are equal but opposed to the external forces producing them, and are, therefore, measured and represented by these forces. Thus, as we have explained before, a force of 1,000 pounds produces a stress of 1,000 pounds. The external force is the force applied to a fixed body; the stress is the resistance offered by the body to a change of form; and when the body ceases to change (as when a rod ceases to elongate), the stress just balances the external force.

COEFFICIENT OF ELASTICITY.

1351. Amongst engineers, the term *elasticity* means the *resistance* which a body offers to a permanent change of form; and by *strength*, the resistance which a body offers to division or separation into parts.

It follows from this that those bodies which have the highest elastic limit are the most elastic.

1352. The **coefficient of elasticity** is the ratio of the unit stress to the unit strain, provided the elastic limit is not exceeded. Let S be the unit stress, s the unit strain, and E the coefficient of elasticity; then, by definition, $E = \frac{S}{s}$. Substituting the values of S and s obtained from formulas **108** and **109**,

$$E = \frac{S}{s} = \frac{P}{A} \div \frac{e}{l} = \frac{P l}{A e}. \qquad \textbf{(110.)}$$

1353. If in this formula we assume $e = l$, and $A = 1$ (1 square inch), then $E = P$. That is, the coefficient of elasticity is that force which, *if stress and strain continued proportional to each other*, would produce in a bar of unit area a strain equal to the original length of the bar ($l = e$). This, however, is never the case, as the elastic limit and the ultimate strength are reached before the applied force reaches the value E.

EXAMPLE.—A wrought iron bar 2 inches square and 10 feet long is stretched .0528 inch by a stress of 44,000 pounds; what is the coefficient of elasticity?

SOLUTION.—Using formula **110**,

$$E = \frac{P l}{A e} = \frac{44{,}000 \times 10 \times 12}{2^2 \times .0528} = 25{,}000{,}000 \text{ lb. per sq. in.} \quad \text{Ans.}$$

1354. The **ultimate strength** of any material is that unit stress which is just sufficient to break it.

1355. The **ultimate elongation** is the total elongation produced in a unit of length of the material having a unit of area, by a stress equal to the ultimate strength of the material.

1356. For the same size, quality, and kind of material, the ultimate strength, ultimate elongation, coefficient of elasticity, and elastic limit are the same for different pieces. Table 25 gives the *average* values of the coefficient of elasticity (E_1), elastic limit (L_1), ultimate strength (S_1), and

TABLE 25.

Material.	Coefficient of Elasticity. E_1.	Elastic Limit. L_1.	Ultimate Tensile Strength. S_1.	Ultimate Elongation. s_1.
	Lb. per Sq. In.	Lb. per Sq. In.	Lb. per Sq. In.	In. per Linear Inch.
Timber......	1,500,000	3,000	10,000	0.015
Cast Iron....	15,000,000	6,000	20,000	0.005
Wrought Iron	25,000,000	25,000	55,000	0.20
Steel........	30,000,000	50,000	100,000	0.10

ultimate elongation (s_1), of different materials, the quantities given being for *tension* only. As brick and stone are never used in tension, their values are not given.

The values in this table are subject to great variation, and cannot be depended upon in designing machine parts. Thus, the ultimate tensile strength of steel varies from less than 60,000 to more than 180,000 pounds per square inch, according to its purity and the amount of carbon it contains; that of cast iron from 12,000, or 13,000, to over 40,000; wrought iron varies from 40,000 to 72,000, according to quality, the latter value being for iron wire. Timber varies fully as much as, if not more than, any of the three preceding materials, its properties depending upon the kind of wood, its degree of dryness, the manner of drying, etc.

All the problems in this section will be solved by using the average values given in the preceding and following tables; the designer, however, should not use them, but either test the materials himself or state in the specifications what strength the material must have. For example, mild steel, for boiler shells, should have a tensile strength of not less than 60,000 or 65,000 pounds per square inch; Bessemer steel, for steel rails, not less than 110,000; open-hearth steel, for locomotive tires, not less than 125,000, and crucible cast steel, for tools, cutlery, etc., not less than 150,000. It is also customary to specify the amount of elongation. This is necessary because, as a rule, the elongation decreases as the tensile strength increases. Having tested the material about to be used, or having specified the lowest limits, the designer can ascertain the strength and stiffness of construction by means of the formulas and rules which are to follow.

EXAMPLE.—How much will a piece of steel 1 inch in diameter and 1 foot long elongate under a steady load of 15,000 pounds?

SOLUTION.—$E_1 = \frac{P\,l}{A\,e}$, or $e = \frac{P\,l}{A\,E_1}$.

From Table 25, $E_1 = 30,000,000$ for steel; hence,

$$e = \frac{15,000 \times 12}{1^2 \times .7854 \times 30,000,000} = .00764''. \text{ Ans.}$$

NOTE.—All lengths given in this treatise on Strength of Materials must be reduced to inches before substituting in the formulas.

EXAMPLE.—A piece of timber has a cross-section $2'' \times 4''$ and is 6 feet long. A certain stress produces an elongation of .144 inch; what is the value of the stress in pounds?

SOLUTION.—

$$E_1 = \frac{P\,l}{A\,e}, \text{ or } P = \frac{E_1\,A\,e}{l} = \frac{1,500,000 \times 2 \times 4 \times .144}{6 \times 12} = 24,000 \text{ lb. Ans.}$$

COMPRESSION.

1357. If the length of the piece is not more than five times its least transverse dimension (its diameter, when round; its shorter side, when rectangular, etc.), the laws of compression are similar to those of tension. The strain is proportional to the stress until the elastic limit has been

reached; after that, it increases more rapidly than the stress, as in the case of tension. The area of the cross-section is slightly enlarged under compression. In Table 26 are given the average compression values of E, L, and S for wood, brick, stone, cast iron, wrought iron, and steel. (See also Table 25.) E is not given for brick, nor L for cast iron, brick, or stone, because these values are not known. To distinguish between tension and compression when applying a formula, E_2, L_2, and S_2 will be used instead of E_1, L_1, and S_1.

TABLE 26.

Material.	Coefficient of Elasticity. E_2.	Elastic Limit. L_2.	Ultimate Compressive Strength. S_2.
	Lb. per Sq. In.	Lb. per Sq. In.	Lb. per Sq. In.
Timber..........	1,500,000	3,000	8,000
Brick	....	......	2,500
Stone............	6,000,000	......	6,000
Cast Iron.........	15,000,000	......	90,000
Wrought Iron.....	25,000,000	25,000	55,000
Steel.............	30,000,000	50,000	150,000

1358. When the length of a piece subjected to compression is greater than ten times its least transverse dimension, it is called a *column*, and the material fails by a sidewise bending or flexure. The preceding table is to be used only for pieces whose length does not exceed *five* times the least dimension of the cross-section. (See Art. **1421***a*.)

EXAMPLE.—How much will a wrought iron bar 4 inches square and 15 inches long shorten under a load of 100,000 pounds?

SOLUTION.—$E_2 = \frac{P\,l}{A\,e}$, or $e = \frac{P\,l}{A\,E_2}$.

Hence, $e = \frac{100{,}000 \times 15}{16 \times 25{,}000{,}000} = .00375''$. Ans.

SHEAR.

1359. When two surfaces move in opposite directions very near together in such a manner as to cut a piece of material, or to pull part of a piece through the remainder, the piece is said to be *sheared*. A good example of a shearing stress is a punch; the two surfaces in this case are the bottom of the punch and the top of the die. Another example is a bolt with a thin head; if the pull on the bolt is great enough, it will be pulled through the head and leave a hole in it, instead of the bolt breaking by pulling apart, as would be the case with a thick head. In this case, the two surfaces are the under side of the head and the surface pressed against. Other examples are a knife cutting a piece of wood, and the ordinary shears from which this kind of stress takes its name.

TABLE 27.

Material.	Coefficient of Elasticity. E_s.	Ultimate Shearing Strength. S_s.
Timber (across the grain).....	....	3,000
Timber (with the grain)......	400,000	600
Cast Iron....................	6,000,000	20,000
Wrought Iron................	15,000,000	50,000
Steel.........................	....	70,000

1360. Formula **108** applies in cases of shearing stress, but formulas **109** and **110** are never used for shearing. In the preceding table, E_s and S_s are used to represent, respectively, the coefficient of shearing elasticity, and ultimate shearing strength.

EXAMPLE.—What force is necessary to punch a one-inch hole in a wrought iron plate $\frac{3}{8}$ of an inch thick?

SOLUTION.—$1'' \times 3.1416 \times \frac{3}{8}'' = 1.1781$ sq. in. = area of punched surface = area of a cylinder 1″ in diameter and $\frac{3}{8}''$ high. Using formula **108**,

$$P = A\,S_s = 1.1781 \times 50{,}000 = 58{,}905 \text{ lb. Ans.}$$

EXAMPLE.—A wooden rod 4 inches in diameter and 2 feet long is turned down to 2 inches diameter in the middle so as to leave the enlarged ends each 6 inches long. Will a steady stress pull the rod apart in the middle, or shear the ends?

SOLUTION.—$P = A\,S_s = 2 \times 3.1416 \times 6 \times 600 = 22{,}620$ lb. to shear off the ends.

The force required to rupture by tension is

$$P = A\,S_t = 2^2 \times .7854 \times 10{,}000 = 31{,}416 \text{ lb.}$$

Since the former is only about ¾ of the latter, the piece will fail through the shearing off of the end. Ans.

Had a transverse stress been used, the force necessary to shear off a section of the end would have been

$$4^2 \times .7854 \times 3{,}000 = 37{,}700 \text{ lb.}$$

FACTORS OF SAFETY.

1361. It was previously stated that no stress should ever be applied to a machine part that would strain it beyond the elastic limit. The usual practice is to divide the ultimate strength of the material by some number depending upon the kind and quality of the material, and upon the nature of the stress; this number is called a **factor of safety.**

The factor of safety for any material is the ratio of its ultimate strength to the actual stress to which it is subjected, or for which it is intended.

In Table 27, 70,000 pounds per square inch is given as the ultimate shearing strength for steel. Now, suppose that the actual stress on a piece of steel is 10,000 pounds per square inch; then, the factor of safety for this piece would be $\frac{70{,}000}{10{,}000} = 7$.

1362. To find the proper allowable working strength of a material, divide the ultimate strength for tension, compression, or shearing, as the case may be, by the proper factor of safety.

Table 28 gives the factors of safety generally used in American practice. Factors of safety will always be denoted by the letter f in the formulas to follow.

TABLE 28.

Material.	For Steady Stress. (Buildings.)	For Varying Stress. (Bridges.)	For Shocks. (Machines.)
Timber	8	10	15
Brick and Stone	15	25	30
Cast Iron	6	10	15
Wrought Iron	4	6	10
Steel	5	7	10

1363. *Twice as much strain is caused by a suddenly applied stress as by one that is gradually applied.* For this reason a larger safety factor is used for shocks than for steady stresses. In general, the factor of safety for a given material must be chosen according to the nature of the stress.

The designer usually chooses his own factors of safety. If the material has been tested, or the specifications call for a certain strength, then the factor of safety can be chosen accordingly.

EXAMPLE.—Assuming the mortar and brick to be of the same strength, how many tons could be safely laid upon a brick column 2 feet square and 8 feet high?

SOLUTION.—$P = A\ S_3 = 2 \times 2 \times 144 \times 2{,}500 = 1{,}440{,}000$ lb. $= 720$ tons. The factor of safety for this case is 15 (see Art. **1362** and Table **28**); hence, $720 \div 15 = 48$ tons. Ans.

EXAMPLE.—What must be the diameter of the journals of a wrought iron locomotive axle to resist shearing safely, the weight on the axle being 40,000 pounds?

SOLUTION.—Let f be the factor of safety; then, $P = \frac{A\ S_s}{f}$, or $A = \frac{P f}{S_s}$. Since the axle has two journals, the stress on each journal is 20,000 lb. Owing to inequalities in the track, the load is not a steady one, but varies; for this reason, the factor of safety will be taken as 6. Then, $A = \frac{20{,}000 \times 6}{50{,}000} = 2.4$ sq. in. Therefore, $d = \sqrt{\frac{2.4}{.7854}} = 1\frac{3}{4}''$. Ans.

EXAMPLE.—Considering the piston rod of a steam engine as if its length were less than ten times its diameter, what must be the diameter of a steel rod, if the piston is 18 inches in diameter and the steam pressure is 110 pounds per square inch ?

SOLUTION.—Area of piston is $18^2 \times .7854 = 254.47$ sq. in. $254.47 \times 110 =$ 27,991.7 lb., or, say, 28,000 lb. = stress in the rod. $A = \frac{Pf}{S_s} = \frac{28,000 \times 10}{150,000}$ $= 1.87''$, nearly. Hence, diameter $= \sqrt{\frac{1.87}{.7854}} = 1.543''$, say $1\frac{9}{16}''$. Ans.

1364. When designing a machine, care should be taken (1) *to make every part strong enough to resist any stress likely to be applied to it; and* (2) *to make all parts of equal strength.* The reason for the first statement is obvious, and the second should be equally clear, since no machine can be stronger than its weakest part (proportioned, of course, for the stress it is to bear), and those parts of the machine which are stronger than others contain an excess of material which is wasted. In actual practice, however, this second rule is frequently modified. Some machines are intended to be massive and rigid, and need an excess of material to make them so; in others, there are difficulties in casting that modify the rule, etc., etc. In most cases, the designer must rely on his own judgment.

EXAMPLES FOR PRACTICE.

1. A cast iron bar is subjected to a steady tensile stress of 120,000 pounds. The cross-section is an ellipse whose axes are 6 and 4 inches. (*a*) What is the stress per square inch? (*b*) What load will the bar carry with safety?
Ans. { (*a*) 6,366.18 lb. per sq. in. (*b*) 62,832 lb.

2. How much will a piece of steel 2 inches square and 10 inches long shorten under a load of 300,000 pounds? Ans. .025".

3. A cylindrical wooden pin $1\frac{1}{2}$ inches in diameter is subjected to a double shearing stress. If the stress is suddenly applied, what total force is necessary to shear the pin? Ans. 7,216 lb.

4. A wrought iron tie rod is $\frac{3}{4}$ inch diameter; how long must it be to lengthen $\frac{3}{8}$ inch under a steady pull of 5,000 pounds? Ans. 69 ft.

5. A steel bar having a cross-section of $5'' \times 4''$ and 14 feet long is lengthened .036 inch by a steady pull of 120,000 pounds; what is its coefficient of elasticity? Ans. 28,000,000 lb. per sq. in.

6. Which is the stronger, weight for weight, a bar of chestnut wood whose tensile strength is 12,000 pounds per square inch and specific gravity .61, or a bar of steel whose tensile strength is 125,000 pounds per square inch?

7. What should be the diameter of a cast-iron pin subjected to a suddenly applied double shearing stress of 40,000 pounds to withstand the shocks with safety? Ans. 4⅜", nearly.

8. What safe steady load may be placed upon a brick column 2 feet square and 9 feet high? Ans. 96,000 lb.

PIPES AND CYLINDERS.

1365. A pipe or cylinder subjected to a pressure of steam or water is strained equally in all its parts, and, when rupture occurs, it is in the direction of its length.

Let $d =$ inside diameter of pipe in inches;
$l =$ length of pipe in inches;
$p =$ pressure in pounds per square inch;
$P =$ total pressure.

Then, $P = p\,l\,d$.

This formula is derived from a principle of hydrostatics that the pressure of water in any direction is equal to the pressure on a plane perpendicular to that direction. In Fig. 317, suppose the direction of pressure to be as shown by the arrows; $A\ B$ would then be the plane perpendicular to this direction, the width of the plane being equal to the diameter, and the length equal to the length of the pipe. The area of the plane would then be $l \times d$, and the total pressure $P = p \times l \times d$, as above.

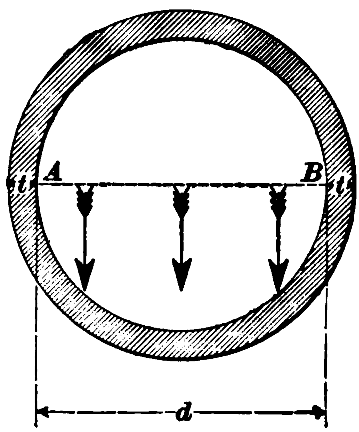


FIG. 317.

Suppose the pipe to have a thickness t, and let S be the working strength of the material; then, the resistance of the pipe on *each* side is $t\,l\,S$. Resistance must equal pressure; therefore, $p\,l\,d = 2\,t\,l\,S$, or

$$p\,d = 2\,t\,S. \qquad \textbf{(111.)}$$

Also, $p = \frac{2 t S}{d}$, which is the maximum pressure a pipe of a given material and of given dimensions can stand.

The pressure of water per square inch may be found by the formula, $p = .434\ h$, where h is the head in feet. In pipes where shocks are likely to occur, the factor of safety should be high. The thickness of a pipe to resist a given pressure varies directly as its diameter, the pressure remaining constant.

EXAMPLE.—Find the factor of safety for a cast-iron water pipe 12 inches in diameter and $\frac{3}{4}$ inch thick, under a head of 350 feet.

SOLUTION.—Here p, pressure per square inch, equals $.434\ h = .434 \times 350 = 151.9$ lb. Substituting, in formula **111**, the values given,

$$151.9 \times 12 = 2 \times \tfrac{3}{4} \times S, \text{ or } S = 1{,}215.2 \text{ lb. per sq. in.}$$

In Table 25, Art. **1356**, the ultimate tensile strength of cast iron is given as 20,000 pounds per square inch; then, the factor of safety is $f = \frac{20{,}000}{1{,}215.2} = 16 +$. Ans. The pipe would, therefore, be secure against shocks.

EXAMPLE.—Find the proper thickness for a wrought-iron steam pipe 18 inches in diameter to resist a pressure of 140 pounds per square inch.

SOLUTION.—Using a factor of safety of 10, the working strength $S = \frac{55{,}000}{10} = 5{,}500$ lb. per sq. in. From formula **111**, $t = \frac{p\,d}{2S} = \frac{140 \times 18}{2 \times 5{,}500} = 0.23$ in. In practice, however, the thickness is made somewhat greater than the formula requires.

CYLINDERS.

1366. The tendency of a cylinder subjected to internal pressure is to fail or rupture in the direction of its length, the same as a pipe.

Let π (pronounced *pi*) be the ratio of the circumference of a circle to its diameter $= 3.1416$; then, $\frac{1}{4}\pi = .7854 =$ ratio of area to the square of the diameter.

The total pressure on the cylinder head $= \frac{1}{4}\pi d^2 p$. Let $S =$ working unit stress $= \frac{S_1}{f}$, then $\pi\, d\, t\, S =$ the resistance to rupture caused by the pressure acting on the opposite

cylinder heads and tending to elongate the cylinder. Since the resistance must equal the force, or pressure, $\frac{1}{4}\pi d^2 p = \pi d t S$, or

$$p d = 4 t S. \qquad (112.)$$

$$\text{Also, } p = \frac{4 t S}{d}.$$

Since, for longitudinal rupture, $p = \frac{2 t S}{d}$, it is seen that a cylinder is twice as strong against transverse rupture as against longitudinal rupture. Hence, other things being equal, the cylinder will always fail by longitudinal rupture.

1367. The foregoing formulas are for comparatively thin pipes and cylinders, in which the thickness is less than about $\frac{1}{10}$ inside radius. For pipes and cylinders whose thickness is greater than $\frac{1}{10}$ radius, use the following formula, in which r = the inner radius, and the other letters have the same meaning as before.

$$p = \frac{S t}{r + t}. \qquad (113.)$$

Substituting the values given in the example in Art. **1365**, in formula **113**, instead of formula **111**,

$$p = \frac{S t}{r + t}, \text{ or } S = \frac{p(r + t)}{t} = \frac{151.9 \times (6 + \frac{3}{4})}{\frac{3}{4}} =$$
$$151.9 \times 6\tfrac{3}{4} \times \tfrac{4}{3} = 1{,}367.1 \text{ lb.}$$

When formula **111** was used, $S = 1{,}215.2$ lb.; hence, formula **113** gives, for this case, a value $12\frac{1}{2}\%$, or $\frac{1}{8}$ greater.

The formula for spheres is the same as that for transverse rupture of cylinders, or $p d = 4 t S$.

1368. A cylinder under external pressure is theoretically in a similar condition to one under internal pressure, so long as its cross-section remains a true circle. A uniform internal pressure tends to preserve the true circular form, but an external pressure tends to increase the slightest variation from the circle, and to render the cross-section elliptical. The distortion, when once begun, increases rapidly, and

failure occurs by the collapsing of the tube rather than by the crushing of the material. The flues of a steam boiler are the most common instances of cylinders subjected to external pressure.

The letters having the same meaning as before, the following formula gives the collapsing pressure in pounds per square inch for wrought-iron pipe:

$$p = 9,600,000 \frac{t^{2.18}}{l\,d}. \qquad \textbf{(114.)}$$

EXAMPLE.—What must be the thickness of a boiler tube 2 inches in diameter and 11 feet long, if the steam pressure is to be not over 160 pounds per square inch?

SOLUTION.—Using formula **114**, with a factor of safety of 10, and solving for t,

$$t = \sqrt[2.18]{\frac{10\,p\,l\,d}{9,600,000}} = \sqrt[2.18]{\frac{10 \times 160 \times 11 \times 12 \times 2}{9,600,000}} = \sqrt[2.18]{\frac{11}{250}}.$$

Hence, $\log t = \dfrac{\log 11 - \log 250}{2.18} = \bar{1}.37773$, or $t = .2386''$, say $\frac{1}{4}''$.

EXAMPLES FOR PRACTICE.

1. What must be the thickness of a 16-inch cast-iron stand pipe which is subjected to a head of water of 250 feet? Assume that the stress is steady. Ans. .26″.

2. What should be the thickness of a wrought-iron boiler flue 15 feet long, 4 inches in diameter, and subjected to an external pressure of 200 pounds per square inch? Ans. .42″.

3. What pressure per square inch can be safely sustained by a cast-iron cylinder 12 inches in diameter and 3 inches thick? Ans. $1,111\frac{1}{9}$ lb. per sq. in.

4. What external pressure per square inch can a wrought iron pipe 20 feet long, 3 inches in diameter, and $\frac{3}{8}$ inch thick, safely sustain and be secure against shocks? Ans. 157.2 lb. per sq. in.

5. A cast-iron cylinder 14 inches in diameter sustains a total pressure of 125 tons; what is the necessary thickness, assuming that the pressure is gradually applied, and that the cylinder is not subjected to shocks? Ans. 6.65″.

6. A cylindrical boiler shell 3 feet in diameter is subjected to a steady hydrostatic pressure of 180 pounds per square inch. What should its thickness be if made of steel having a tensile strength of 60,000 pounds per square inch? Ans. .27″.

ELEMENTARY GRAPHICAL STATICS.

Before taking up the subject of flexure, some fundamental principles of Graphical Statics not heretofore considered will be explained and applied to the case of beams.

FORCE DIAGRAM AND EQUILIBRIUM POLYGON.

1369. In Arts. **878** and **879,** the polygon of forces was used to find the resultant of several forces having a common point of application, or whose lines of action passed through a common point. A method of finding the resultant will now be given when the forces lie, or may be considered as lying, in the same plane, but their lines of action do not pass through a common point.

In Fig. 318, let F_1, F_2, and F_3 be three forces whose magnitudes are represented by the lengths of their respective lines, and their directions by the positions of the lines and by

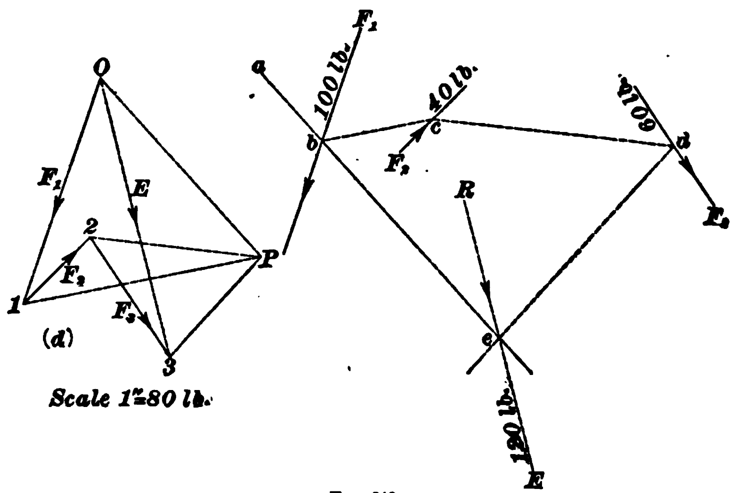


FIG. 318.

the arrow-heads. Construct the polygon of forces *O 1 2 3 O* as shown at (*d*), in the same manner as described in Art. **878,** *O 3* representing the direction and magnitude of the resultant. Everything is now known except the line on

which the point of application of the resultant must lie. To find this, proceed as follows:

Choose *any* point P, and draw $P\ O$, $P\ 1$, $P\ 2$, and $P\ 3$. Choose any point b, on the line of direction of one of the forces as F_1, and draw lines through b parallel to $P\ O$ and $P\ 1$, the latter intersecting F_2, or F_2 prolonged, in c. Draw $c\ d$ parallel to $P\ 2$, and intersecting F_3, or F_3 prolonged, in d. Draw $d\ e$ parallel to $P\ 3$, intersecting the line $a\ b\ e$, parallel to $P\ O$, in e. The point e is a point on the line of direction of the resultant of the three forces. Hence, through e, draw R parallel and equal to $O\ 3$ and acting in the same direction; it will be the resultant.

The method just described is applicable to any number of forces considered as acting in the same plane. The resultant can also be found when the forces act in different planes, but the method of finding it will not be described here.

The point P is called the **pole**; the lines $P\ O$, $P\ 1$, $P\ 2$, $P\ 3$ joining the pole with the vertexes of the force polygon are called the **strings** or **rays**; the **force diagram** is the figure composed of the force polygon, $O\ 1\ 2\ 3\ O$, the pole, and the strings. The polygon $b\ e\ d\ c\ b$ is called the **equilibrium polygon.**

Since the pole P may be taken anywhere, any number of force diagrams and equilibrium polygons may be drawn, all of which will give the same value for the resultant, and whose lines $d\ e$ and $a\ e$ will intersect on the resultant R. To test the accuracy of the work, take a new pole and proceed as before. If the work has been done correctly, $d\ e$ and $a\ e$ will intersect on R.

The equilibrium polygon gives an easy method of resolving a force into two components.

EXAMPLE.—In Fig. 319, let $F = 16$ pounds be the force, and let it be required to resolve it into two *parallel* components, A and B, at distances respectively of 5 feet and 15 feet from F. What will be the magnitudes of A and B?

SOLUTION.—Draw $O\ 1$ to represent $F = 16$ lb. Choose any convenient pole P, and draw the rays $P\ O$ and $P\ 1$. Take any point a on F

and draw $a b$ parallel to $P O$, intersecting A in b, and $a c$ parallel to $P 1$, intersecting B in c. Join b and c by the line $b c$. Through the pole P, draw $P d$ parallel to $b c$, intersecting $O 1$ in d. Then $O d$ is the magnitude of A, measure, to the scale to which $O 1$ was drawn, and $d 1$ is the magnitude of B to the same scale.

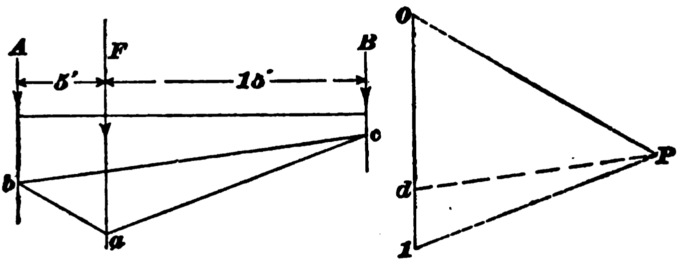


FIG. 319.

1370. If the components are not parallel to the given force, they must intersect its line of direction in a common point.

In Fig. 320, let $F = 16$ pounds be the force; it is required to resolve it into two components A and B, intersecting at a, as shown. Draw O 1 to some convenient scale equal to 16 pounds; then draw $O P$ and $1 P$ parallel to A and B, and $O P$ and $P 1$ are the values of the components A and B, respectively, both in magnitude and direction.

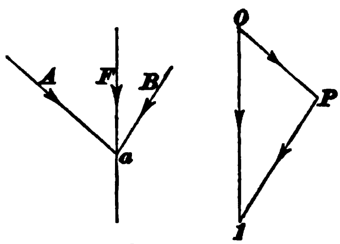


FIG. 320.

EXAMPLE.—Let F_1, F_2, F_3, F_4, and F_5, Fig. 321, be five forces whose magnitudes are 7, 10, 5, 12, and 15 pounds, respectively. It is required to find their resultant and to resolve this resultant into two components parallel to it and passing through the points a and b.

SOLUTION.—Choose any point O, Fig. 321, and draw $O 1$ parallel and equal to F_1; 1-2 parallel and equal to F_2, etc.; $O 5$ will be the value of the resultant, and its direction will be from O to 5, opposed to the other forces acting around the polygon. Choose a pole P, and complete the force diagram. Choose a point c on F_1, and draw the equilibrium polygon $c d e f g h c$; the intersection of $c h$, parallel to $P O$, and $g h$, parallel to $P 5$, gives a point h, on the resultant R. Through h, draw R parallel to $O 5$, and it will be the position of the line of action

of the resultant of the five forces. The components must pass through the points a and b, according to the conditions; hence, through a and b, draw V_1 and V_2 parallel to R. Since $O\,5$ represents the magnitude of R, draw $h\,k$ and $h\,l$ parallel to $P\,O$ and $P\,5$, respectively, as in Fig.

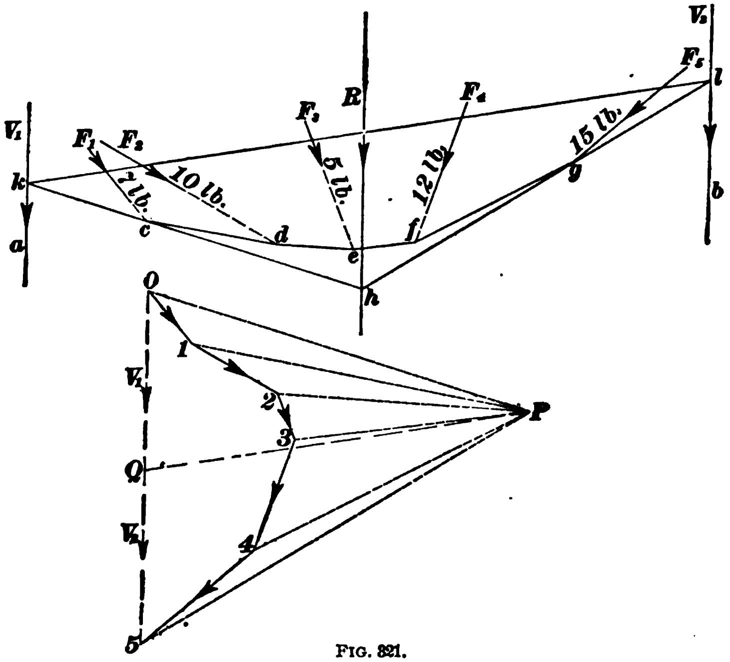


FIG. 821.

819 (they, of course, coincide with $c\,h$ and $g\,h$, since the same pole P is used), intersecting V_1 and V_2 in k and l. Join k and l, and draw $P\,Q$ parallel to $k\,l$. Then, $O\,Q = V_1$, and $Q\,5 = V_2$.

COMPOSITION OF MOMENTS.

1371. In Art. **906** it was stated that the moment of a force about a point is the product of the magnitude of the force by the perpendicular distance from the point to the line of action of the force. A force can act in two ways upon a body: it can either produce a motion of translation—that is, cause all the points of the body to move in straight parallel lines—or it can produce a motion of rotation—that is, make the body turn. A *moment* measures the capacity of a force to produce rotation about a given point. For example,

suppose, in Fig. 322, that $A\ C$ is a lever 30 inches long, having a fulcrum at B 10 inches from A. If a weight is suspended from C, it will cause the bar to rotate about B in the direction of the arrow. A weight suspended from A will cause it to revolve in the opposite direction, as indicated by the arrow. Suppose, for simplicity, that the bar itself weighs nothing. If two weights of 12 pounds each are hung at A and C, it is evident that the bar will revolve in the direction of the arrow at C, on account of the arm $B\ C$ being longer than the arm $A\ B$. Let the weight at A be increased until it equals 24 pounds. The bar will then balance exactly, and any additional weight at A will cause the bar to rotate in the opposite direction, as shown by the arrow at that point. When the lever is balanced, it will be found that $24 \times 10 = 12 \times 20$, or, considering B as the center of moments, $24 \times$ perpendicular distance $A\ B = 12 \times$ perpendicular distance $B\ C$. In other words, the moment of W about B must equal the moment of P about B—that is, the two moments must be equal. Further, P tends to cause rotation in the direction in which the hands of a watch move, and will, for convenience, be considered positive, or $+$; W tends to cause rotation in a direction opposite to the hands of a watch, and will be considered negative, or $-$. Adding the two algebraically, $P \times B\ C + (-W \times A\ B) = P \times B\ C - W \times A\ B = 0$, since the two moments are equal. Hence the following general

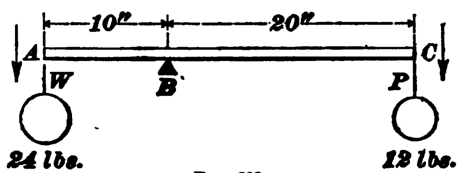


FIG. 322.

Rule.—*One of the necessary conditions of equilibrium is that the* **algebraic** *sum of the moments of all the forces about a given point should equal zero.*

Applications of this rule will occur farther on.

GRAPHICAL EXPRESSIONS FOR MOMENTS.

1372. The moment of a single force may be expressed graphically in the following manner: Let $F = 10$ pounds be the given force (see Fig. 323), and c, at a distance from

$F = f\,c = 7\frac{1}{2}$ feet, be the center of rotation (center of moments). Draw $O\,1$ parallel to F and equal to 10 pounds. Choose any point P as a pole, and draw the rays $P\,O$ and $P\,1$; also draw $P\,2$ perpendicular to $O\,1$. Through any point b on F, draw the sides $a\,b$ and $g\,b$ of the equilibrium polygon; they correspond to $b\,e$ and $d\,e$, Fig. 318, through the intersection of which the resultant must pass, the resultant F being given in the present case. Prolong $a\,b$, and draw

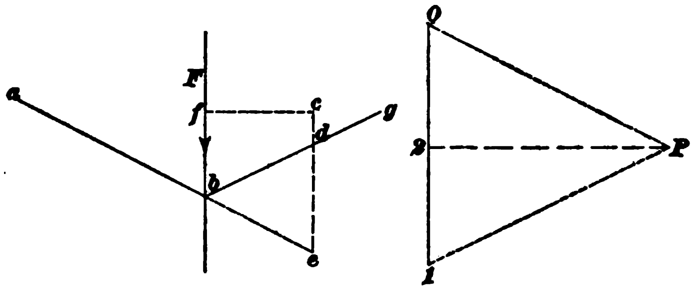


FIG. 323.

$e\,d$ through c parallel to F, intersecting $b\,g$ and $a\,b$ in d and e. It can now be shown that the moment of F about $c = P2 \times d\,e$, when $P\,2$ is measured to the same scale as $O\,1$, and $e\,d$ is measured to the same scale as $c\,f$.

The line $P\,2$ is called the **pole distance** and will hereafter be always denoted by the letter H. The line $d\,e$ is called the **intercept.** Hence, the pole distance multiplied by the intercept equals the moment, or, denoting the intercept by y, moment $= H\,y$.

The statement made in the last sentence is one of the most important facts in Graphical Statics, and should be thoroughly understood. In the triangle $P\,O\,1$, the lines $P\,O$ and $P1$ represent the components of the force F in the directions $a\,b$ and $g\,b$, respectively, while the lines of action of those components are $a\,b$ and $g\,b$, meeting at b. As $d\,e$ is limited by $g\,b$ and $a\,b$ (produced), we may give the following definition: The *intercept* of a force whose moment about a point is to be found is the segment (or portion) which the two components (produced, if necessary) cut off

from a line drawn through the center of moments parallel to the direction of the force.

1373. The pole distance and intercept for the moment of several forces about a given point may be determined in a similar manner, by first finding the magnitude and position of the resultant of all the forces; the moment of this resultant about the given point will equal the value of the resultant moment of all the forces which tend to produce rotation about that point.

EXAMPLE.—Let $F_1 = 20$ pounds, $F_2 = 25$ pounds, and $F_3 = 18$ pounds

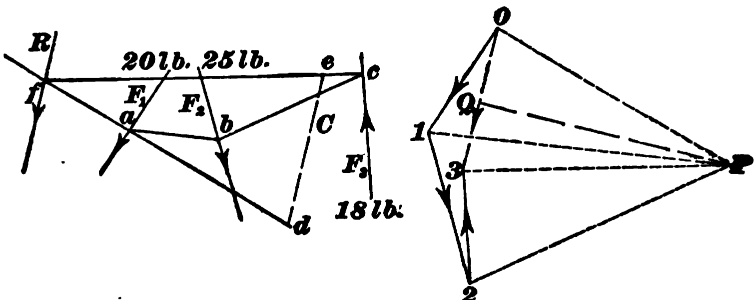


FIG. 824.

be three forces acting as shown in Fig. 824; find their resultant moment about the point C.

SOLUTION.—Draw the force diagram, equilibrium polygon, and resultant R as previously described. Draw $d\,C\,e$ parallel to R. The intercept $d\,e$, multiplied by the pole distance $P\,Q$ = the resultant moment.

1374. If all the forces are parallel, the force polygon is a straight line. This is evident, since if a line of the force polygon be drawn parallel to one of the forces, and from one end of this line a second line be drawn parallel to another force, the second line will coincide with the first.

EXAMPLE.—Let $F_1 = 30$, $F_2 = 20$, and $F_3 = 20$, all in pounds, be three parallel forces acting downwards, as shown in Fig. 825. It is required to find their resultant moment (algebraic sum of the moments) and the moment of their resultant, all moments to be taken about the point C.

SOLUTION.—Lay off $O\,1 = 30$ lb. $= F_1$, $1\text{-}2 = 20$ lb. $= F_2$ and $2\text{-}3 = 20$ lb. $= F_3$, and $O\,3$ is the value of the resultant. Choose some point P as a pole and draw the rays. Take any point, as b, on any force, as F_1, and complete the equilibrium polygon $b\,c\,d\,e\,b$; then, the line of action of the resultant must pass through e. Through C draw $C\,i$ parallel to R and prolong $d\,e$. The moment of R about C equals the pole distance H, multiplied by the intercept $h\,i$, since $h\,i$ is that part of the line drawn through C parallel to R, and included between the lines $b\,e$ and $d\,e$ of the equilibrium polygon which meet upon R. Measuring $h\,i$ to the scale of distances (1 in. $= 40$ ft.), it equals 23 ft. Measuring H to the scale of forces, it equals 40 lb. Consequently, the moment of R about

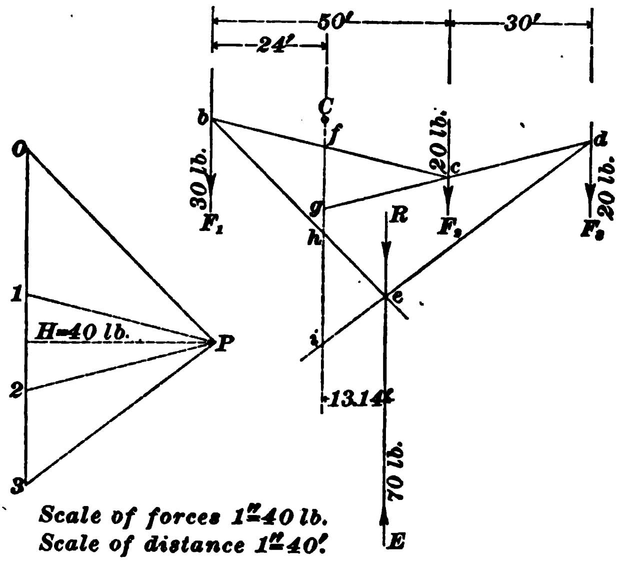


FIG. 325.

$C = 40 \times 23 = 920$ ft.-lb. Considering the force F_3, the intercept is $g\,i$, since F_3 is parallel to R, and, consequently, to $C\,i$; also, $g\,i$ is that part of the line $C\,i$ included between the sides $c\,d$ and $d\,e$, which meet on F_3. Measuring $g\,i$, it is found to equal 28 ft. Hence, the moment of F_3 about $C = 40 \times 28 = 1{,}120$ ft.-lb. The moment of F_2 about $C = H \times f\,g = 40 \times 13 = 520$ ft.-lb. The moment of $F_1 = H \times f\,h = 40 \times 18 = 720$ ft.-lb. Now F_3 and F_2 have positive moments, since they tend to cause rotation in the direction of the hands of a watch, while F_1 has a negative moment, since it tends to cause rotation in the opposite direction. Consequently, adding the moments algebraically, the resultant moment $= 1{,}120 + 520 - 720 = 920$ ft.-lb., the same as the moment of the resultant.

Having described the fundamental principles of Graphical Statics, the subject of Strength of Materials will now be continued, and the stresses due to flexure and torsion discussed.

BEAMS.

1375. Any bar resting upon supports in a horizontal position is called a **beam.**

1376. A **simple beam** is a beam resting upon two supports very near its ends.

1377. A **cantilever** is a beam resting upon one support in its middle, or which has one end fixed (as in a wall) and the other end free.

1378. A **restrained beam** is one which has both ends fixed (as a plate riveted to its supports at both ends).

1379. A **continuous beam** is one which rests upon more than two supports.

In this Course, the continuous beam will not be discussed, as the subject requires a knowledge of higher mathematics.

REACTIONS OF SUPPORTS.

1380. All forces which act upon beams will be considered as vertical, unless distinctly stated otherwise. According to the third law of motion, every action has an equal and opposite reaction. Hence, when a beam is acted upon by downward forces, the supports react upwards. It is required to find the value of the reaction at each support. If a simple beam is uniformly loaded or has a load in the middle, it is evident that the reaction of each support is one-half the load, plus one-half the weight of the beam. If the load is not uniformly distributed over the beam, or if the load or loads are not in the middle, the reactions of the two supports will be different. The upward reactions are considered positive, and the downward forces negative. In

order that a beam may be in equilibrium, three conditions must be fulfilled:

I. The algebraic sum of all vertical forces = 0.

II. The algebraic sum of all horizontal forces = 0.

III. The algebraic sum of the moments of all forces about any point = 0.

Since the loads act downwards and the reactions upwards, the first condition states that the sum of all the loads must equal the sum of the reactions of the supports.

EXAMPLE.—Let R_1 be the reaction of the left support, R_2 the reaction of the right support, and the distance between the two supports 14 feet. Suppose that loads of 50, 80, 100, 70, and 30 pounds are placed

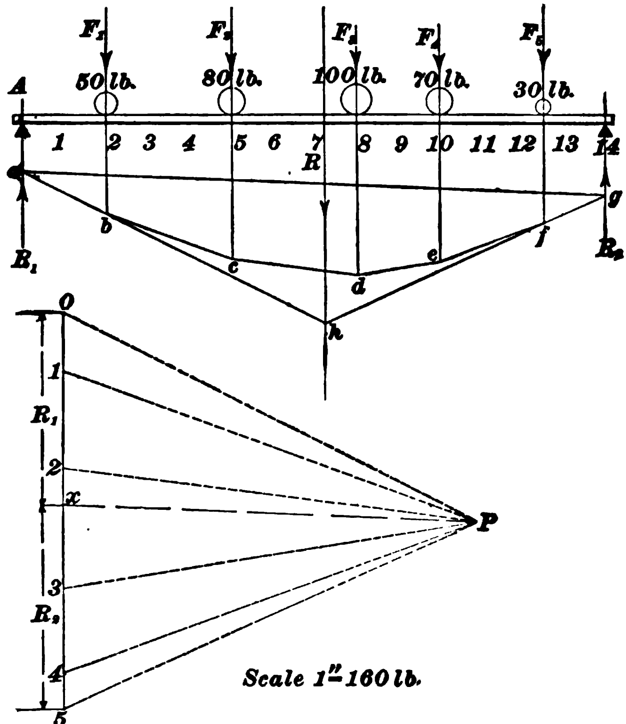


FIG. 326.

on the beam at distances from the left support equal to 2, 5, 8, 10, and 12½ feet, respectively. Required the reactions of the supports, neglecting the weight of the beam. See Fig. 326.

SOLUTION.—The reactions may be found graphically by resolving the resultant of the weights (which, in this case, acts vertically downwards) into two parallel components, passing through the points of support. The sum of the reactions is equal to the sum of the components, but the two sums have different signs. Draw the beam to some convenient scale, and locate the loads as shown in the figure. Draw the force diagram, making *O 1* = 50 lb., *1-2* = 80 lb., etc., *O 5* representing the force polygon.

Choose a point *b* on the line of action of the force F_1, and draw the equilibrium polygon *a b c d e f g a*; *a b* and *f g* intersect at *h*, the point through which the resultant *R* must pass. Draw *P x* parallel to *a g*, and *O x* will be the reaction (= component) R_1, and *x 5* the reaction R_2. Measuring *O x* and *x 5* to the same scale used to draw *O 5*, we find $R_1 = 161$ lb., and $R_2 = 169$ lb. By calculation, $R_1 = 160.4$ lb., and $R_2 = 169.6$ lb. This shows that the graphical method is accurate enough for all practical purposes. The larger the scale used, the more accurate will be the results.

The reactions and forces acting upwards will always be considered as positive, or +, and the downward weights as negative, or —. It is plain that the first condition of equilibrium is satisfied when the sum of the positive forces and reactions is equal to the sum of the negative forces.

THE VERTICAL SHEAR.

1381. In Fig. 326, imagine that part of the beam at a minute distance to the left of a vertical line passing through the point of support *A*, to be acted upon by the reaction R_1 = 160 pounds, and that part to the right of the line to be acted upon by the equal downward force due to the load. The two forces acting in opposite directions tend to shear the beam.

Suppose the line had been situated at the point marked *3* instead of at *A*; the reaction upwards would then be partly counterbalanced by the 50-pound weight, and the total reaction at this point would be 160 — 50 = 110 pounds. Since the upward reaction must equal the downward load at the same point, the downward force at *3* also equals 110 pounds, and the shear at this point is 110 pounds. At the point *6*, or any point between *5* and *8*, the downward force due to the weight at the left is 50 + 80 = 130 pounds, and the upward reaction is 160 pounds. The resultant shear is

therefore, $160 - 130 = 30$ pounds. At any point between *8* and *10*, the shear is $160 - (50 + 80 + 100) = -70$ pounds. The negative sign means nothing more than that the weights exceed the reaction of the left-hand support.

The vertical shear equals the reaction of the left-hand support, minus all the loads on the beam to the left of the point considered.

For a simple beam, the greatest positive shear is at the left-hand support, and the greatest negative shear is at the right-hand support, and both shears are equal to the reactions at those points.

1382. The vertical shear may be represented graphi-

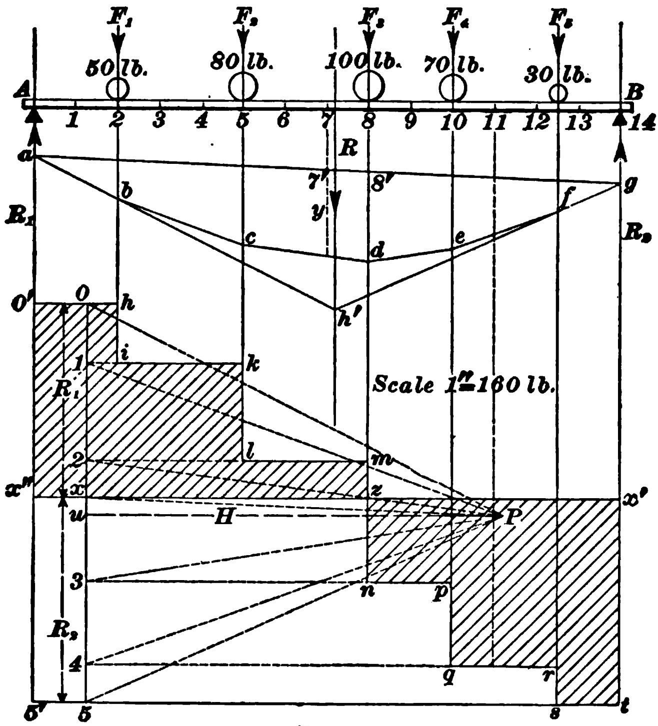


FIG. 327.

cally, as shown in Fig. 327, which is Fig. 326 repeated. Draw the force diagram, continue the lines of action of R_1 and

R_2 downwards and make $O'\,5' = O\,5$. Through x draw the horizontal line $x\,x'$, called the **shear axis.** The vertical shear is the same for any point between A and 2 and $= O\,x = O'\,x'' = 160$; hence, draw $O'\,h$ parallel to $x\,x'$, and any ordinate measured from $x\,x'$ to $O'\,h$, between O' and $h =$ 160 pounds = the vertical shear at any point between O' and h when $O'\,h = A\,2$. Through 1, draw $1\,k$ and project the points 2 and 5 upon it, in i and k. Then, the length of the ordinate between $x\,x'$ and $i\,k =$ the vertical shear between 2 and 5. In the same way, find the remaining points l, m, etc. The broken line $O'\,h\,i$........t is called the **shear line;** and the figure $O'\,h\,i$————$r\,s\,t\,x'\,x''$ is called the **shear diagram.** To find the shear at any point, as 11, project the point upon the shear axis and measure the ordinate to the shear line, drawn through the projected point. If the ordinate is measured from the shear axis upwards, the vertical shear is positive; if downwards, it is negative. For the point 11, the vertical shear $= -140$ lb. The maximum negative vertical shear is -170 lb. $= x'\,t = x\,5$. The greatest shear, whether positive or negative, is the one which the beam must be designed to withstand.

1383. A beam seldom fails through shearing, but generally breaks by reason of the load bending and breaking it; that is, through flexure. In order to design a beam to resist flexure, the greatest (maximum) bending moment must be known.

The ***bending moment*** *at any point of a beam is the algebraic sum of the moments of all of the forces (the reaction included) acting upon the beam, on either side of that point, the point being considered as the center of moments.*

The expression "algebraic sum" refers to the fact that, when considering the forces acting at the left of the point taken, the moments of all the forces acting upwards are considered positive, and the moments of all the forces acting downwards are considered negative. Hence, the algebraic sum is the moment of the left reaction about the given point, minus the sum of the moments about the same point of all

the downward forces between the reaction and the given point. Should there be any force or forces acting upwards, their moments must be *added*, since they are positive. If the forces on the right of the point are considered, all lever arms are negative, distances to the left of the point being +, to the right, − (see Art. **1371**). Hence, the downward forces give positive moments, and the right reaction gives a negative moment. This is as it should be, for the downward forces on the right and the upward forces on the left tend to rotate the beam in the direction of the hands of a watch, while the downward forces on the left and the upward forces on the right tend to rotate the beam in the opposite direction.

1384. To find the bending moment for any point of a beam, as *7* in Fig. 327, by the graphical method, draw the vertical line *7-7′* through the point. Let $y =$ that part of the line included between $a\ g$ and $a\ c\ f\ g$ of the equilibrium polygon (= vertical intercept). Then, $H \times y =$ the bending moment. H, of course, = the pole distance = $P\ u$. For any other point on the beam, the bending moment is found in the same manner—i. e., by drawing a vertical line through the point and measuring that part of it included between the upper and lower lines of the equilibrium polygon. The scale to which the intercept y is measured is the same as that used in drawing the length $A\ B$ of the beam. The pole distance H is measured to the same scale as $O\ 5$. In the present case, $y = 2.05$ feet, and $H = 349$ pounds; hence, the bending moment for the point *7* is $H\ y = 349 \times 2.05 = 715.45$ foot-pounds.

NOTE.—The expression "foot-pounds," used in stating the value of a moment, must not be confounded with foot-pounds of work. The former means simply that a force has been multiplied by a distance, while the latter means that a resistance has been overcome through a distance. In expressing the value of a moment, the force is usually measured in pounds or tons, and the distance in inches or feet; hence, the moment may be inch-pounds or inch-tons and foot-pounds or foot-tons. Unless otherwise stated, the bending moment will always be expressed in inch-pounds, the length of the beam being always measured in inches, and, consequently, also the length of the intercept y.

1385. If expressed in inch-pounds, the value of the moment just found is $715.45 \times 12 = 8,585.4$ inch-pounds.

It will be noticed that after the force diagram and equilibrium polygon have been drawn the value of the bending moment depends solely upon the value of y, since the length $P u = H$ is fixed. At the points a and g, directly under the points of support of the beam, $y = 0$; hence, for these two points, bending moment $= H y = H \times 0 = 0$; that is, for any simple beam, the bending moment at either support is zero. The greatest value for the bending moment will evidently be at the point *8*, since $d\,8'$ is the longest vertical line which can be included between $a\,g$ and $a\,c\,f\,g$.

The figure $a\,c\,e\,g\,a$ is called the **diagram of bending moments.**

1386. Consider now the case of a simple beam uniformly loaded. Let the distance between the supports in Fig. 328 be 12 feet, and let the total load uniformly distributed over the beam be 216 pounds. Divide the load into a convenient number of equal parts, the more the better, say 12, in this case. The load which each part represents is $216 \div 12 = 18$ pounds. For convenience, lay off $O\ C$ on the vertical through the left-hand support, equal to 216 pounds to the scale chosen, and divide it into 12 equal parts, $O\,a$, $a\,b$, etc.; each part will represent 18 pounds to the same scale. Choose a pole P, and draw the rays $P\,O$, $P\,a$, $P\,b$, etc. Through the points d, e, etc., the centers of gravity of the equal subdivisions of the load, draw the verticals d *1*, e *3*, f *5*, etc., intersecting the horizontals through O, a, b, etc., in *1*, *3*, *5*, etc. Draw O *1*, *1-2*, *2-3*, *3-4*, *4-5*, etc., and the broken line thus found will be the shear line. In drawing the shear line for a uniform load in this manner, it is assumed that each part of the total load is concentrated at its center of gravity, or, in other words, that a force equal to each small load (18 lb.) acts upon the beam at each of the points d, e, f, etc.

Construct the diagram of bending moments in the ordinary manner by drawing $g\,i$ parallel to $P\,O$, $i\,k$ parallel to $P\,a$, etc. Draw $P\,M$ parallel to $g\,h$, and $M\,q$ horizontal; $M\,q$ is the shear axis. When the load is uniform and the

work has been done correctly, *O M* should equal *M C*—that is, the reactions of the two supports are equal.

1387. The shear line is not a broken line in reality, as shown, since the load is distributed evenly over the entire beam, and not divided into small loads concentrated at *d*, *e*, *f*, etc., as was assumed. The points *1*, *3*, etc., are evidently

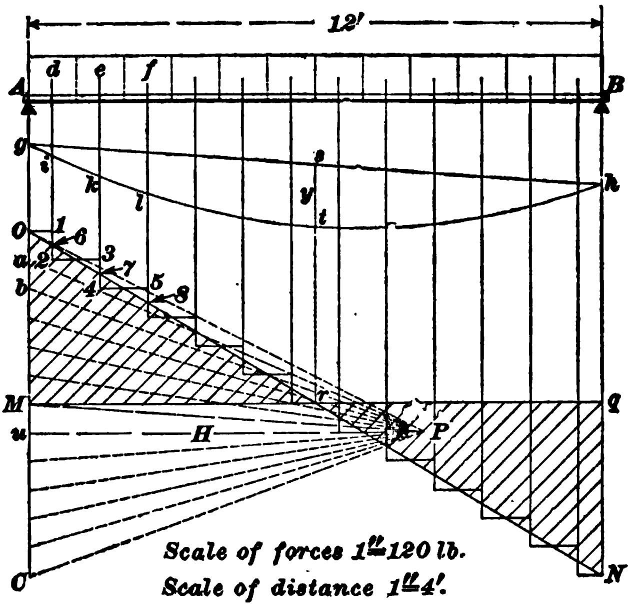


FIG. 828.

too high, and the points *2*, *4*, etc., too low. To find the real shear line, bisect the lines *1-2*, *3-4*, etc., locating the points *6*, *7*, *8*, etc. Trace a line through *O*, *6*, *7*, *8*_______*N*, and it will be the real shear line. For all cases of a uniform load, the shear line will be straight and may be drawn from *O* to *N* directly.

The diagram of bending moments is also not quite exact, but may be corrected by tracing a curve through the points *g* and *h*, which shall be tangent to *g i*, *i k*, *k l*, etc., at their middle points, as shown in the figure.

1388. To find the maximum bending moment for any beam having two supports, draw a vertical line through r, where the shear line cuts the shear axis, and the intercept, $s\,t = y$, on the diagram of bending moments, will be the greatest value of y, and, consequently, the greatest bending moment $= H \times y$. In the present example, $H = P\,u = 247.5$ pounds and maximum $y = s\,t = 15.72$ inches; hence, the maximum bending moment $= H\,y = 247.5 \times 15.72 = 3{,}890.7$ inch-pounds. The above is true, no matter how the beam may be loaded. In Fig. 327, the shear line cuts the shear axis at z, and $d\,8'$, on the vertical through z, was shown previously to be the maximum intercept.

1389. If there is a uniform load on the beam and one or more concentrated loads, as in Fig. 329, the method of finding the moment diagram and shear line is similar to that used in the last case. In Fig. 329, let the length $A\,B$ of the beam be 15 feet; the uniformly distributed load 180 pounds, with two concentrated loads, one of 24 pounds, 5 feet from A, and the other of 30 pounds, 11 feet from A. Draw the beam and loads as shown, the length of the beam and the distances of the weights from A being drawn to scale. Divide the uniform load into a convenient number of equal parts, say 10 in this case; each part will then represent $\frac{180}{10} = 18$ pounds. Draw $A\,O\,C$, as usual, and lay off 3 of the 18-pound subdivisions from O downwards; then lay off 24 pounds, to represent the first weight. Lay off four more of the equal subdivisions, and then the 30-pound weight. Finally, lay off the remaining three equal subdivisions, the point C being the end of the last 18-pound subdivision. $O\,C$ should then equal $180 + 24 + 30 = 234$ pounds to the scale to which the weights were laid off. It will be noticed in the above that the equal subdivisions of the load were laid off on $O\,C$ until that one was reached on which the concentrated loads rested, and that the concentrated loads were laid off before the equal subdivision on which the concentrated load rested was laid off. Had one of the concentrated loads been to the right of the center of gravity of the subdivision on which it

rests, the weight of the subdivision would have been laid off first. Locate the centers of gravity of the equal subdivisions and draw the verticals through them as in the previous case. Choose a pole P, draw the rays, the diagram of bending moments, and the shear axis $M\,q$, as previously described.

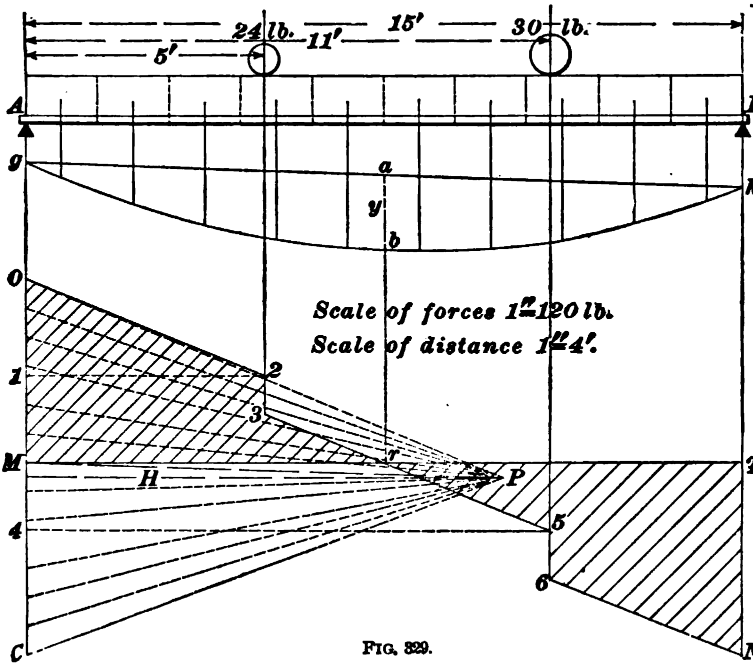


FIG. 329.

To find the maximum bending moment, the shear line must be drawn and its intersection with the shear axis determined.

The weight of the uniform load per foot of length is $\frac{180}{15} = 12$ lb. The weight of that part between A and the center of the 24-lb. weight is $12 \times 5 = 60$ lb. Lay off $O\,1 = 60$ lb. and draw *1-2* horizontal, cutting the vertical through the center of the 24-lb. weight in *2*. Draw $O\,2$ and it will be a part of the shear line. Lay off *2-3* vertically downwards, equal to 24 lb., locating the point *3*. Lay off $O\,4$ equal to $12 \times 11 + 24 = 156$ lb., and draw the horizontal *4-5*, cutting the vertical through the center of the 30-lb.

weight in *5*. Join *3* and *5* by the straight line *3-5*. Lay off *5-6* vertically downwards equal to 30 lb. Draw the horizontal $C N$, intersecting the vertical $B q N$ in N and join *6* and N by the straight line *6* N. The broken line O *2-3-5-6* N is the shear line and cuts the shear axis in the point r. Drawing the vertical $r b a$, through r, it intersects the moment diagram in a and b; hence, $a b$ is the maximum y. For this case, the maximum bending moment $= H \times y = 300 \times 18.36 = 5{,}508$ in.-lb.

It is better, in ordinary practice, to choose the pole P on a line perpendicular to $O C$ and at some distance from $O C$ easily measured with the scale used to lay off $O C$. Thus, suppose $O C$ to be laid off to a scale of $1'' = 60$ lb. At some convenient point, as M, on $O C$, draw a perpendicular line and choose a point on this line whose distance from $O C$ shall be easily measurable, say $3\frac{1}{2}''$. Then, H is known to be exactly $60 \times 3\frac{1}{2} = 210$ lb. and will not have to be measured when finding the bending moment $H y$.

1390. If the student has familiarized himself with the method of constructing the shear and moment diagrams for concentrated loads, he will find no difficulty in understanding the preceding operations, which may be condensed into the following

Rule.—*Divide the beam into an even number of parts* (the greater the better), *and the uniform load into half as many. Consider the divisions of the load as concentrated loads applied, alternately, at the various points of division of the beam* (the ends included); *that is, the first point of division (the support) carries no load, the next one does, the following one does not*, etc. *Then proceed as in the ordinary case of concentrated loads.*

EXAMPLE.—Find the reactions of the supports, the maximum bending moments, and the maximum vertical shear of the beam shown in Fig. 830, which has one overhanging end.

SOLUTION.—Draw $O C$ and the force diagram in the usual manner. Construct the bottom curve of the moment diagram in the same manner as in the preceding cases. The side $d e$ is parallel to $P 4$; $e h$ is parallel to $P C$, and cuts the vertical through the right reaction in h.

Join h and g by the straight line $g\,h$, and draw $P\,M$ parallel to $g\,h$. Then, $O\,M = 77$ lb. = left reaction and $M\,C = 173$ lb. = right reaction. The shear line is drawn as in the previous cases until the point n, on the vertical $h\,n$, is reached; $k\,n$ here denotes the vertical shear for any point between the 50-lb. weight and the right support, and this shear is negative. The point k denotes the intersection of the shear axis and the vertical through the right support. For any point to the

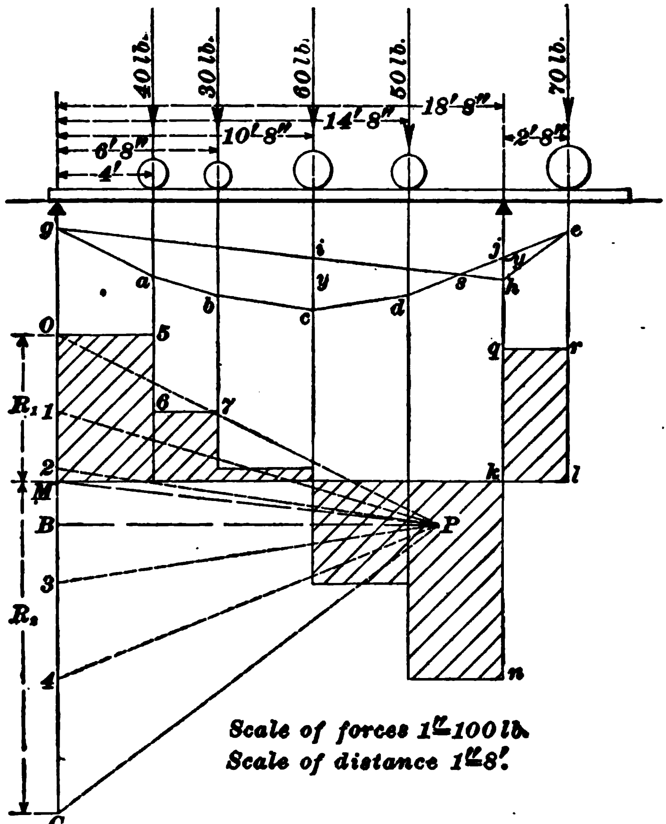


FIG. 330.

right of k, between k and l, the vertical shear is positive, and is equal to 70 lb.; hence, lay off $k\,q$ upwards equal to 70 lb., and draw $q\,r$ horizontal. The line, O 5-6-7 .... $n\,q\,r$ is the shear line. Measuring $O\,M$, $k\,n$, and $k\,q$, it is found that $O\,M = 77$ lb., $k\,n = -103$ lb., and $k\,q = 70$ lb.; therefore, $k\,n = -103$ lb. = maximum vertical shear; $c\,i$ is evidently the maximum y; hence, the maximum bending moment =

$Hy = PB \times ci = 200 \times 26'' = 5{,}200$ in.-lb. Any value of y measured in the polygon $g\,a\,b\,c\,d\,s\,g$ is positive, and any value measured in the triangle $e\,h\,s$ is negative. Consequently, the bending moment for any point between s and the vertical, through the center of the 70-lb. weight, is negative, since $H \times (-y) = -Hy$. In all cases, when design-

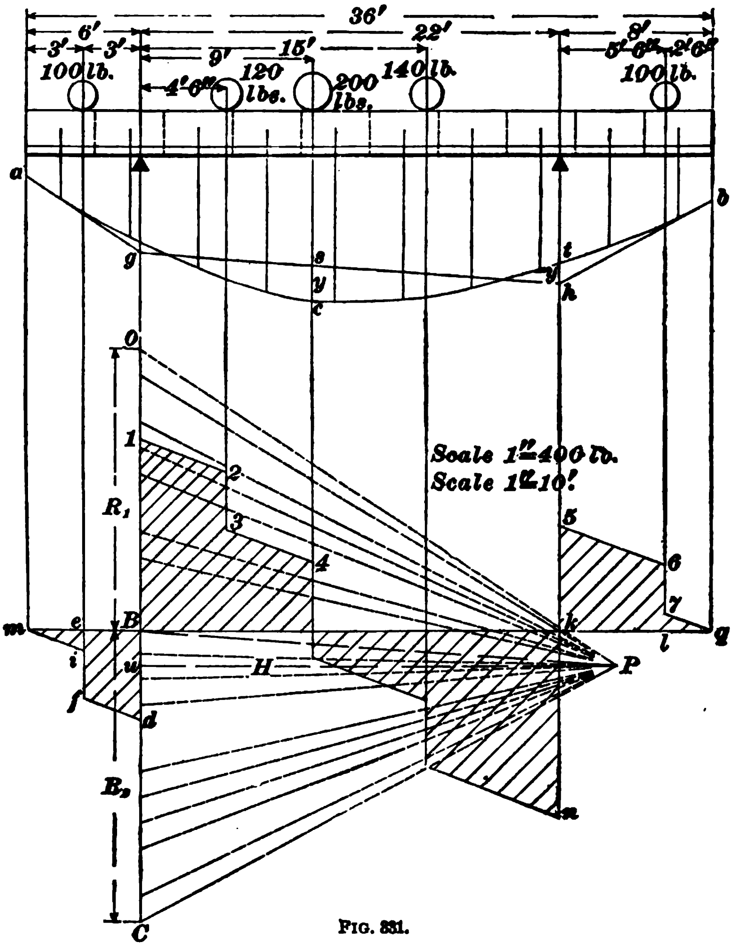


FIG. 331.

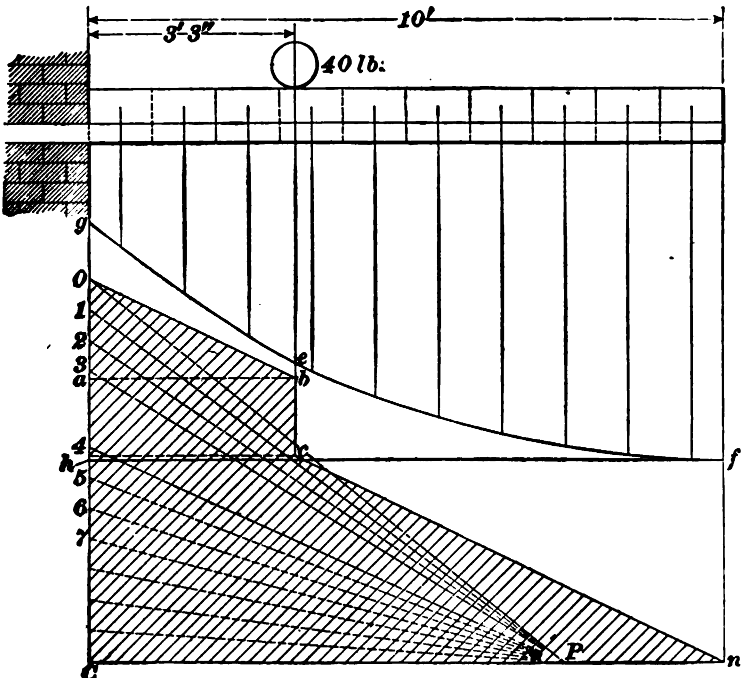
ing beams with overhanging studs, the maximum bending moment, whether positive or negative, should be used. In the present case, the maximum negative y, or $h\,j$, is less than the maximum positive y, or $c\,i$; therefore, the maximum negative bending moment is also less than the maximum positive moment.

EXAMPLE.—Fig. 331 shows a beam overhanging both supports, which carries a uniform load of 15 pounds per foot of length, and has five concentrated loads at distances from the supports as marked in the figure. Required the reactions of the supports, the maximum positive and negative bending moments, and the maximum vertical shear.

SOLUTION.—Construct the force diagram and equilibrium polygon in the ordinary manner, continuing the latter to b and a, points on the verticals passing through the ends of the beam. Draw $b\,h$ and $a\,g$ parallel to $P\,C$ and $P\,O$ respectively, intersecting the verticals through the points of support in h and g. Join g and h, and draw $P\,B$ parallel to $g\,h$. Then, $O\,B = 588$ lb. $= R_1$, and $B\,C = 612$ lb. $= R_2$. Through B, draw the shear axis $m\,q$. To draw the shear line, proceed as follows: The shear for any point to the left of the left support is negative, and for any point to the right of the right support is positive; between the two supports it is positive or negative, according to the manner of loading, and the point considered. The negative shear at the left support $= 15 \times 6 + 100 = 190$ lb.; hence, lay off $B\,d$ downwards equal to 190 lb. For a point to a minute distance to the right of e, the shear is $15 \times 3 + 100 = 145$ lb. $= e\,f$, and for a minute distance to the left, it is $15 \times 3 = 45$ lb. $= e\,i$; at m, it is 0. Consequently, $m\,i\,f\,d$ is the shear line between the end of the beam and the left support. Lay off $O\,1 = B\,d = 190$ lb. and draw the shear line *1-2-3-4*———*n* in the usual manner. Draw the shear line *5-6-7* q, laying off $k\,5 = 15 \times 8 + 100 = 220$ lb.; $l\,6 = 100 + 15 \times 2\frac{1}{2} = 137\frac{1}{2}$ lb., and $6\text{-}7 = 100$ lb. At q, the vertical shear is again 0. The broken line $m\,i\,f\,d$ *1-2-3*———*n 5-6-7* q is the shear line. The maximum positive bending moment is $H \times y = P\,u \times s\,c = 1{,}000 \times 22.5 = 22{,}500$ in.-lb. The greatest maximum negative moment is $H \times (-y) = P\,u \times -t\,h = 1{,}000 \times -11.6 = -11{,}600$ in.-lb. It will be noticed that there are two negative and one positive maximum bending moments.

1391. The student should now be able to find the bending moment for any beam having but two supports, whatever the character of the loading. The bending moment plays a very important part in the flexure of beams, which is the next subject to be considered. In all cases of loading heretofore considered, no other forces than the loads themselves have been considered. Should forces act upon the beam which are not vertical, the force polygon will be no longer a straight line, but a broken line somewhat similar in character to *O 1-2-3-4-5* in Fig. 321.

In Fig. 332 is shown a cantilever beam projecting 10 ft. from the wall. It carries a uniform load of 16 pounds per foot of length, and a concentrated load of 40 pounds at a distance of $3\frac{1}{4}$ feet from the wall. The maximum bending moment is required. The method is similar to the last, except that, as there is but one support, there can be but one reaction. Since the beam is 10 feet long, the total



Scale of forces 1" = 100 lb.
Scale of distance 1" = 3'.
FIG. 332.

weight of the uniform load is $16 \times 10 = 160$ pounds. Hence, the reaction $= 160 + 40 = 200$ pounds.

Draw $O\ C$ equal to 200 lb., to some convenient scale. Draw $P\ C$ perpendicular to $O\ C$ at C and choose the pole P at a convenient distance from $O\ C$. For convenience, divide the uniform load into 10 equal parts, as shown; then,

each part will represent 16 lb. Lay off *O 1*, *1-2*, *2-3*, each equal to 16 lb., and *3-4* equal to 40 lb. Also, *4-5*, *5-6*, *6-7*, etc., equal to 16 lb. each. 3 ft. 3 in. = $3\frac{1}{4}$ ft., and $16 \times 3\frac{1}{4} = 52$ lb. Lay off $O\,a = 52$ lb., and draw *a b*, meeting the vertical through the center of the weight in *b*. Draw *O b*; lay off *b c* equal to 40 lb. and draw *C n*. *O b c n* is the shear line. The perpendicular through the point *C* coincides with *P C*; hence, *n P C* is the shear axis. Draw the line *g c f* of the moment diagram as in the previous cases. In Fig. 329, and in the preceding figures, the line *g h* was drawn connecting the extreme ends of the bottom line; in other words, it joined the points where the equilibrium polygon cut the lines of direction of the reactions of the supports. This cannot be done in this case, because there is no right reaction; therefore, *g h* must be drawn by means of some other property of the polygon. In the previous cases, the shear axis was drawn perpendicular to *O C* at the point where a line through the pole *P* parallel to *g h* cut *O C*, or, in other words, the shear axis was drawn through the point which marked the end *M* of the left reaction *O M*. In the present case, Fig. 332, the point *C* is the end of the left reaction; hence, *f h* is parallel to *P C*, *h g* is the maximum *y*, and the bending moment $= H\,y = P\,C \times h\,g = 250 \times 45 =$ 11,250 inch-pounds.

It will also be noticed that the distance $y = h\,g$ is measured from the line *h f* upwards, while, for all points between the supports in the previous examples, this distance was measured downwards. The same observation is true for any point between *h* and *f*; hence, for a cantilever, all bending moments are negative.

1392. All the cases of beams heretofore given might have been solved by analytical methods—that is, by algebraic processes and formulas; but the graphical method is to be preferred, as it is usually shorter, very nearly as accurate, and less liable to error. Moreover, when the diagram has once been drawn, both the bending moment and the shear for any point can be instantly determined. Ex-

cept in special cases, the graphical method will be employed to determine the maximum bending moment, but in some particular cases formulas will be used, by which the result can be obtained more quickly than by the graphical method.

EXAMPLES FOR PRACTICE.

1. A simple beam 24 feet long carries 4 concentrated loads of 160, 180, 240, and 120 pounds at distances from the left support of 4, 10, 16, and 21 feet, respectively. (*a*) What are the values of the reactions? (*b*) What is the maximum bending moment in inch-pounds?

Ans. $\begin{cases} (a)\ R_1 = 333\frac{1}{3} \text{ lb.};\ R_2 = 366\frac{2}{3} \text{ lb.} \\ (b)\ 28{,}480 \text{ in.-lb.} \end{cases}$

2. A simple beam carries a uniform load of 40 pounds per foot, and supports two concentrated loads of 500 and 400 pounds at distances from the left support of 5 and 12 feet, respectively. The length of the beam is 18 feet. What are (*a*) the reactions? (*b*) The maximum bending moment in inch-pounds?

Ans. $\begin{cases} (a)\ R_1 = 854\frac{4}{9} \text{ lb.};\ R_2 = 765\frac{5}{9} \text{ lb.} \\ (b)\ 48{,}846 \text{ in.-lb.} \end{cases}$

3. A cantilever projects 10 feet from a wall and carries a uniform load of 60 pounds per foot; it also supports three concentrated loads of 100, 300, and 500 pounds at distances from the wall of 2, 5, and 9 feet, respectively. Required, (*a*) the maximum vertical shear, and (*b*) the maximum bending moment in inch-pounds.

Ans. $\begin{cases} (a)\ -1{,}500 \text{ lb.} \\ (b)\ -110{,}400 \text{ in.-lb.} \end{cases}$

4. A beam which overhangs one support sustains six concentrated loads of 160 lb. each at distances from the left support of 4 ft. 9 in., 7 ft., 9 ft. 6 in., 12 ft., 15 ft., and 18 ft. 3 in., respectively, the distance between the supports being 16 ft. What are (*a*) the reactions? (*b*) The maximum bending moment?

Ans. $\begin{cases} (a)\ R_1 = 295 \text{ lb.};\ R_2 = 665 \text{ lb.} \\ (b)\ 20{,}460 \text{ in.-lb.} \end{cases}$

5. A beam which overhangs both supports equally carries a uniform load of 80 pounds per foot, and has a load of 1,000 pounds in the middle, the length of the beam being 15 feet, and the distance between the supports 8 feet. What is (*a*) the vertical shear? (*b*) The maximum bending moment?

Ans. $\begin{cases} (a)\ 820 \text{ lb.} \\ (b)\ 25{,}800 \text{ in.-lb.} \end{cases}$

NOTE.—The student may not obtain the exact answers given above, but if his results do not differ by more than 1%, he will know that his method is right.

NEUTRAL AXIS.

1393. In Fig. 333, let $A\ B\ C\ D$ represent a cantilever. Suppose that a force F acts upon it at its extremity A. The beam will then be bent into the shape shown by $A'\ B\ C\ D'$.

It is evident from the cut that the upper part $A' B$ is now longer than it was before the force was applied; i. e., $A' B$ is longer than $A B$. It is also evident that $D' C$ is shorter than $D C$. Hence, the effect of the force F in bending the beam is to lengthen the upper fibers and to shorten

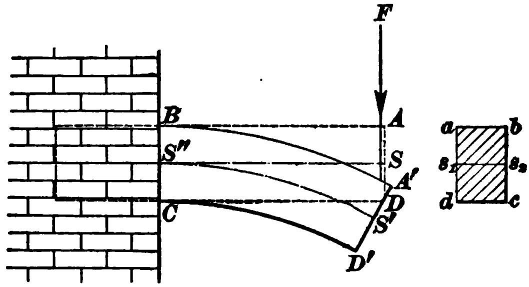


FIG. 333.

the lower ones. In other words, when a cantilever is bent through the action of a load, the upper fibers are in tension and the lower fibers in compression. The reverse is the case in a simple beam in which the upper fibers are in compression and the lower fibers in tension. Further consider-

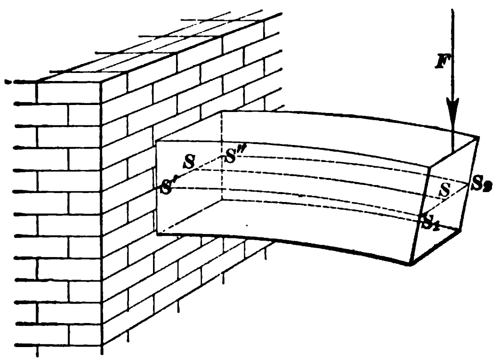


FIG. 334.

ation will show that there must be a fiber, $S S''$, which is neither lengthened nor shortened when the beam is bent, i. e., $S' S'' = S S''$. When the beam is straight the fiber $S S''$, which is neither lengthened nor shortened when the beam is bent, is called the **neutral line.** There may be

any number of neutral lines dependent only on the width of the beam. For, let $b\,a\,d\,c$, Fig. 333, be a cross-section of the beam. Project s upon it in s_1. Make $b\,s_2 = a\,s_1$ and draw $s_1\,s_2$; then, any line in the beam which touches $s_1\,s_2$ and is parallel to $S\,S'$ is a neutral line. Thus, in Fig. 334, $S_1\,S'$, $S\,S$, $S_2\,S''$, etc., are all neutral lines. The line $S_1\,S_2$ is called the **neutral axis**, and the surface $S_1\,S'\,S''\,S_2$ is called the **neutral surface.** The neutral axis, then, is the line of intersection of a cross-section with the neutral surface. It is shown in works on mechanics that *the neutral axis always passes through the center of gravity of the cross-section of the beam.*

1394. Experimental Law.—*When a beam is bent, the horizontal elongation (or compression) of any fiber is directly proportional to its distance from the neutral surface, and, since the strains are directly proportional to the horizontal stresses in each fiber, they are also directly proportional to their distances from the neutral surface, provided the elastic limit is not exceeded.*

1395. Suppose the beam to be a rectangular prism, then every cross-section will be a rectangle, and the neutral axis will pass through the center o. See Fig. 335.

Let the perpendicular distance from the neutral axis $M\,N$ to the *outermost fiber* be denoted by c, and the horizontal unit stress (stress per square inch) at the distance c from the axis by S. If a is the area of a fiber, the stress on the outermost fibers will be $a\,S$. The stress on a fiber at the distance unity (1 inch) from $M\,N$ is $\frac{a\,S}{c}$; and the stress on a fiber at the distance r_1 is $\frac{a\,S}{c} \times r_1 = \frac{a\,S\,r_1}{c}$. The moment of this stress about the axis $M\,N$ is $\frac{a\,S\,r_1}{c} \times r_1 = \frac{a\,S\,r_1^2}{c} = \frac{S}{c}\,a\,r_1^2$. The moment of the

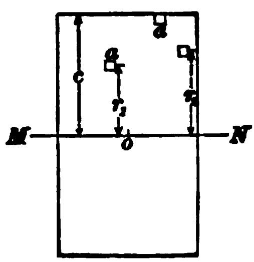


FIG. 335.

stress on any other fiber at a distance r_2 from MN is evidently $\frac{S}{c} a r_2^2$, and for a distance r_3, $\frac{S}{c} a r_3^2$, etc. If n is the number of fibers, the sum M of the moments of the horizontal stresses on all the fibers is

$$M = \frac{S}{c} a r_1^2 + \frac{S}{c} a r_2^2 + \frac{S}{c} a r_3^2 +, \text{ etc.}, =$$

$$\frac{S}{c}(a r_1^2 + a r_2^2 + a r_3^2 + \text{——} a r_n^2) = \frac{S}{c} a (r_1^2 + r_2^2 + r_3^2 + \text{——} r_n^2).$$

Now, let r be a quantity whose square equals the mean of the squares of $r_1, r_2, r_3, \text{——} r_n$. Then, $r^2 = \frac{r_1^2 + r_2^2 + \text{——} + r_n^2}{n}$; and, therefore, $r_1^2 + r_2^2 + r_3^2 + \text{——} + r_n^2 = n r^2$. Substituting above, we get $M = \frac{S}{c} n a r^2$. But, since a is the area of one fiber, $n a$ is the area of all the fibers—that is, the area A of the cross-section; hence, the sum of the moments of all the horizontal stresses $= \frac{S}{c} A r^2$.

1396. The expression $A r^2$, which is found by dividing a section into a large number of minute areas (a, a, etc.), multiplying each area by the square of its distance from an axis (r_1^2, r_2^2, r_3^2, etc.), and then adding the products thus obtained, is called the **moment of inertia** of the section with respect to that axis, and is usually denoted by the letter I. Hence,

$$I = A r^2. \qquad \textbf{(115.)}$$

1397. The quantity r, whose square is the mean of the squares of all the distances of the minute areas from the axis, is called the **radius of gyration.**

1398. The sum of the moments of all the horizontal stresses may then be written as $\frac{S}{c} A r^2 = \frac{S}{c} I$, or $S \frac{I}{c}$. This expression is called the **resisting moment,** since it is the measure of the resistance of the beam to bending (and, consequently, to breaking) when loaded. The resisting moment must equal the bending moment when the beam

is in equilibrium; hence, denoting the bending moment by M,

$$M = S_4 \frac{I}{c}. \qquad (116.)$$

1399. The values of I and c depend wholly upon the size and form of the cross-section of the beam, and S_4 is the ultimate strength of flexure of the material.

In Table 29, the average ultimate strength of flexure S_4 is given for a number of different materials.

TABLE 29.

Material.	Ultimate Strength of Flexure in Lb. per Sq. In. S_4.
Cast Iron	38,000
Wrought Iron	45,000
Steel	120,000
Brass	17,000
Ash	14,000
Brick	1,000
Stone	2,000
Hemlock	7,200
Oak, white	12,500
Pine, white	9,000
Pine, yellow	11,000
Hickory	16,000

1400. Exact values of I for most cross-sections can only be determined by the aid of the calculus. The least value of I occurs when the axis passes through the center of gravity of the cross-section—that is, when I is found with reference to the neutral axis.

The least moments of inertia for a number of different sections are given in the Table of Moments of Inertia; also, the area of the sections and the values of c. The dotted line indicates the position of the neutral axis, about which the moment of inertia is taken.

In the Table of Moments of Inertia, A is the area of the section, and $\pi =$ ratio of the circumference of a circle to its diameter $= 3.1416$. It will be noticed that d is always taken vertically.

1401. To use formula **116**, find the bending moment in inch-pounds by the graphical method previously described, or calculate it by means of the Table of Bending Moments. If it is desired to find the size of a beam that will safely resist a given bending moment, take S_4 from Table 29, Art. **1399**, and divide it by the proper factor of safety taken from Table 28. Then, formula **116** becomes

$$M = \frac{S_4 I}{f c}. \qquad \textbf{(117.)}$$

From this $\frac{I}{c} = \frac{M f}{S_4}$. Substituting the values of M, f, and S_4, the value of $\frac{I}{c}$ is found. The kind and shape of beam having been decided upon, the size can be so proportioned that $\frac{I}{c}$ for the section shall not be less than the value calculated above. An example will make this clear.

EXAMPLE.—What should be the size of an ash girder to resist safely a bending moment of 28,000 inch-pounds, the cross-section to be rectangular and the load steady?

SOLUTION.—$M = \frac{S_4 I}{f c}$, or $\frac{I}{c} = \frac{M f}{S_4}$.

$M = 28{,}000$; from Table 29, Art. **1399**, $S_4 = 14{,}000$; from Table 28, Art. **1362**, $f = 8$.

Then, $\frac{I}{c} = \frac{28{,}000 \times 8}{14{,}000} = 16.$

From the table of Moments of Inertia, $I = \frac{b d^3}{12}$ and $c = \frac{d}{2}$ for a rectangle; hence, $\frac{I}{c} = \frac{b d^3}{12} \times \frac{2}{d} = \frac{b d^2}{6} = 16$, or $b d^2 = 96$. Any number of values of b and d can be found that will satisfy this equation. If b is taken as 6 inches, $d^2 = \frac{96}{6} = 16$ and $d = \sqrt{16} = 4$. Hence, the beam may be a 6×4, with the short side vertical. When possible, it is always better to have the longer side vertical. If b is taken as 2 inches, $d^2 = 48$ and $d = \sqrt{48} = 7$ inches, nearly; hence, a 2×7 will also

answer the purpose. The advantage of using a 2×7 instead of a 6×4 is evident, since the 6×4 contains nearly twice as much material as the 2×7. Thus, the area of the cross-section of a 6×4 is 24 square inches, and of a 2×7, 14 square inches. Moreover, the 2×7, with its long side vertical, is slightly stronger than the 6×4, with its short side vertical, since $\frac{I}{c} = \frac{2 \times 7^2}{6} = 16\frac{1}{3}$ for the former, and $\frac{6 \times 4^2}{6} = 16$ for the latter. If the 6×4 had its longer side vertical, thus making it a 4×6, $\frac{I}{c}$ would then equal $\frac{4 \times 6^2}{6} = 24$, and the safe bending moment could be increased to $M = \frac{S_4 I}{f c} = \frac{14{,}000 \times 24}{8} = 42{,}000$ in.-lb.

1402. If the breaking bending moment, form and size of the cross-section of the beam are known, the ultimate strength of flexure S_4 can be readily found from formula **116**, by substituting the values of M, I, and c, and solving for S_4.

EXAMPLE.—A cast iron bar, 2 inches square, breaks when the maximum bending moment $= 63{,}360$ inch-pounds; what is its ultimate strength of flexure?

SOLUTION.—$M = S_4 \frac{I}{c}$, or $S_4 = \frac{M c}{I}$. $c = \frac{d}{2} = 1''$.

$I = \frac{d^4}{12} = \frac{2^4}{12}$; therefore, $S_4 = \frac{63{,}360 \times 1}{\frac{1}{12} \times 2^4} = 47{,}520$ lb. per sq. in.

1403. In order to save time in calculating, the bending moments for cases of simple loading are given in the Table of Bending Moments. W denotes a concentrated load, and w the uniform load per inch of length. All dimensions are to be taken in inches when using the formulas.

For any other manner of loading than is described in the Table of Bending Moments, the maximum bending moment must be found by the graphical method.

EXAMPLE.—A wrought iron cantilever, 6 feet long, carries a uniform load of 50 pounds per inch. The cross-section of the beam is an equilateral triangle, with the vertex downwards; what should be the length of a side?

SOLUTION.—$M = \frac{w l^2}{2}$, from the Table of Bending Moments, $=$ $\frac{50 \times (6 \times 12)^2}{2} = 129{,}600$ in.-lb. $I = \frac{b d^3}{36}$ and $c = \frac{2}{3} d$, from the Table

of Moments of Inertia; hence, $\frac{I}{c}=\frac{b\,d^2}{24}$. $S_4=45{,}000$, from Table 29, Art. **1399**, and $f=4$, from Table 28, Art. **1362.** Therefore, $129{,}600 = \frac{45{,}000}{4}\times\frac{b\,d^2}{24}$, or $b\,d^2=\frac{129{,}600\times4\times24}{45{,}000}=276.48$. Since an equilateral triangle has been specified, b can not be given *any* convenient value in order to find d. For an equilateral triangle, $d=b\sin 60°=.866\ b$. Hence, $b\,d^2=b\ (.866\ b)^2=.75\ b^3$. Therefore, $b\,d^2=.75\ b^3=276.48$ or $b=\sqrt[3]{\frac{276.48}{.75}}$ $\log b=\frac{\log 276.48-\log .75}{3}=.85553$, or $b=7.17''$, nearly.

EXAMPLE.—What weight would be required to break a round steel bar 4 inches in diameter, 16 feet long, fixed at both ends and loaded in the middle?

SOLUTION.—Use formula **116**; $M=\frac{S_4 I}{c}$. Here $M=\frac{Wl}{8}$, from the Table of Bending Moments, Vol. V; $S_4=120{,}000$; $\frac{I}{c}=\frac{\frac{1}{64}\pi d^4}{\frac{1}{2}d}=\frac{\pi d^3}{32}$. Hence, $\frac{Wl}{8}=\frac{W\times(16\times12)}{8}=\frac{120{,}000\times3.1416\times4^3}{32}$, or $W=\frac{120{,}000\times3.1416\times64\times8}{16\times12\times32}=31{,}416$ lb.

DEFLECTION OF BEAMS.

1404. The deflection, or amount of bending, produced in a beam by one or more loads is given by certain general formulas, whose derivation is too complicated to be given here. We shall give the formulas only, illustrating their application by examples.

In the third column of the Table of Bending Moments are given expressions for the value of the greatest deflection of a beam when loaded as shown in the first column. From this it is seen that the deflection s equals a constant (depending upon the manner of loading the beam and upon the condition of the ends—whether fixed or free), multiplied by $\frac{Wl^3}{EI}$. Let a represent the constant and s the deflection; then,

$$s=a\frac{Wl^3}{EI}. \qquad \textbf{(118.)}$$

In the above formula, E = coefficient of elasticity, and is to be taken from Table 25, Art. **1356,** l = length in inches, W = concentrated load in pounds, W' = total uniform load in pounds, and I is the moment of inertia about the neutral axis; a has values varying from $\frac{1}{384}$ to $\frac{1}{3}$.

It will be noticed that the deflection is given for only nine cases; for any other manner of loading a beam than those here given, it is necessary to use the calculus to obtain the deflection.

EXAMPLE.—What will be the maximum deflection of a simple wooden beam 9 feet long, whose cross-section is an ellipse, having axes of 6 inches and 4 inches (short axis vertical), under a concentrated load of 1,000 pounds?

SOLUTION.—Use formula **118,** $s = a\frac{Wl^3}{EI}$. From the Table of Bending Moments,

$$a = \frac{1}{48},\ W = 1{,}000 \text{ lb.},\ l = 9 \times 12 = 108 \text{ in.},$$

$$E = 1{,}500{,}000 \text{ and } I = \frac{\pi b d^3}{64} = \frac{\pi \times 6 \times 4^3}{64}.$$

$$\text{Hence, } s = \frac{1{,}000 \times 108^3 \times 64}{48 \times 1{,}500{,}000 \times \pi \times 6 \times 64} = .9282''.$$

1405. The principal use of the formula for deflection is to determine by its means the stiffness of a beam or shaft. In designing machinery, it frequently occurs that a piece may be strong enough to sustain the load with perfect safety, but the deflection may be more than circumstances will permit; in this case, the piece must be made larger than is really necessary for mere strength. An example of this occurs in the case of locomotive guides, and the upper guides of a steam engine when the engine runs under. It is obvious that they must be very stiff. In such cases it is usual to allow a certain deflection, and then proportion the piece so that the deflection shall not exceed the amount decided upon.

EXAMPLE.—The breadth of the guides of a certain locomotive is not to exceed $2\frac{1}{4}$ inches. Regarding the guides as fixed at both ends, (a) what must be their depth to resist a load of 10,000 pounds at the middle? The guides are made of cast iron and are 38 inches long between the points of support. (b) What weight would these guides be able to support with safety? The deflection must not exceed $\frac{1}{100}$ of an inch. The cross-section is, of course, rectangular.

SOLUTION.—Since the load comes on two guides, each piece must support $10{,}000 \div 2 = 5{,}000$ lb. In the formula,

$$s = \frac{1}{200} = a\frac{W l^3}{E I},\ a = \frac{1}{192} \text{ for this case, } W = 5{,}000,\ l = 38,$$

$$E = 15{,}000{,}000, \text{ and } I = \frac{b d^3}{12} = \frac{2\frac{1}{4} d^3}{12} = \frac{3}{16} d^3. \text{ Substituting,}$$

$$(a)\ s = \frac{5{,}000 \times 38^3 \times 16}{192 \times 15{,}000{,}000 \times 3d^3} = \frac{1}{200},$$

$$\text{or } d = \sqrt[3]{\frac{5{,}000 \times 38^3 \times 16 \times 200}{192 \times 15{,}000{,}000 \times 3}} = 4.67'', \text{ nearly, say } 4\tfrac{11}{16}''. \quad \text{Ans.}$$

(*b*) To find the weight which these guides could support with safety, use formula **117**, $M = \frac{S_4 I}{f c}$, in which $M = \frac{W l}{8}$, $S_4 = 38{,}000$, $f = 10$, $\frac{I}{c} = \frac{b d^2}{6} = \frac{2\frac{1}{4} \times (4\frac{11}{16})^2}{6} = \frac{50{,}625}{6{,}144}$. Substituting,

$$W = \frac{38{,}000 \times 50{,}625 \times 8}{10 \times 6{,}144 \times 38} = 6{,}592 \text{ lb.} \quad \text{Ans.}$$

Hence, the beam is over 30% stronger than necessary, the extra depth being required for stiffness.

EXAMPLES FOR PRACTICE.

1. How much will a simple wooden beam 16 ft. long, 2 in. wide and 4 in. deep deflect under a load in the middle of 120 lb.? Ans. 1.106″.

2. What should be the size of a rectangular yellow pine girder 20 ft. long, to sustain a uniformly distributed load of 1,800 lb.? Assume a factor of safety for a varying stress. Ans. 5″ × 8″.

3. A hollow cylindrical beam, fixed at both ends, has diameters of 8 in. and 10 in. If the beam is 30 ft. long and is made of cast iron, (*a*) what steady load will it safely support at 15 ft. from one of the supports? (*b*) What force will be required to rupture the beam if applied at this point?

Ans. $\begin{cases} (a)\ 8{,}158 \text{ lb.} \\ (b)\ 48{,}946 \text{ lb.} \end{cases}$

4. A simple cylindrical wrought-iron beam, resting upon supports 24 ft. apart, sustains three concentrated loads of 350 lb. each, at distances from one of the supports of 5, 12, and 19 ft.; what should be the diameter of the beam to withstand shocks safely? Ans. 4.71″, say 4¾″.

5. Find the value of $\frac{I}{c}$ for a hollow rectangle whose outside dimensions are 10 in. and 13 in., and inside dimensions are 8 in. and 10 in.; (*a*) when the long side is vertical; (*b*) when the short side is vertical.

Ans. $\begin{cases} (a)\ 179.103. \\ (b)\ 131\frac{1}{3}. \end{cases}$

6. What is the deflection of a steel bar 1 in. square and 6 ft. long, which supports a load of 100 lb. at the center? Ans. .31104".

7. Which will be the stronger, a beam whose cross-section is an equilateral triangle, one side measuring 15 in., or one whose cross-section is a square, one side measuring 9 in.? Both beams are of the same length. Ans. The one having the square cross-section.

8. A wooden beam of rectangular cross-section sustains a uniform load of 50 lb. per foot. If the beam is 8" × 14" and 16 ft. long, how much more will it deflect when the short side is vertical than when the long side is vertical? Ans. .055417".

COMPARISON OF STRENGTH AND STIFFNESS OF BEAMS.

1406. Consider two rectangular beams, loaded in the same manner, having the same lengths and bending moments, but different breadths and depths. Then,

$M = S\frac{I}{c} = S\frac{b\,d^2}{6}$ (1) and $M = S\frac{b_1 d_1^2}{6}$ (2). Dividing (1) by (2), $\frac{M}{M} = \frac{6\,S\,b\,d^2}{6\,S\,b_1\,d_1^2} = \frac{b\,d^2}{b_1\,d_1^2} = 1$, or $b\,d^2 = b_1\,d_1^2$ (3).

Equation 3 shows that, if both beams have the same depth, their strengths will vary directly as their breadths, i. e., if the breadths are increased 2, 3, 4, etc., times, their strengths will also be increased 2, 3, 4, etc., times. It also shows that, if the breadths are the same and the depths are increased, the strengths will vary as the square of the depth, i. e., if the depths are increased 2, 3, 4, etc., times, the strengths will be increased 4, 9, 16, etc., times. Hence, it is always best, when possible, to have the long side of a beam vertical. If the bending moments are the same, but the weights and lengths are different, $M = g\,W\,l$ (1) and $M = g\,W_1\,l_1$ (2), when g denotes the fraction $\frac{1}{2}$, $\frac{1}{4}$, etc., according to the manner in which the ends are secured, and the manner of loading. Dividing (1) by (2) $\frac{M}{M} = \frac{g\,W\,l}{g\,W_1\,l_1}$, or $W\,l = W_1\,l_1$ (3).

Equation 3 shows that if the load W or W_1 be increased, the length l or l_1 must be decreased; consequently, the strength of a beam loaded with a given weight varies inversely as its length, i. e., if the load be increased 2, 3, 4.

etc., times, the length must be shortened 2, 3, 4, etc., times, the breadth and depth remaining the same.

EXAMPLE.—If a simple beam, loaded in the middle, has its breadth and depth reduced one-half, what proportion of the original load could it carry?

SOLUTION.—In the preceding paragraphs, it was shown that the strength varied as the product of the breadth and the square of the depth, or $b_1 d_1^2 = \frac{1}{2} \times (\frac{1}{2})^2 = \frac{1}{8}$. Consequently, the beam can support only $\frac{1}{8}$ of the original load. Had the breadth remained the same, $(\frac{1}{2})^2 = \frac{1}{4}$ of the original load could have been supported. Had the depth remained the same, $\frac{1}{2}$ of the original load could have been supported.

EXAMPLE.—A beam 10 ft. long, loaded in the middle, has a breadth of 4 in. and a depth of 6 in. The length is increased to 12 ft., the breadth to 6 in., and the depth to 8 in.; how many times the original load can it now support?

SOLUTION.—The strength varies directly as the product of the breadth and square of the depth and inversely as the length, or as $\frac{b d^2}{l}$. If b, d, and l denote the original sizes, the strength of the two beams will be to each other as $\frac{b_1 d_1^2}{l} : \frac{b d^2}{l}$, or as $\frac{6 \times 8^2}{12} : \frac{4 \times 6^2}{10}$; $\frac{6 \times 8^2}{12} = 32$ and $\frac{4 \times 6^2}{10} = 14.4$. $\frac{32}{14.4} = 2\frac{2}{9}$. Consequently, the beam will support a load $2\frac{2}{9}$ times as great as the original beam.

1407. By a process of reasoning similar to that employed above, it can be shown that the maximum deflection of a beam varies inversely as the cube of the depth and directly as the cube of the length. In other words, if the depth be increased 2, 3, 4, etc., times, the deflection will be decreased 8, 27, 64, etc., times; and, if the length be increased 2, 3, 4, etc., times, the deflection will be increased 8, 27, 64, etc., times. Hence, if a beam is required to be very stiff, the length should be made as short and the depth as great as circumstances will permit.

COLUMNS.

1408. When a piece ten or more times as long as its least diameter or side (in general, its least transverse dimension) is subjected to compression, it is called a **column** or **pillar.**

The ordinary rules for compression do not apply to columns, for the reason that when a long piece is loaded beyond a certain amount, it buckles and tends to fail by flexure. This combination of flexure and compression causes the column to break under a load considerably less than that required to merely crush the material. It is likewise evident that the strength of a column is principally dependent upon its diameter, since that part having the least thickness is the part that buckles, or bends. A column free to turn in any direction, having a cross-section of $3'' \times 8''$, is not nearly so strong as one whose cross-section is $4'' \times 6''$. The strength of a very long column varies, practically, inversely as the square of the length; i. e., if a column *b* is twice as long as a column *a*, the strength of $b = (\frac{1}{2})^2 = \frac{1}{4}$ the strength of *a*, the cross-sections being equal.

1409. The conditions of the ends of a column play a very important part in determining their strength, and must always be taken into consideration. In Fig. 336, are shown three classes of columns. The column marked *a* is used in architecture, while the columns similar to *b* and *c* are used in bridge and machine construction.

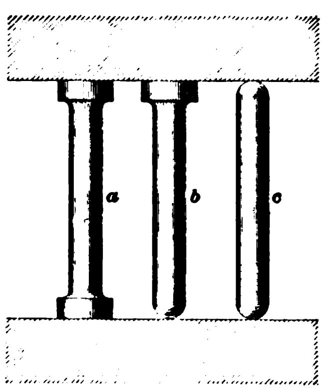


FIG. 336.

According to theory, which is confirmed by experiment, a column having one end flat and the other rounded, like *b*, is $2\frac{1}{4}$ times as strong as a column having both ends rounded, like *c*.

One having both ends flat, like *a*, is 4 times as strong as *c*, which has both ends rounded, the three columns being of the same length. If the length of *c* be taken as 1, the length of *b* may be $1\frac{1}{2}$, and that of *a* may be 2 for equal strength, the cross-sections all being the same; for, since the strengths vary inversely as the squares of the lengths, the strength of *c* is to that of *b* as $1 : \frac{1}{(1\frac{1}{2})^2}$ or as $1 : \frac{4}{9}$. But,

since b is $2\frac{1}{4} = \frac{9}{4}$ times as strong as c, $\frac{4}{9} \times \frac{9}{4} = 1$, or, the length of b being $1\frac{1}{2}$ times that of c, its strength is the same. Similarly, when a is twice as long as c its strength is the same.

1410. Columns like b and c do not actually occur in practice, an eye being formed at the end of the column and a pin inserted, forming what may be termed a *hinged end.* A steam engine connecting-rod is a good example of a column having two hinged ends, and a piston rod of a column having one end hinged and one end flat.

1411. There are numerous formulas for calculating the strength of columns, but the one that gives the most satisfactory results for columns of all lengths is the following:

$$W = \frac{S_c A}{f\left(1 + \frac{A l^2}{g I}\right)}. \qquad \textbf{(119.)}$$

In this formula, W = load, S_c = ultimate strength for compression, taken from Table 26, Art. **1357**, A = area of section of a column in square inches, f = factor of safety, l = length in inches, g = constant, to be taken from Table 30, and I = least moment of inertia of the cross-section—that is, the moment of inertia about an axis passing through the center of gravity of the cross-section and parallel to the longest side. In other words, if the column has a rectangular cross-section, whose longer side is b and shorter side d, the least moment of inertia is $\frac{b d^3}{12}$, the axis in this case being parallel to the long sides b. The values of g are given in the following table:

TABLE 30.

Material.	Both Ends Fixed.	One End Hinged.	Both Ends Hinged.
Timber	3,000	1,690	750
Cast Iron	5,000	2,810	1,250
Wrought Iron............	36,000	20,250	9,000
Steel	25,000	14,060	6,250

EXAMPLE.—The section of a hollow rectangular cast-iron column has the following dimensions (see Table of Moments of Inertia): $d = 8''$, $d_1 = 6''$, $b = 6''$, and $b_1 = 3\frac{1}{2}''$. If the length is 10 feet and the ends are fixed, what steady load will the column sustain with safety?

SOLUTION.—From the Table of Moments of Inertia, least $I = \frac{1}{12}(d b^3 - d_1 b_1^3) = \frac{8 \times 6^3 - 6 \times 3.5^3}{12} = 122.5625$. $A = b\,d - b_1 d_1 = 6 \times 8 - 3.5 \times 6 = 27$ sq. in. $S_2 = 90{,}000$, $f = 6$, $l = 10 \times 12 = 120''$, and $g = 5{,}000$. Therefore, $W = \dfrac{90{,}000 \times 27}{6\left(1 + \dfrac{27 \times 120^2}{5{,}000 \times 122.5625}\right)} = \dfrac{2{,}430{,}000}{9.806} = 247{,}800$ lb., nearly. Ans.

Had the column been less than $10 \times 6 = 60$ in. $= 5$ ft. long, the safe load would have been $\dfrac{90{,}000 \times 27}{6} = 405{,}000$ lb. Had it been twice as long, it would have supported a safe load of only $\dfrac{90{,}000 \times 27}{6\left(1 + \dfrac{27 \times 240^2}{5{,}000 \times 122.5625}\right)} = 114{,}500$ lb., nearly.

1412. In the actual designing of a column, the size of the cross-section is not known, but the form (square, round, etc.) is known, also the length, material, condition of ends and load it is to carry. To find the size of the cross-section, substitute $\frac{S_2}{f}$ in formula **108,** for S, and solve for A, obtaining $A = \frac{Pf}{S_2}$. Substituting in this equation the values of $P (= W)$, f, and S_2, this gives the value of A for a short piece less than 10 times the length of the shortest side, or diameter. Assume a value of A somewhat larger than that just found, and dimension a cross-section of the form chosen so that its area shall equal that assumed. Calculate the moment of inertia and substitute the values of W, A, I, l, and g in formula **119,** and solve for $\frac{S_2}{f}$. If the result last found equals the value of $\frac{S_2}{f}$ taken from Tables 26 and 28, the assumed dimensions are correct; if larger, the assumed dimensions must be increased; if smaller, they should be

diminished, and in both cases the value of $\frac{S_2}{f}$ should be recalculated. An example will serve to illustrate the process.

EXAMPLE.—What should be the diameter of a steel piston rod 5 feet long, the diameter of the piston being 18 inches and the greatest pressure 130 pounds per square inch?

SOLUTION.—S_2 for this case = 150,000 lb. Since the piston rod is liable to shocks, a factor of safety of 10 should be used; hence, $\frac{S_2}{f} = \frac{150,000}{10} = 15,000$ lb. The load $W = 18^2 \times .7854 \times 130 = 33,081$ lb. $A = \frac{Pf}{S_2} = \frac{33,081}{15,000} = 2.2$ sq. in., nearly.

Assume that 3 sq. in. are needed. The diameter of a circle corresponding to an area of 3 sq. in. is $\sqrt{\frac{3}{.7854}} = 1.9544''$. Assume the diameter to be $1\frac{15}{16}'' = 1.9375$; the area will be $1.9375^2 \times .7854 = 2.9483$ sq. in. The value of $I = \frac{\pi d^4}{64} = \frac{3.1416 \times (1\frac{15}{16})^4}{64} = .69173$. $W = \frac{S_2 A}{f\left(1 + \frac{A l^2}{g I}\right)}$.

Consequently, $\frac{S_2}{f} = \frac{W}{A}\left(1 + \frac{A l^2}{g I}\right) = \frac{33,081}{2.9483}\left(1 + \frac{2.9483 \times (5 \times 12)^2}{14,060 \times .69173}\right) = 23,465$ lb.

As this value exceeds 15,000 lb., the diameter of the rod must be increased. Trying $2\frac{1}{8}''$ as the diameter, the area is 3.5466 sq. in., and $I = 1.00093$. Substituting these values as before, $\frac{S^2}{f} = \frac{33,081}{3.5466}\left(1 + \frac{3.5466 \times 60^2}{14,060 \times 1.00093}\right) = 17,790$ lb. This is still too large; hence, trying $2\frac{1}{4}''$, the area = 3.976 sq. in. $I = 1.258$ and

$$\frac{S_2}{f} = \frac{33,081}{3.976}\left(1 + \frac{3.976 \times 60^2}{14,060 \times 1.258}\right) = 15,052 \text{ lb.}$$

Consequently, the diameter should be $2\frac{1}{4}''$.

EXAMPLES FOR PRACTICE.

1. What safe steady load will a hollow cylindrical cast-iron column support, which is 14 feet long, outside diameter 10 inches, inside diameter 8 inches, and which has flat ends? Ans. 273,500 lb.

2. A hollow wooden column having a square cross-section is to support a steady load of 15,155 pounds. If the thickness of the side is $1\frac{1}{2}$ inches, length of column 20 feet, and the ends flat, what should be the length of the sides of the cross-section, outside and inside?

Ans. Outside, 9"; inside, 6".

3. Suppose a wrought-iron connecting-rod to have a rectangular cross-section of uniform size throughout its length. If the diameter of the steam cylinder is 40 inches, steam pressure 110 pounds per square inch, and the length of the rod is $12\frac{1}{2}$ feet, what should be the dimensions of the cross-section of the rod?

Ans. $5\frac{1}{4}'' \times 9''$.

1413. The preceding method for determining the dimensions of the cross-section, when the load and length are given, is perfectly general, and can, therefore, be used in every case. It is, however, somewhat long and cumbersome. For the special cases of square, circular, and rectangular columns, the following formulas may be applied, if preferred. They seem complicated, but, when substitutions are made for the quantities given, the formulas will be found of relatively easy application.

For square columns, the side c of the square is given by the formula

$$c = \sqrt{\frac{Wf}{2S_s} + \sqrt{\frac{Wf}{S_s}\left(\frac{Wf}{4S_s} + \frac{12l^2}{g}\right)}}. \qquad \textbf{(120.)}$$

For circular columns, the diameter d of the circle is given by the formula

$$d = 1.4142\sqrt{\frac{.3183\,Wf}{S_s} + \sqrt{\frac{.3183\,Wf}{S_s}\left(\frac{.3183\,Wf}{S_s} + \frac{16\,l^2}{g}\right)}}. \qquad \textbf{(121.)}$$

For rectangular columns, assume the shorter dimension (depth $= d$). Then the longer dimension (breadth $= b$) is given by the formula

$$b = \frac{Wf\left(1 + \frac{12\,l^2}{d^2 g}\right)}{d\,S_s}. \qquad \textbf{(122.)}$$

Should the dimensions given by the last formula be too much out of proportion, a new value may be assumed for d, and a new value found for b.

EXAMPLE.—Required the section of a square timber pillar to stand a steady load of 20 tons, the length of the column being 30 feet, and its ends both flat.

SOLUTION.—Here $S_2 = 8{,}000$ lb., $f = 8$, $g = 3{,}000$, $W = 40{,}000$ lb., $l = 30 \times 12 = 360$ in. These values, substituted in formula **120**, give

$$c = \sqrt{\frac{40{,}000 \times 8}{2 \times 8{,}000} + \sqrt{\frac{40{,}000 \times 8}{8{,}000}\left(\frac{40{,}000 \times 8}{4 \times 8{,}000} + \frac{12 \times 360^2}{3{,}000}\right)}}$$

$$= \sqrt{20 + \sqrt{40\left(10 + \frac{4 \times 36^2}{10}\right)}}$$

$$= \sqrt{20 + \sqrt{21{,}136}} = \sqrt{20 + 145.35}$$

$$= \sqrt{165.35} = 12.90 = 12\tfrac{7}{8}'', \text{ nearly, or say } 13''.$$

EXAMPLE.—Let it be required to solve the problem worked out by the general method in Art. **1412.**

SOLUTION.—Here $S_2 = 150{,}000$ lb., $f = 10$, $g = 14{,}060$, $W = 33{,}000$ lb., nearly, and $l = 5 \times 12 = 60$ in. From these data we have

$$\frac{.3183\,Wf}{S_2} = \frac{.3183 \times 33{,}000 \times 10}{150{,}000} = \frac{.3183 \times 11}{5} = .7003,$$

$$\frac{16\,l^2}{g} = \frac{16 \times 60^2}{14{,}060} = \frac{8 \times 360}{703} = 4.0967.$$

Then, formula **121**,

$$d = 1.4142\sqrt{.7003 + \sqrt{.7003 \times 4.7970}}$$

$$= 1.4142\sqrt{.7003 + \sqrt{3.3593}} = 1.4142\sqrt{.7003 + 1.8328}$$

$$= 1.4142 \times 1.5916 = 2.25 = 2\tfrac{1}{4}'',$$

as found by the general or trial method.

The student may apply formula **122** to the solution of example 3 in the preceding article.

TORSION AND SHAFTS.

1414. When a force is applied to a beam in such a manner that it tends to twist it, the stress thus produced is termed **torsion.** In Fig. 337, bc represents a beam fixed at one end; a load W is applied at the end of a lever arm on, which twists the beam. If a straight line cb is drawn parallel to the axis before the load is applied, it will be found, after the weight W has been hung from n, that the line cb will take a position ca, forming a spiral. If the load

does not strain the material beyond its elastic limit, *c a* will return to its original position *c b* when *W* is removed. It will also be found that the angles *a c b* and *a o b* are directly proportional to the loads.

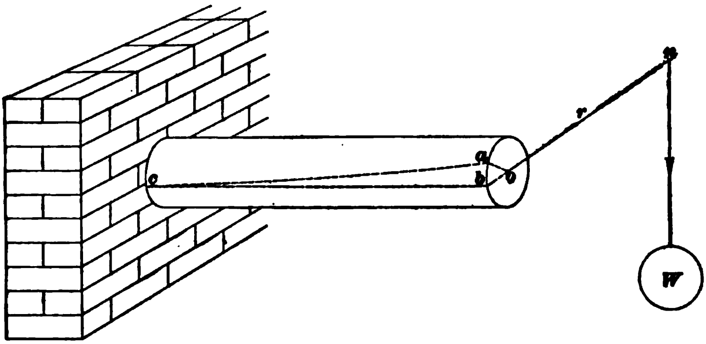


FIG. 887.

Torsion manifests itself in the case of rotating shafts. Instead of one end being fixed, as in the previous case, the reisistance which the shaft has to overcome takes the place of the force which before was necessary for fixing one end. Should the shaft be too small, the resistance will overcome the strength of the material and rupture it.

1415. The angle *a o b*, which may be called the angle of twist, plays an important part in the designing of shafts. For all solid shafts below 11 inches in diameter, the following formula may be used:

$$d = c\sqrt[4]{Pr} = c_1\sqrt[4]{\frac{H}{N}}, \qquad (123.)$$

in which d = diameter of round shaft or the side of a square shaft in inches; c = constant from Table 31; P = force or weight applied to the end of the lever arm, in pounds; r = length of lever arm in inches, from center of shaft to point of application of P; c_1 = constant from same table; H = horsepower transmitted, and N = number of revolutions per minute.

TABLE 31.

Material.	c.		c_1.	
	Round.	Square.	Round.	Square.
Wrought Iron......	.31	.272	4.92	4.31
Cast Iron..........	.353	.309	5.59	4.89
Steel..............	.297	.26	4.7	4.11

EXAMPLE.—What should be the diameter of a wrought-iron crank shaft for a 16" × 20" steam engine, if the greatest steam pressure is to be 90 lb. per sq. in.? (Assume that the entire steam pressure is transmitted through the crank-pin at some point of the stroke).

SOLUTION.—Total pressure on piston $= 16^2 \times .7854 \times 90 = 18{,}095.616$, say 18,000 lb. $= P$ in formula **123.** $r = \frac{20}{2} = 10''$. Therefore,

$$d = c\sqrt[3]{Pr} = .31\sqrt[3]{18{,}000 \times 10} = 6.385''.$$

A $6\frac{1}{2}''$ shaft would be sufficiently large.

EXAMPLE.—What horsepower could be safely transmitted by a 7-inch cast iron square shaft making 80 revolutions per minute?

SOLUTION.—Formula **123** gives, $d = c_1\sqrt[4]{\frac{H}{N}}$, or $H = \frac{Nd^4}{c_1^4} = \frac{80 \times 7^4}{4.89^4}$ $= 335.93$, say 336 horsepower.

1416. If the diameter of a wrought-iron shaft is greater than 12.4 inches, of a cast-iron shaft greater than 10.3 inches, or of a steel shaft greater than 13.6 inches, the following formula should be used:

$$d = k\sqrt[3]{Pr} = k_1\sqrt[3]{\frac{H}{N}}, \qquad \textbf{(124.)}$$

k and k_1 being taken from Table 32. If the shaft is hollow (round), either of the two following formulas may be used:

$$P = q\left(\frac{d_1^4 - d_2^4}{d_1 r}\right), \qquad \textbf{(125.)}$$

or

$$H = q_1 N\left(\frac{d_1^4 - d_2^4}{d_1}\right), \qquad \textbf{(126.)}$$

d_1 and d_2 being the outside and inside diameters respectively and q and q_1 constants to be taken from Table 32.

TABLE 32.

Material.	k	k_1	q	q_1
Wrought Iron.......	.0909	3.62	1,335	.0212
Cast Iron...........	.1145	4.56	669	.0106
Steel	.0828	3.3	1,767	.028

EXAMPLE.—What horsepower can be safely transmitted by a hollow wrought-iron shaft making 60 revolutions per minute, and whose diameters are $9\frac{1}{2}$ and 12 inches?

SOLUTION.—

$$H = q_1 N\left(\frac{d_1^4 - d_2^4}{d_1}\right) = .0212 \times 60\left(\frac{12^4 - 9.5^4}{12}\right) = 1{,}884.65 \text{ H.P.}$$

EXAMPLES FOR PRACTICE.

1. What should be the diameter of a steel shaft to transmit 500 horsepower at 200 revolutions per minute? Ans. 5.91", say $5\frac{15}{16}$".

2. How many horsepower will an 8" round wrought-iron shaft transmit with safety, running at 150 R. P. M.? Ans. 1,048.5 H.P.

3. A hollow cast-iron shaft has an outside diameter of 10 inches and an inside diameter of 6 inches; at what speed should it be run to transmit 750 horsepower? Ans. 81.29 R. P. M.

4. A wrought-iron shaft 4 inches square runs at 110 revolutions per minute; what horsepower will it safely transmit? Ans. 81.6 H.P.

5. What should be the diameter of a wrought-iron shaft to transmit 6,000 horsepower at 100 revolutions per minute?

Ans. 14.172", say $14\frac{3}{16}$".

ROPES.

1417. The strength of hemp and manila ropes varies greatly, depending not so much upon the material and area of cross-section as upon the method of manufacture and the amount of twisting.

Hemp ropes are about 25% to 30% stronger than manila ropes or tarred hemp ropes. Ropes laid with tar wear better than those laid without tar, but their strength and flexibility are greatly reduced. For most purposes, the following formula may be used for the safe working load of any of the three ropes mentioned above:

$$P = 100\,C^2, \qquad \textbf{(127.)}$$

in which $P=$ working load in pounds and $C=$ circumference of rope in inches. This formula gives a factor of safety of from $7\frac{1}{2}$ for manila or tarred hemp rope to about 11 for best three strand hemp rope. When excessive wear is likely to occur, it is better to make the circumference of the rope considerably larger than that given by the formula.

1418. Wire rope is made by twisting a number of wires (usually 19) together into a strand and then twisting several strands (usually 7) together to form the rope. It is very much stronger than hemp rope, and may be much smaller in size to carry the same load.

For iron wire rope of 7 strands, 19 wires to the strand, the following formula may be used, the letters having the same meaning as in formula **127**:

$$P = 600\,C^2. \qquad \textbf{(128.)}$$

Steel wire ropes should be made of the best quality of steel wire; when so made they are superior to the best iron wire ropes. If made from an inferior quality of steel wire, the ropes are not as good as the better class of iron wire ropes. When substituting steel for iron ropes, the object in view should be to gain an increase of wear rather than to reduce the size. The following formula may be used in computing the size or working strength of the best steel wire rope, 7 strands, 19 wires to the strand:

$$P = 1{,}000\,C^2. \qquad \textbf{(129.)}$$

Formulas **128** and **129** are based on a factor of safety of 6.

1419. When using ropes for the purpose of raising loads to a considerable height, the weight of the rope itself must also be considered and added to the load. The weight of the rope per running foot, for different sizes, may be obtained from the manufacturer's catalogue.

EXAMPLE.—What should be the allowable working load of an iron wire rope whose circumference is $6\frac{3}{4}$ inches? Weight of rope not to be considered.

SOLUTION.—Using formula **128**,

$$P = 600 \times (6\tfrac{3}{4})^2 = 27{,}337.5 \text{ lb.}$$

EXAMPLE.—The working load, including weight, of a hemp rope is to be 900 pounds; what should be its circumference?

SOLUTION.—Using formula **127**,

$$C = \sqrt{\frac{P}{100}} = \sqrt{\frac{900}{100}} = 3''.$$

1420. In measuring ropes, the circumference is used instead of the diameter, because the ropes are not round and the circumference is not equal to 3.1416 times the diameter. For three strands the circumference is about 2.86 d, for seven strands about 3 d, d being the diameter.

CHAINS.

1421. The size of a chain is always specified by giving the diameter of the iron from which the link is made. The two kinds of chain most generally used are the **open link** chain and the **stud link** chain. The former is shown by (a), Fig. 338, and the latter by (b). The stud prevents the two sides of a link from coming together when under a heavy pull, and thus strengthens the chain.

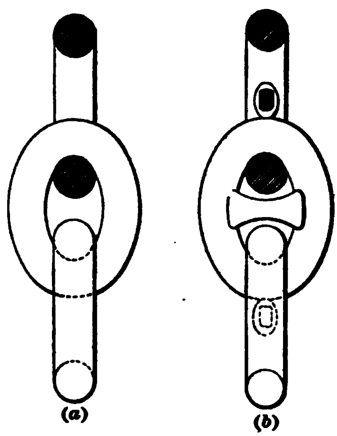


FIG. 338.

It is a good practice to anneal old chains which have become brittle by overstraining. This renders them less liable to snap from sudden jerks. The annealing process reduces their tensile strength, but increases their toughness and ductility, two qualities which are sometimes more important than mere strength.

Let P = safe load in pounds;

d = diameter of link in inches.

Then, for open link chains, made from a good quality of wrought iron,

$$P = 12,000\, d^2, \qquad \textbf{(130.)}$$

and, for stud link chains,

$$P = 18,000\, d^2. \qquad \textbf{(131.)}$$

Example.—What load will be safely sustained by a ¾-inch open link chain?

Solution.—Using formula 130,

$$P = 12,000\ d^2 = 12,000 \times (\tfrac{3}{4})^2 = 6,750 \text{ lb.}$$

Example.—What must be the diameter of a stud link chain to carry a load of 28,125 pounds?

Solution.—Using formula 131, $P = 18,000\ d^2$. Hence,

$$d = \sqrt{\frac{P}{18,000}} = \sqrt{\frac{28,125}{18,000}} = 1\tfrac{1}{4}''.$$

1421*a*. The statement in the last sentence of Art. **1358** may be modified in practice to include pieces whose lengths are not greater than *ten* times their least transverse dimensions, without material error. Although the stress is pure compression only for pieces whose lengths do not exceed *five* times their least transverse dimensions, the results obtained by formula **119** agree so closely with those obtained by formula **110,** when the length does not exceed ten times the least transverse dimension, that the latter may be used in all such cases. When the length is greater than ten times the least transverse dimension, the piece becomes a column, and formula **119** must be used.

APPLIED MECHANICS.

1422. A **machine** is an assemblage of moving parts, together with a supporting frame so arranged as to utilize some external source of energy for the purpose of doing work.

In the operation of machinery, motion and force are communicated to one of the moving parts, and transmitted to the part where the work is done. During the transmission, both the motion and force are modified in direction and amount, so as to be rendered suitable for the purpose to which they are to be applied.

The moving parts are arranged to have certain definite motions relative to each other, the effect of which is to compel the piece where the work is done to have the required motion. The nature of these movements is independent of the amount of force transmitted; in other words, in a model of a machine, operated by hand, the relative motions of the parts will be precisely the same as in the machine itself, although, in the latter case, a great amount of power may be transmitted and much work done.

1423. **Kinematics** is that branch of Applied Mechanics which treats of the motions of the parts of a machine, without regard to the forces acting.

1424. The **dynamics of machinery** is that branch which treats of the forces acting in the operation of machinery. The dynamics of machinery depends upon the subject of Kinematics, since every change in force in a machine is the result of a change in motion. Thus, in the steam engine, if the motion of the piston be transmitted and modified so as to cause the periphery of the fly-wheel to move eight times as fast as the piston, the force exerted upon the belt would be only one-eighth of the steam pressure upon the piston.

In what follows, Kinematics will be treated of mainly, though in some cases the forces acting will be considered.

1425. Mechanism is a term applied to three or more parts of a machine, so combined that the motion of the first compels the motion of the other movable parts, according to the law depending upon the nature of the combination.

The terms *mechanical movement* and *mechanical motion* are often used as having the same meaning as *mechanism.* A machine is made up of a number of mechanisms.

1426. Driver and Follower.—That piece of a mechanism which causes motion is called the **driver,** and the one whose motion is effected is called the **follower.** In the case of belt or toothed gearing, the follower is often called the *driven* wheel.

1427. Right-handed rotation is rotation in the direction of the motion of the hands of a watch. **Left-handed rotation** is in the opposite direction Looking at a rotating pulley from one side, its rotation would be right-handed, if it turned in the same direction as the hands of a watch, held and looked at by the observer. Viewing the pulley from the other side, its rotation would be left-handed.

1428. Cycle of Motions.—When a mechanism is set in motion, and its parts go through a series of movements, which are repeated over and over in the same order, each series is called a **cycle of motions.**

1429. Velocity ratio is a term used to signify the comparative velocities of two pieces. Thus, if two gear wheels are so proportioned that one turns three times as fast as the other, their velocity ratio would be 3 or $\frac{1}{3}$, according as the more rapidly revolving gear was mentioned first or last.

LINK MECHANISMS.

1430. A bar, or other rigid body, connecting two elements or parts of a mechanism, is termed a **link.** In the steam engine, the crank and connecting-rod are links, and the engine frame may be considered to be a third or closing

link, one end of which supports the crank-shaft, the other end being slotted to guide the cross-head.

Links have special names, according to the machine or location in which they are used. Thus, a link which vibrates about a point is called a **beam, rocker,** or **lever,** and one which turns completely around a point is called a **crank.** A link connecting with an oscillating, or rotating, link is called by various names, as **connecting-rod, crank-rod, pitman, eccentric-rod, coupling-rod, parallel-rod,** etc. In practice, the word "link" is applied mainly to *slotted* links, such as the link in "Stephenson's link motion."

In what follows, when speaking of the length of a link, the distance between centers will be understood, or the distance A, in Fig. 339.

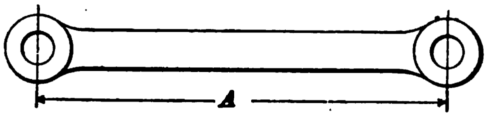


FIG. 339.

1431. The **center line of motion** of any mechanism is a straight line so drawn as to represent the general or mean direction in which one or more of the parts move. When the motion of the parts is not in a straight line, their deviation occurs equally on each side of this center line. Thus, in the steam engine, the center line of motion runs from the center of the cylinder to the center of the shaft, the connecting-rod vibrating equally on each side of the line.

1432. Levers.—Levers are used in mechanisms to guide a moving point, as the end of a moving rod, or to transfer motion from one line to another. There are three cases: (I) Levers whose lines of motion are parallel; (II) levers whose lines of motion intersect, and (III) levers having arms whose center lines do not lie in the same plane.

In proportioning levers, the following points should in general be observed. They apply to all three cases just mentioned:

(1.) When in mid position, the center lines of the arms should be perpendicular to the lines along which they give

or take their motions, so that the lever will vibrate equally each way.

(2.) If a vibrating link is connected to the lever, its point of attachment should be so located as to move equally on each side of the center line of motion of the link.

(3.) The lengths of the lever arms must be proportional to the distances through which they are to vibrate.

1433. Case I.—Reversing Levers.—An example of a lever which illustrates the foregoing principles is shown in Fig. 340. The crank $R\ S$ is the driver, and gives a

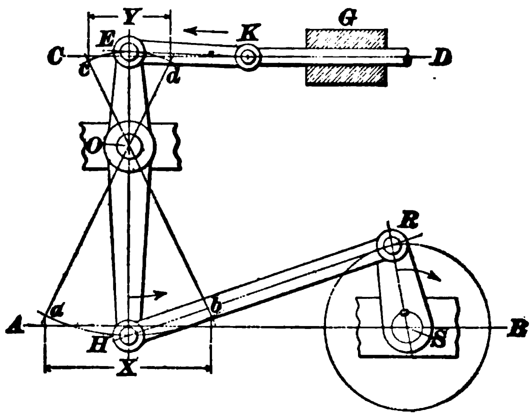


FIG. 340.

motion along the center line $A\ B$, which is transferred by the lever $E\ H$ to the line $C\ D$. The lever vibrates equally each way about its fulcrum or center O, as indicated by the lines $c\ b$ and $d\ a$. When in mid position, its center line $E\ H$ is perpendicular to the lines of motion $C\ D$ and $A\ B$. The horizontal distances traversed by points E and H, respectively, are proportional to the arms $E\ O$ and $H\ O$, or $Y : E\ O = X : H\ O$. The vibrating link $E\ K$ connects the point E with the rod $K\ D$, which is constrained to move in a straight line by the guide G, and in accordance with principle 2, the lever is so proportioned that point E will be as far above the center line of motion $C\ D$, when in mid position, as it will be below it in the two extreme positions;

that is, points c and d are as far below the line as point E is above it.

At the bottom of the lever, where the rod $H R$ connects with the crank, the same principle holds, point H being as far below the line $A B$ as points a and b are above it.

Frequently, the distance between the center lines $C D$ and $A B$ is given, and the extent of the motion along these lines, from which to proportion the lever. A correct solution to this problem is troublesome by calculation, because it is not known at the start how far above and below their respective lines of motion the points E and H should be.

1434. It may easily be done graphically, however, as shown in Fig. 341. Draw the center lines of motion $C D$ and $A B$ and a center line $S T$ perpendicular to them. Draw $M E$ parallel to $S T$ at a distance from it equal to $\frac{1}{2} Y$, or half the stroke along $C D$; also, the parallel line $H N$, on the other side of $S T$, and at a distance from it equal to $\frac{1}{2} X$, or half the stroke along $A B$. Connect points M and N by a straight line; where this line intersects $S T$, as at O, will be the center or fulcrum of the lever. With O as a center, find by trial the radius of an arc which will cut $S T$ as far below the line $A B$ as it does $H N$ above this line, or so that the distance m will equal the distance n. As an aid in determining the correct radius, describe an arc cutting $S T$, with O as a center and a radius $O N$. The

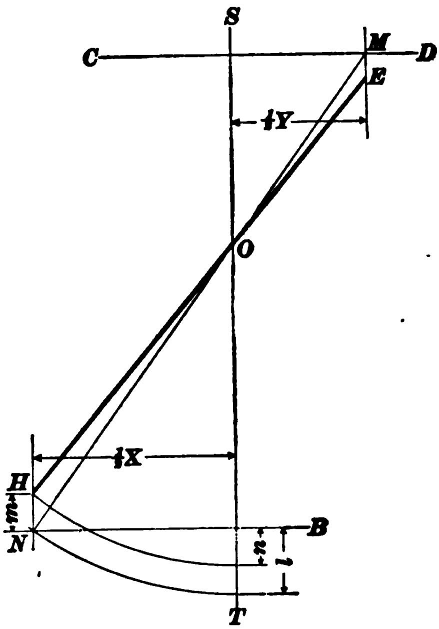


FIG. 341.

distance n will be a little more than $\frac{1}{2}$ l. Now, draw a straight line through points H and O. The part included between $H N$ and $M E$ determines the length of the lever. In this case, the length of the shorter arm is equal to $O E$, and of the longer arm, $O H$.

1435. Non-Reversing Levers.—Fig. 342 shows the same construction applied to a lever in which the center O is at one end of the lever. This lever does not reverse the motion like the previous one, since when the motion along $A B$ is to the right or left, the motion along $C D$ will be in the same direction. The figure is lettered like the preceding one, so that the construction will be easily understood.

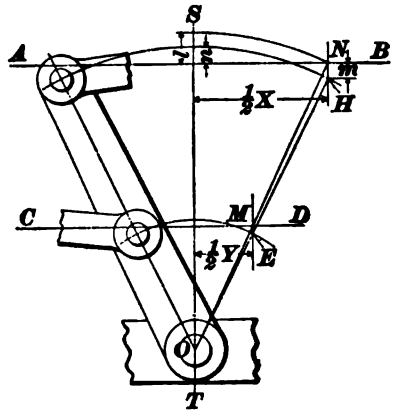


FIG. 342.

1436. It is often desirable that a lever mechanism shall reproduce upon a greater or smaller scale, along one line, the exact motion that occurs along another line; that is, that every change in the rate of the motion along one line shall reproduce a corresponding change along the other line. Figs. 343 and 344 illustrate three indicator-reducing motions that accomplish this.

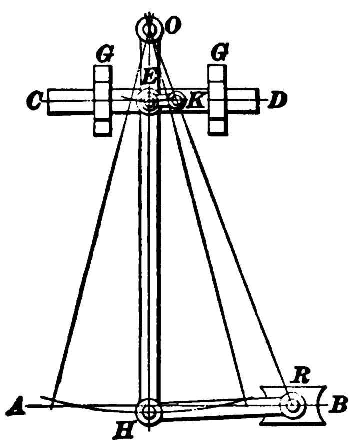


FIG. 343.

In Fig. 343 the lower end of the lever attaches to the cross-head of the engine through the swinging link $H R$. The indicator string is fastened to the bar $C D$, which receives its motion from the lever through

the link $E K$, and slides through the guides G, G, in a direction parallel to the line of motion A B of the cross-head. In order that the bar C D shall have the same kind of motion

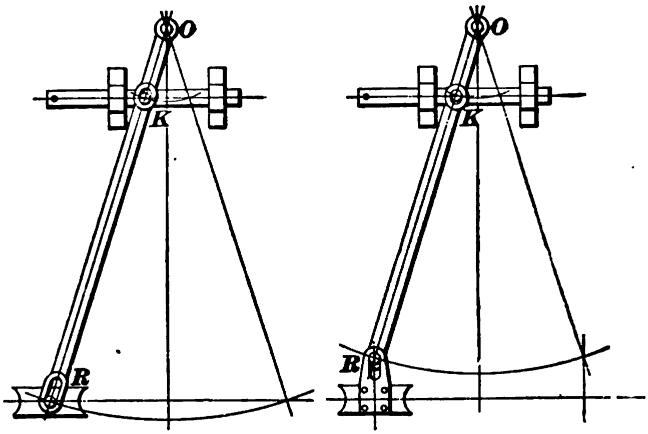


FIG. 344.

as the cross-head, it is only necessary that the links E K and H R shall be proportional to their respective lever arms; thus $O H : H R = O E : E K$. The pins must be so placed that the connecting links will be parallel; if parallel at one point of the stroke, they will be so at all points.

1437. It is to be observed that the pins O, K, and R are in one straight line, and, in general, it may be said that any arrangement of the lever which will keep these three pins in a straight line for all points of the stroke will be a correct one. In Fig. 344 are two such arrangements. In the first, the pins K and R are fast to the slide and cross-head, respectively, and slide in slots in the lever. In the second, they are fast to the lever, the slots being in the cross-head and slide. In both, the pins K and R are in a straight line with the pin O during the whole stroke.

1438. Case II.—Bell-Crank Levers.—Levers whose lines of motion intersect are termed **bell-crank** levers.

In Fig. 345, suppose the angle $C A B$, made by the lines

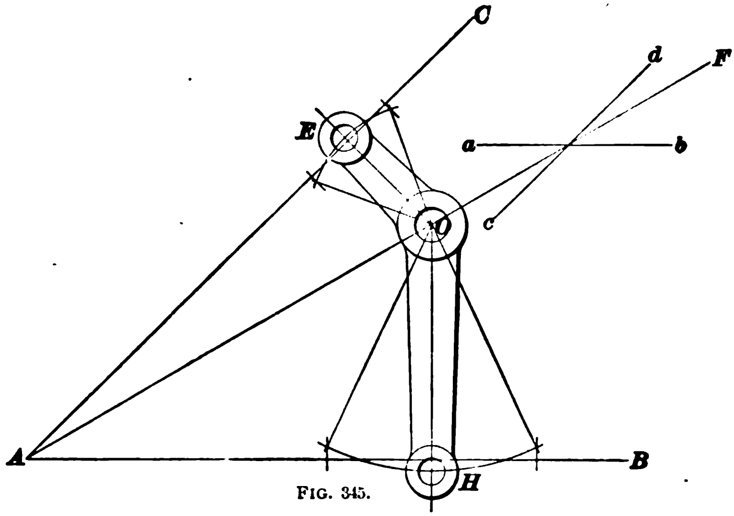


FIG. 345.

of motion, to be given, and that the motion along $A\ B$ is to be twice that along $A\ C$. Draw $c\ d$ parallel to $A\ C$ at any

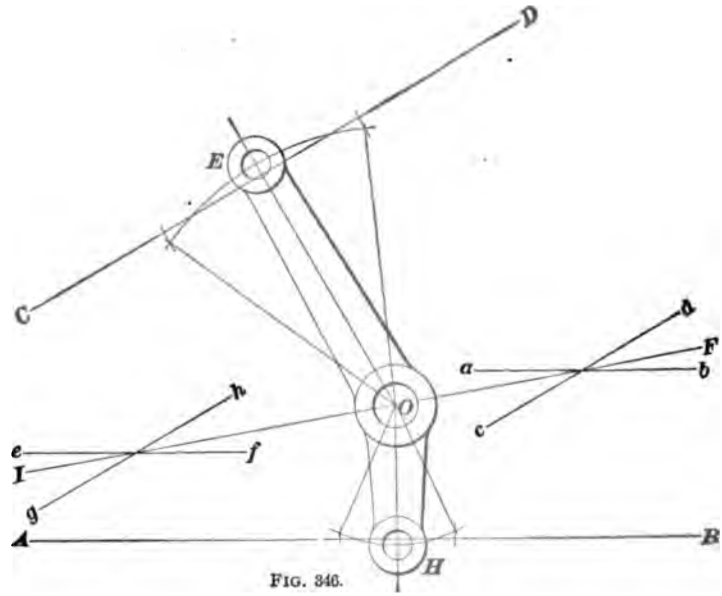


FIG. 346.

convenient distance from *A C*. Draw *a b* parallel to *A B* and at a distance from *A B* equal to twice the distance of *c d* from *A C*. Through the intersection of these two lines and the apex *A* of the angle, draw the line *A F*. Then the center *O* of the bell-crank may be taken at any point on *A F* suited to the design of the machine. Having chosen point *O*, draw the perpendiculars *O E* and *O H*, which will be the center lines of the lever arms.

1439. In Fig. 346 a construction is shown that may be employed when the two lines *C D* and *A B* do not intersect

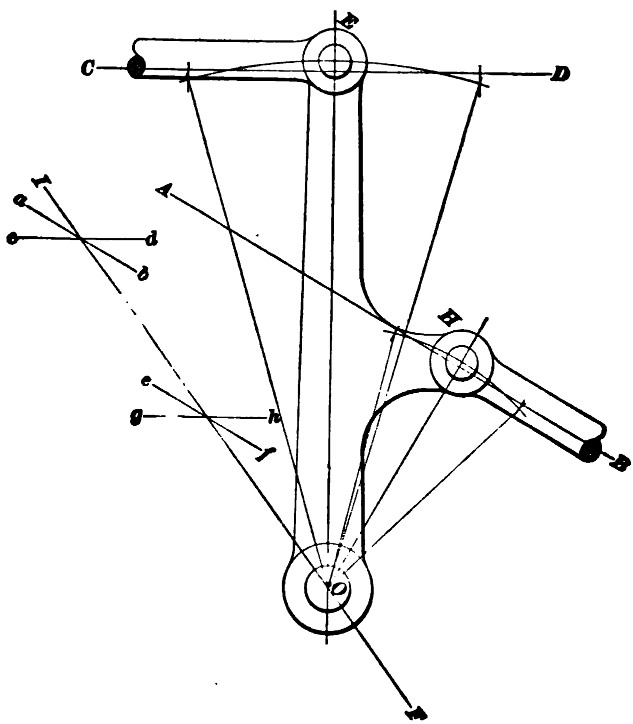


FIG. 347.

within the limits of the drawing. In Fig. 347 the same construction is applied to a non-reversing lever, in which the center *O* falls outside of the lines *A B* and *C D*. The figures

are lettered alike, and the following explanation applies to either: Draw $c\ d$ parallel to $C\ D$, and $a\ b$ parallel to $A\ B$, as before, so that the distance of $c\ d$ from $C\ D$: distance of $a\ b$ from $A\ B$ = amount of motion along $C\ D$: amount of motion along $A\ B$. Again, draw lines $g\ h$ and $e\ f$ in exactly the same way, but taking care to get their distances from $C\ D$ and $A\ B$ different from those of the lines just drawn. Thus, if $c\ d$ should be six inches from $C\ D$, make $g\ h$ some other distance, as four inches, or eight inches, and then draw $e\ f$ at a proportionate distance from $A\ B$. Through the intersections of $a\ b$ with $c\ d$, and of $e\ f$ with $g\ h$, draw the line $I\ F$, which will be the line of centers for the fulcrum O.

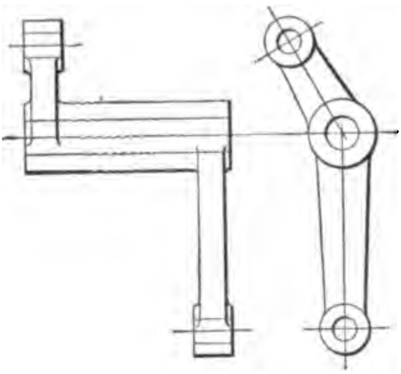


FIG. 348.

1440. Case III.—Levers falling under this case are usually bell-crank levers, with their arms separated by a long hub, so as to lie in different planes. They introduce no new principle. See Fig. 348.

1441. Crank and Connecting-Rod.—Fig. 349 is a diagram of the crank mechanism used in steam engines,

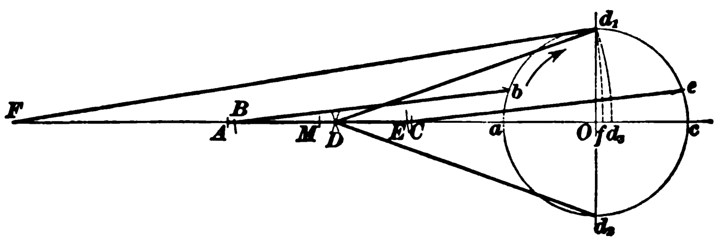


FIG. 349.

power pumps, etc. $A\ C$ represents the stroke of the crosshead, O the center of the crank-shaft, and $a\ d_1\ c\ d_2$ the path described by the crank-pin, called the crank-pin circle.

In the crank motion, the relative positions of the cross-head and crank-pin vary at every point of the stroke. This irregularity, which has an important influence in the design of steam-engine valve gears, may readily be observed by plotting a few points of the motion. Having drawn the crank-pin circle and the center line of motion *A c*, set the compasses to a radius equal to the length of the connecting-rod, which, in this case, is three times the length of the crank, or $3 \times O a$. With *a* as a center, strike an arc at *A*, and with *c* as a center, an arc at *C*. *A C* equals the length of the stroke of the cross-head, *A* and *C* being the extreme positions of the stroke. The corresponding positions *a* and *c* of the crank-pin are known as the **dead points** or **dead centers**, because the crank can not be started at these points by a direct pressure on the connecting-rod.

With some point near *a* on the crank-pin circle, as *b*, for a center, and with the same radius as before, strike arc *B* on the stroke line. Mark point *e* on the crank-pin circle, so that arc $e\,c = b\,a$; with *e* as a center, and with the same radius, strike arc *E*. It will be seen that the distance *E C* is less than *A B*, which shows that the relative motion during the first half stroke is different from that during the second half. The reason for this is that one-half the crank-pin circle curves *towards* the cross-head, and the other half *away* from it.

The greatest irregularity occurs when the crank is in its middle positions, or at points d_1 and d_2. With either point as a center, strike an arc, as before, which will fall at *D*, a distance equal to *M D* from the mid-stroke position of the cross-head. Suppose, the crank to turn in the direction of the arrow, the cross-head will have *passed* mid position *M*, when the crank-pin reaches its middle point at d_1; on the return stroke the reverse will be true, the crank-pin reaching point d_2 *before* the cross-head reaches *M*. Hence, during the forward stroke, the cross-head *moves ahead* of the crank; on the return stroke, the cross-head *lags behind* the crank.

The common way of plotting the motion is as follows: Take for example, the point d_1. With d_1 as a center, strike the arc at D, as before; with D as a center and same radius, describe arc $d_1 d_2$. $O d_2$ is the displacement of the cross-head from mid stroke. Point f was obtained in the same way by increasing the length of the connecting-rod to $F d_1$, *showing that the longer the connecting-rod, the less the irregularity.*

1442. Crank and Slotted Cross-Head.—If the connecting-rod in Fig. 349 be increased to a very great length, an arc drawn through d_1, corresponding to the arcs $d_1 d_2$, and $d_1 f$, would be nearly a straight line coinciding with $d_1 O$, and the horizontal movement of the crank-pin would, therefore, be practically the same as that of the cross-head. If the connecting-rod were increased to an *infinite* length, the two movements would be exactly the same. Fig. 350 shows the crank and slotted cross-head mechanism by which this

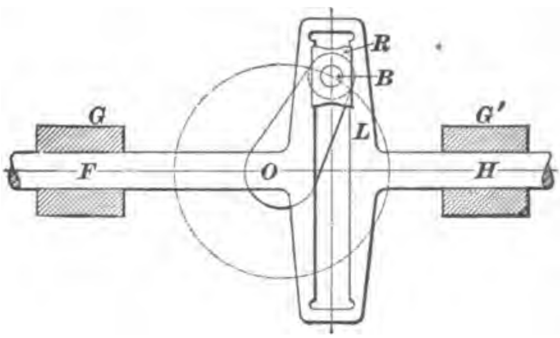


FIG. 350.

is accomplished. Consider the crank $O B$ as the driver. The crank-pin B is a working fit in the block R which is arranged to slide in the slotted link L. The rods F and H are rigidly attached to the link, and are compelled to move in a straight line by the guides G, G'. As the crank revolves, the rods F and H are given a horizontal motion exactly equal to the horizontal motion of the pin B.

This mechanism is often applied to steam pumps, where one of the rods, as F, is the steam piston rod, and the other

is the plunger rod. A fly-wheel is driven by means of the slotted link and crank, and its kinetic energy makes it possible to cut off the steam before the end of the stroke. Without the fly-wheel full steam pressure must be carried throughout the stroke.

1443. In Fig. 350, if the crank rotates uniformly, the motion of the sliding rods is said to be **harmonic**, and the mechanism itself is often called the **harmonic-motion** mechanism. Harmonic motion may be defined as the motion executed by the foot of a perpendicular let fall on the diameter of a circle from a point moving with uniform velocity along the circumference.

1444. Slow-Motion Mechanism.—A mechanism consisting of two connected levers, or of a crank and lever, can be proportioned to produce a slow motion of one of the levers.

Such a combination is shown in Fig. 351, where two levers, A and B, are arranged to turn on fixed centers, and are connected by the rod R. Lever A is actuated by the handle H, secured to the same shaft. If H and lever A be turned left-handed, lever B will turn right-handed, but with a decreasing velocity, which will become zero when the lever A reaches position A_1 in line with the rod, which will then be in position R_1. Any further motion of A will cause B to return towards its first position, its motion being slow at first and then faster. The mechanism, it will be observed, is proportioned contrary to the principles stated in Art. **1432,** and produces a motion that is variable, but very powerful.

To obtain the greatest advantage, the lever B should be so placed that it will occupy a position perpendicular to the link R at the instant when A and R are in line. To lay out the motion, therefore, supposing the positions of the centers and lengths of the levers to be known, describe arc $b\ a$ about the center O, with a radius equal to the length of lever B. Through C, the center of lever A, draw the line $M\ N$ tangent to the arc just drawn. R and A must then be in line

along the line $M N$, and lever B must be perpendicular to it when in position B_1. Generally, there will be a certain required amount of movement for lever B. To secure this, draw B in its extreme left-hand position; then with C as a center, and a radius equal to the length of A, strike the arc

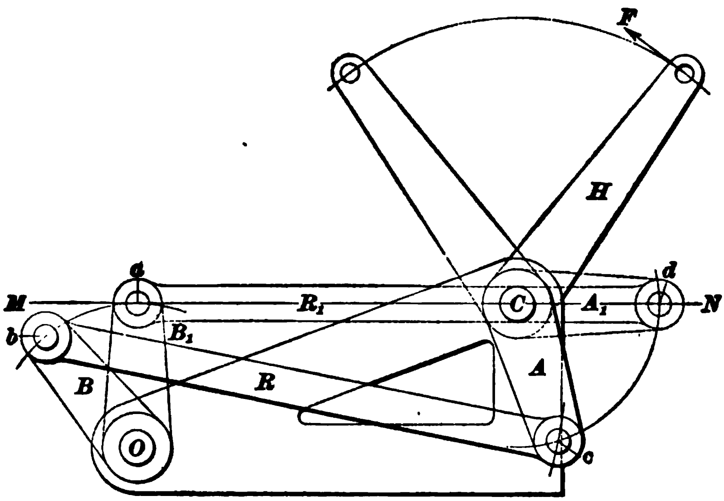


FIG. 351.

$d c$. Set the compasses to length $a d$, and from b as a center strike an arc cutting the arc $c d$. $C c$ will be the second position of lever A.

In the above combination, if the parts were proportioned to allow A to rotate like a crank, R and A would come into line twice during each revolution.

This mechanism has been used to operate platen printing presses, where oscillation of the handle H moves the platen to and from the type, through the lever B. It is also used to operate the exhaust valves in Corliss engines, in a manner which will be explained later.

1445. The **toggle-joint,** shown in Fig. 352, is a modification of the foregoing mechanism, and is much used in presses, punching and shearing machinery, etc. It consists of the two links $O R$ and $C R$, point O being fixed and point

C constrained to move in a horizontal line. A considerable movement of point *R* produces a very small movement in *C*,

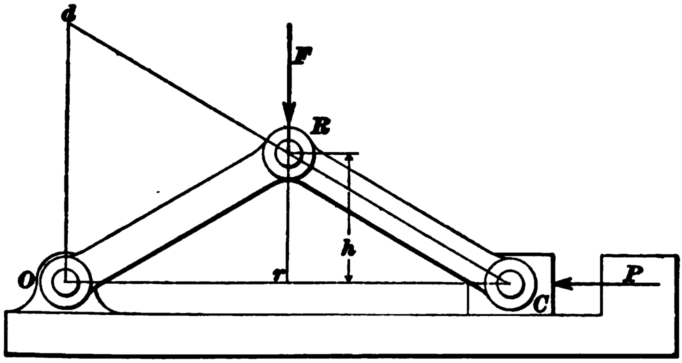


FIG. 352.

and consequently a pressure *F*, applied at *R*, can be made to produce a powerful pressure, or thrust, in a direction opposite to that of the arrow *P*.

THE FORCES ACTING IN LINK MECHANISMS.

1446. In order to understand how the forces act in a link mechanism, it will be necessary to again refer to the subject of moments, which is treated of in Elementary Mechanics.

The **moment of a force** *about any point is the product of the magnitude of the force and the perpendicular distance from the point to the line of action of the force.*

The tendency of a force to rotate a body about a point is measured by the *moment* of the force about that point. For example, let the crank *C*, in Fig. 353, be pivoted at *O*, and suppose a force *P* to act upon the crank-pin in the direction shown. Then, if the perpendicular distance from *O* to the line of action of the force be *O A*, the *moment of the force* tending to rotate the crank about *O* is $P \times O\,A$. If the force be stated in pounds and the perpendicular distance in inches, the product will be in *inch-pounds;* if the perpendicular distance be given in

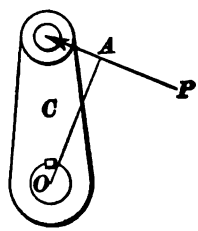


FIG. 353.

feet, the product will be in foot-pounds. These expressions, however, bear no relation to *foot-pounds of work*, and the student must avoid confusing the two.

In the case of a shaft having a pulley, gear, crank, or lever attached to it upon which a force acts tending to cause rotation of the shaft, the moment of the force is generally called the **twisting moment.**

1447. If two or more forces in one plane act upon a body, and are in equilibrium, then *the sum of the moments which tend to turn the body in one direction about a point is equal to the sum of the moments of the forces which tend to turn the body in the opposite direction about the same point.* Or, to state the principle more concisely, *the opposing moments about the point are equal.* This is called the **principle of moments.**

In the crank and connecting-rod mechanism, shown in outline in Fig. 354, the tendency of the force P to cause

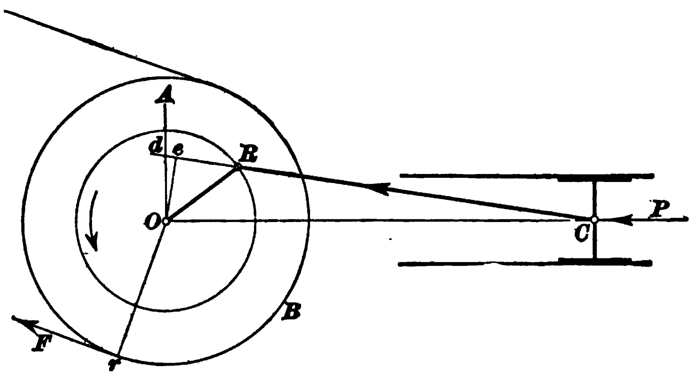


FIG. 354.

rotation of the crank about point O may be determined by resolving this force into two forces, one of which acts along the connecting-rod in the direction of the arrow, and which we will call P', and the other of which acts in a direction perpendicular to the guides. Draw $O\ e$ perpendicular to $C\ R$ produced; then, assuming the parts to be stationary for the instant, so that the effects of inertia may be neglected,

the twisting moment $= P' \times O\,e$. A simpler method, however, is the following: Let the force P act in the direction of the line $O\,C$. From O draw $O\,A$ perpendicular to $O\,C$, and note the point d where it intersects the center line of the connecting-rod, or of the center line produced. Then, it can be proved that the twisting moment about O due to the force P is $P \times O\,d$.

If a belt pulley B is attached to the shaft, the force P will be resisted by the pull F of the belt; and, by the *principle of moments*, $F \times O\,r = P \times O\,d$; whence,

$$F = \frac{P \times O\,d}{O\,r}. \qquad \textbf{(132.)}$$

EXAMPLE.—In a power pump, if a belt pull of 120 pounds is exerted upon the rim of the driving pulley, 24 inches in diameter, and the crank is in such a position that the distance $O\,d$ (Fig. 354) = 2 inches, what is the water pressure per square inch upon the pump plunger, if its diameter is 4 inches?

SOLUTION.—Radius of pulley = 12" = $O\,r$. From formula **132**, $120 = \frac{P \times 2}{12}$, whence $P = 720$ lb. Area of piston = 12.57 sq. in. $720 \div 12.57 = 57.28$ lb. per sq. in. Ans.

1448. A drawing in which the arms, rods, and links of a mechanism are indicated by their center lines only is called **a skeleton diagram.**

Fig. 355 is a skeleton diagram of the motion shown in Fig. 351, the different parts being in the same position. Draw the line $C\,s$ perpendicular to $b\,c$, and suppose a force F to act at the end of the lever $C\,h$. By the principle of moments, the pull along $b\,c$ would then be such that $F \times C\,h$ = pull on $b\,c \times C\,s$, or

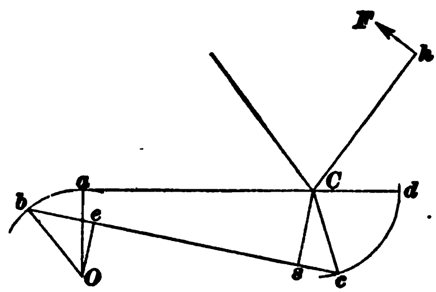


FIG. 355.

$$\text{Pull on } b\,c = \frac{F \times C\,h}{C\,s}. \qquad \textbf{(133.)}$$

The force necessary to balance the pull on $b\ c$ would be supplied by the resistance to motion by whatever force might be applied to the shaft O or the lever $b\ O$, and would be felt at the point b. If it were desired to find the twisting moment about the point O, we would multiply the pull on $b\ c$ by the perpendicular distance $O\ e$.

EXAMPLE.—In Fig. 355, if $F = 25$ lb., $C\ h = 2$ ft., $C\ s = \frac{1}{10}''$, and $O\ e = 3''$, (*a*) what would be the pull at b? (*b*) What is the twisting moment about O?

SOLUTION.—(*a*) Pull $= \dfrac{25 \times 24}{\frac{1}{10}} = 25 \times 24 \times 10 = 6{,}000$ lb. Ans.

(*b*) Twisting moment $= 6{,}000 \times 3 = 18{,}000$ inch-pounds. Ans.

1449. We will now consider the forces acting in the toggle-joint. In Fig. 356, let a pressure F be exerted upon

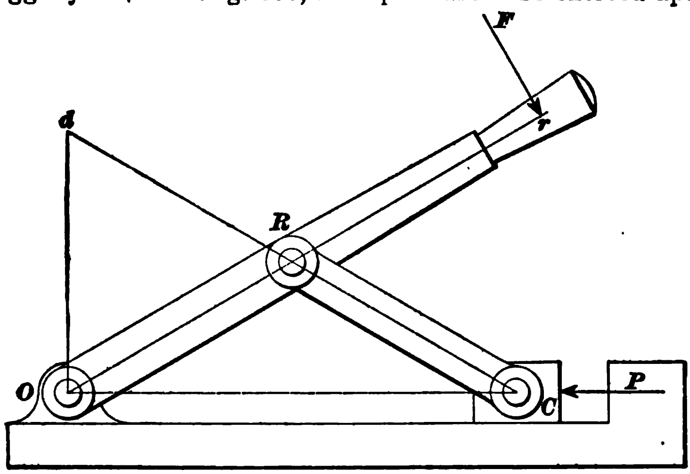


FIG. 356.

the handle in a direction at right angles to $O\ r$. The twisting moment about O is $F \times O\ r$, and the case becomes exactly similar to that of the connecting-rod crank, $O\ R$ corresponding to the crank and C to the cross-head. From O erect the perpendicular which intersects the center line of the link $C\ R$, extended, at d. Then, as before, $P \times O\ d = F \times O\ r$. It will here be more convenient to have the formula in terms of P. Hence,

$$P = \frac{F \times O\ r}{O\ d}. \qquad \textbf{(134.)}$$

The same formula applies when the force F acts as in Fig. 352. Drawing the line $O\ r$ perpendicular to the line of action of F, we have the twisting moment $F \times O\ r$ resisted by $P \times O\ d$.

1450. When the two links $O\ R$ and $R\ C$ are equal in length, the height h of the point R above a straight line drawn through points O and C will equal $\frac{1}{2}\ O\ d$. Hence, for equal links, formula **134** may be written,

$$P = \frac{F \times O\ r}{2\ h}. \qquad \textbf{(135.)}$$

EXAMPLE.—In Fig. 18, if $O\ r = 30''$, $O\ d = 6''$, and $F = 100$ lb., what thrust would be produced by the block C?

SOLUTION.—$P = \frac{100 \times 30}{6} = 500$ lb. Ans.

EXAMPLES FOR PRACTICE.

1. In Fig. 355, let $F = 250$ lb., $C\ h = 2$ ft., $C\ s = 2$ in., and $b\ O = 10$ in. If, in this position, the lever $b\ O$ is perpendicular to $b\ c$, what force would have to be exerted at a point midway between b and O, in order to resist the force F? Ans. 6,000 lb.

2. If the piston of an engine is 6 inches in diameter and the steam pressure 45 lb. per sq. in., what would be the tangential pressure at point R, in Fig. 354, when the crank is in such a position that $O\ d =$ 3 in., the length of the crank being 8 in.? Ans. 477.13 lb.

3. What thrust would be exerted by the block C, in Fig. 356, if the force F were to act in a vertical instead of a slanting direction? Take $F = 100$ lb., $O\ r = 60$ in., $O\ d = 1$ in., and distance of point r above the line $O\ C = 8$ in. Ans. 5,946¼ lb., nearly.

QUICK-RETURN MOTIONS.

1451. Quick-return motions are used in shapers, slotters and other machines, where all the useful work is done during the stroke of a reciprocating piece in one direction. During the working stroke the tool must move at a suitable cutting speed, while on the return stroke, when no work is performed, it is desirable that it should travel as rapidly as possible.

Vibrating-Link Motion.—The mechanism shown in Fig. 357 has been applied to shaping machines operating on

metal. Motion is received from the pinion *P*, which drives the gear *G*. The pin *b* is fast to the gear, and pivoted to it is the block *B*, which is fitted to the slot of the link *C D*. As the gear rotates, the pin describes the circle *b e d c*, the block sliding in the slot of the link *C D*, causing *C D* to oscil-

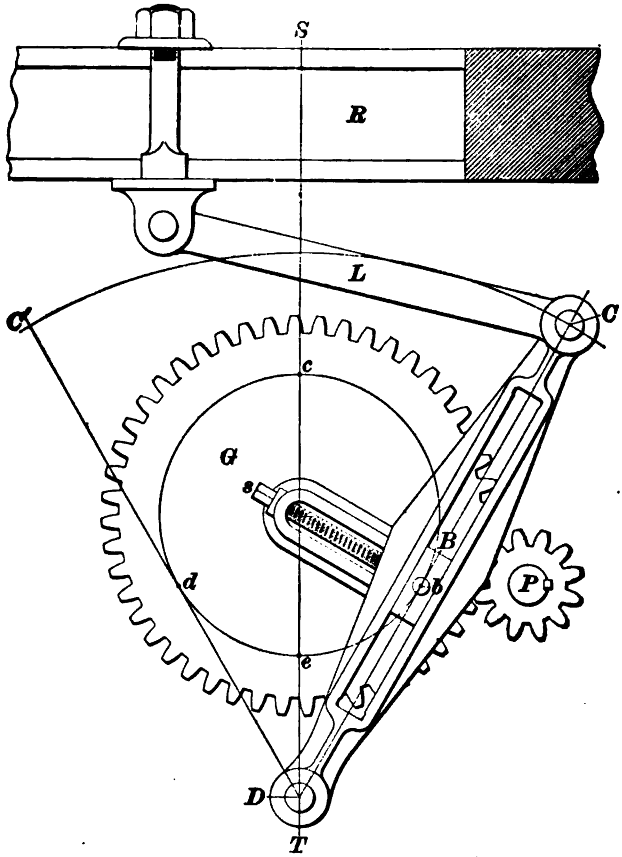


FIG. 857.

late about the point *D*, as indicated by the line *C C'* in the figure. The rod *L* connects the upper end of the link with the tool slide, or "ram," *R*, which, therefore, oscillates with it, but is constrained by guides (not shown) to move in a straight, horizontal line.

During the cutting stroke, the pin *b* travels over the arc *d c b*, or around the greater arc included between the points of tangency of the center lines *C' D* and *C D*. During the return stroke the pin passes over the shorter arc *b e d*, and as the wheel *G* revolves with a uniform velocity, the lines of the forward and return strokes will be to each other as the length of the arc *d c b* is to the length of the arc *b e d*. The throw of the slotted link and the travel of the tool can be varied by the screw *s*, which moves the block *B* to and from the center of the gear. The rod *L*, instead of vibrating equally above and below a center line of motion, is so arranged that the force moving the ram during the cutting stroke will always be downwards, causing it to rest firmly on the guide.

1452. To lay out the motion, proceed as follows: Draw the center line *S T*, Fig. 358, and parallel to it the line *m n*, the distance between the two being equal to one-half the longest stroke of the tool. About *O*, which is assumed to be the center of the gear, describe the circle *b d c* with a radius equal to the distance from the center of *G* to *b* (Fig. 357) when set for the longest stroke. Divide the circumference of the circle into an upper and lower arc, extending equally on each side of the center line, and having the same ratio as the forward and backward strokes. In this case the return is 2 to 1, and the circle is divided into three equal parts, as shown at *b*, *d*, and *c*, thus making the arc *d b* equal to one-half *d c b*. Draw the radial lines *O b* and *O d*. Through *b* draw *C D*, perpendicular to *O b*; the point *D*, where it intersects *S T*, will be the fulcrum, and the point *C*, where it intersects *m n*, the upper end of the slotted lever. Through *C* draw the horizontal line *C C'*, making *C' E* equal to *C E*. Draw *C' D*, which should be tangent to the circle at *d*, thus giving the other extreme position of the lever. It is to be observed that for a quick return of 2 to 1, the only condition is that lines *C D* and *C' D* shall be perpendicular to *O b* and *O d*, respectively; points *C* and *D* and the length of radius *O b* can be varied considerably.

1453. To plot the motion, draw the center line of motion *R L* through the point at which the connecting-rod attaches to the tool slide. Divide the circle *d c b* into a number of equal parts, as *1*, *2*, *3*, etc., and from *D* draw lines through these points, extending to the arc *C' C*. Number the points of division on the circle, and give corresponding numbers to the points of intersection on the arc *C' C*. With these last points as centers, and with a radius equal to the length of the connecting-rod *L*, in Fig. 357, strike arcs

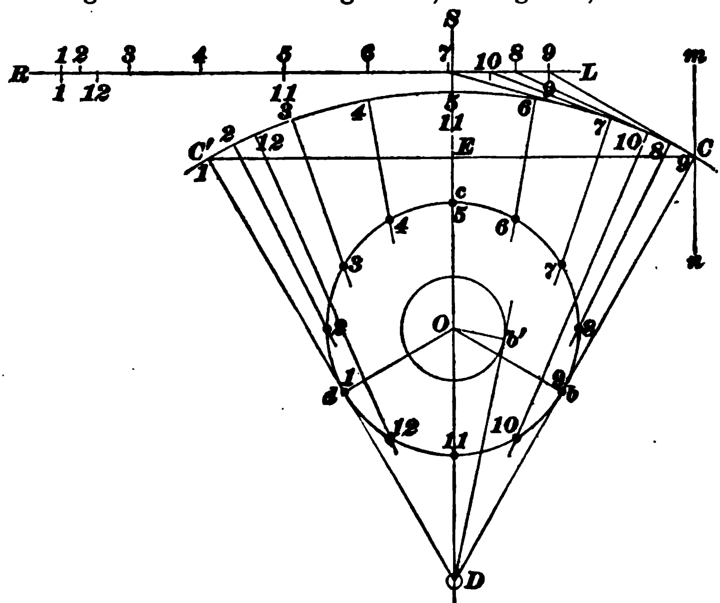


FIG. 358.

cutting the line *R L*, and number these intersections so that they will correspond to the other points. In Fig. 358, supposing the gear to turn with a uniform motion, the tool slide will move along *R L* from point *1* to *2* during the first $\frac{1}{12}$ of a revolution; during the next $\frac{1}{12}$ revolution, from point *2* to *3*, etc., on the forward stroke. On the return stroke, from point *9* to *10*, *11*, *12*, and *1*, the motion is much less uniform.

Another point about this motion is that, as the radius *O b* is diminished to shorten the stroke, the return becomes less

rapid, as can be seen from the dotted lines in the figure, which show the radius $O\ b$ shortened to $O\ b'$, and the corresponding position of $C\ D$.

1454. Whitworth Quick-Return Motion.—This mechanism is shown in principle in Fig. 359. The pin b, inserted in the side of the gear G, gives motion to the slotted link $C\ D$, as in the vibrating link motion. This motion closely resembles the previous one, the difference being that the center D, of the slotted link, lies *within* the circle described by the pin b, while in the previous case it lies *without* it. To accomplish this result, a pin P is provided for the gear to turn upon, and is made large enough to include another pin D, placed eccentrically within it, which acts as the center for $C\ D$. With this arrangement, the slotted link follows the crank-pin during the complete revolution, instead of

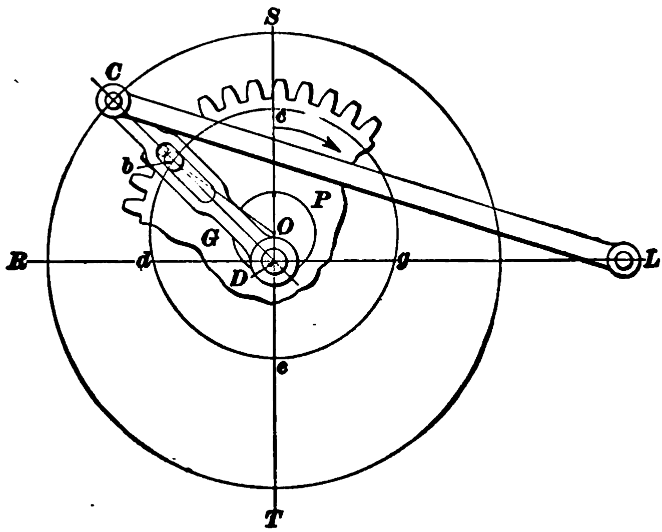


FIG. 359.

vibrating, and thus becomes a crank. The stroke line $R\ L$ passes through the center of D, which is below the center of P. The forward, or working, stroke occurs while the crank-pin b passes over the arc $d\ c\ g$, and the quick return occurs while it travels over the arc $g\ e\ d$. During each of these

intervals, the link $C\ D$ completes a half revolution, and, consequently, must move more rapidly, while the crank-pin describes the shorter arc.

1455. To proportion the motion, it is only necessary to so locate the stroke line $R\ L$ that it will divide the crank-pin circle $d\ c\ g\ e$ into two parts, $d\ c\ g$ and $g\ e\ d$, in the proportions of the forward and return strokes. The point D, where this line cuts the center line $S\ T$, is the position for the center of the slotted crank. The motion is plotted as in Fig. 360. Divide the crank-pin circle $d\ c\ g\ e$ into a number

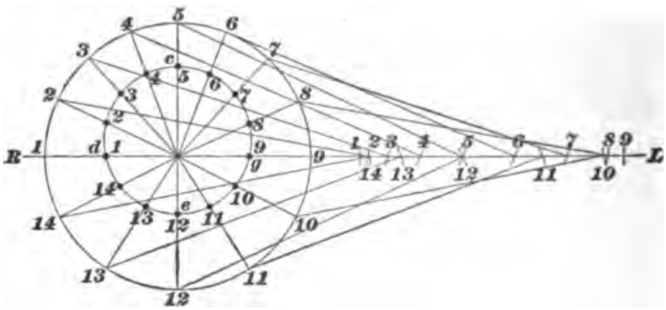


FIG. 360.

of equal parts. From D, the center of the slotted crank, draw radial lines through these points to the outer circle, which represents the path of pin C (Fig. 359), using the latter points of intersections as centers, and with a radius equal to the length of the connecting-rod, strike off points on the stroke line, which will show the movement of the tool for equal amounts of rotation of the driving gear.

1456. Fig. 361 shows the mechanism as practically constructed. The gear G is driven with a uniform velocity, in the direction of the arrow, by the pinion H. It rotates upon the large pin P (which is a part of the frame of the machine), and carries the pin b which turns in the block k and is capable of sliding in a radial slot in the piece B, as clearly shown in the sectional view. This piece B is supported by the shaft D, which turns in a bearing extending through the lower part of the large pin. $R\ L$, drawn through the center of D, is the line of motion of the tool slide. The

connecting-rod, actuating the tool slide, is pivoted to the stud *C*, which is clamped to piece *B*. The parts are lettered

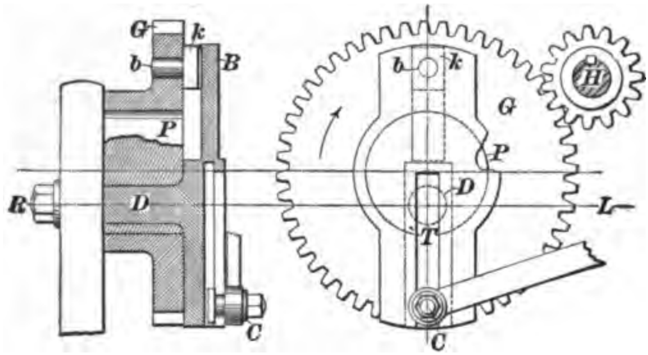


FIG. 361.

as in the two previous cuts, and the student should be able to study out the working of the mechanism without further explanation.

1457. The Adjustment of the Stroke.—The radial slot *T*, in Fig. 361, to which the connecting-rod is attached, provides for the adjustment of the *length* of the stroke, and if the point of attachment of the rod to the tool slide is also made adjustable, the *position* of the stroke, as well as its length, can be changed. Thus, in the case of a shaping machine, it is not only desirable to regulate the *distance* passed over by the tool, but to have the stroke extend *exactly to a certain point*. These two adjustments are often required in mechanism where reciprocating pieces are employed.

OTHER LINKAGES.

1458. The **universal joint,** shown in Fig. 362, is used to connect two shafts, the center lines of which are in the same plane, but make an angle with one another. It is generally constructed in the following manner: Forks *F*, *F* are fastened to the ends of the shafts *A* and *B*, and have burrs *a*, *a* tapped out to receive the studs *S*, *S*. The ends of these studs are turned cylindrical, and are a working fit, in

corresponding bearings in the ring *R*. The details of construction may be seen in the right-hand part of this figure.

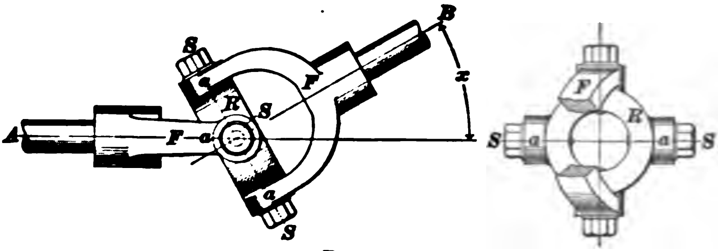


FIG. 362.

In heavy machinery the forks are forged and welded to the shafts.

1459. An objection to the universal joint is that the motion transmitted is not uniform. During one revolution the speed of the driven shaft varies twice between a velocity that is greater, and one that is less than the velocity of the driving shaft, and between these points there are four positions where the velocity of the two shafts is the same. Suppose the driving shaft to revolve uniformly; the *least* speed of the driven shaft will be equal to the speed of the driving shaft, multiplied by the cosine of the angle between the two axes produced (the angle x in Fig. 362). The *greatest* speed will be equal to the speed of the driver, multiplied by $\frac{1}{\cos x}$. Thus, if the shafts revolve at the rate of 100 revolutions per minute, and the angle between them is 30°, the least speed of the driven shaft will momentarily be at the rate of $100 \times \cos 30° = 100 \times .86603 = 86.6$ revolutions per minute, and the greatest speed, $\frac{100}{\cos 30°} = \frac{100}{.86603} = 115.47$ revolutions per minute.

1460. To obviate this trouble, which brings excessive wear and stresses on the working parts, the **double universal joint** is used, as shown in Fig. 363. Let *A* and *B* be the two shafts to be connected. Draw their center lines, intersecting at *o*, and bisect the angle *A o d* by the line *a o*.

The center line $e f$ of the connecting shaft D must now be drawn perpendicular to $a\ o$. Care must be taken that the forks on the intermediate shaft lie in the same plane. Thus constructed, a uniform motion of A will give a varying motion to D, which in turn will transmit to B the same motion as that of A. This arrangement is often

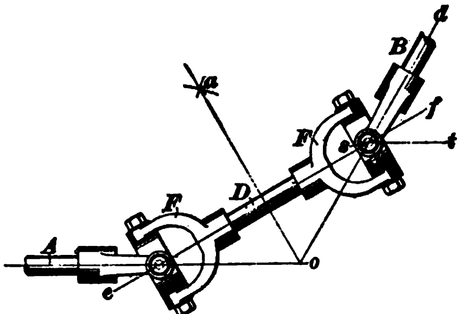


FIG. 363.

employed to connect parallel shafts, as would be the case in the figure if $o\ d$ took the direction $s\ t$. In such a case, it makes no difference what angle is made by $e\ f$, except that, if the joints are expected to wear well, it should not be too great.

1461. Watt's Parallel Motion.—A **parallel motion,** more properly called a **straight-line motion,** is a link mechanism designed to guide a reciprocating piece, as a piston rod, in a straight line. In the early days of the steam engine, parallel motions were extensively used to guide the pump and piston rods, but are now seldom met with, except on steam-engine indicators, where they are employed to give a straight-line motion to the pencil. Very few parallel motions produce an absolutely straight line, and it is customary to design them so that the middle and two extreme positions of the guided point will be in line. The best known motion is the one shown in Fig. 364, which was invented by James Watt, in 1784. The links $A\ B$ and $C\ D$ vibrate on their fixed centers A and D. The other ends, B and C, are connected by the link $C\ B$, which has the point O so chosen that it will pass through three points, O_1, O, and O_2, in the straight line $S\ S$ perpendicular to the links $C\ D$ and $A\ B$ when in their middle positions. When the point O is at the upper extremity of its motion at O_1, the

linkage assumes the position $A\ B_1\ C_1\ D$; at the lower extremity it assumes the position $A\ B_2\ C_2\ D$.

1462. Having given the length $O_1\ O_2$ of the stroke, and O, the middle position of the guided point, the center of one lever as A, and the perpendicular distance between

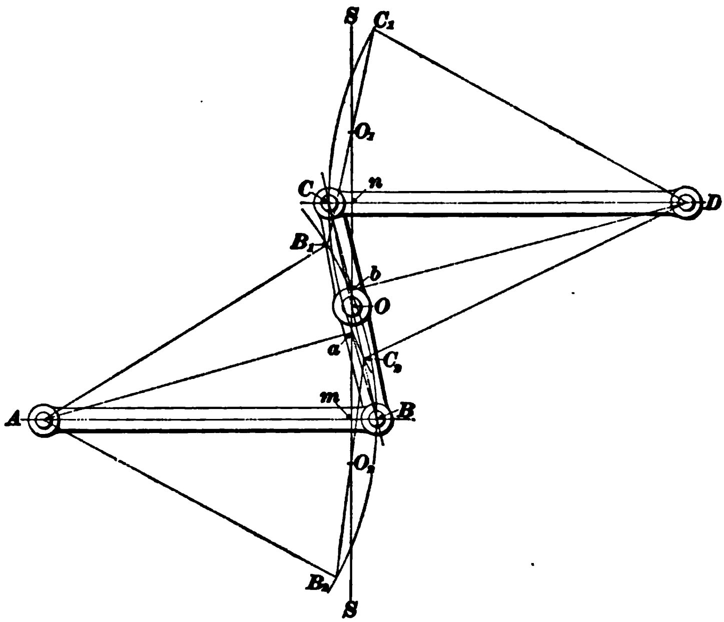


FIG. 864.

the levers when in mid position, the motion may be laid out as follows: Let $S\ S$ be the path of the guided point, O its middle position, A the given center, and $A\ B$ and $C\ D$ indefinite parallel lines, representing the middle positions of the levers. From m, where $A\ B$ intersects $S\ S$, lay off upon $S\ S$ the distance $m\ a$, equal to $\frac{1}{4}$ of the stroke. Join A with a and draw an indefinite line $a\ B$, perpendicular to $A\ a$. The point B, where $a\ B$ intersects line $A\ B$, is the right-hand extremity of lever $A\ B$, and the lower extremity of link $C\ B$. The point C is obtained by drawing an indefinite line through B and O; where it intersects the line $C\ D$ will be the point.

To find center D lay off $n\ b = \frac{1}{2}$ stroke; connect C and b, and from b draw an indefinite line perpendicular to $C\ b$; the center will be at its intersection with $C\ D$.

If the positions of both centers should be known, mark points a and b as before. Draw $A\ a$ and $b\ D$, and through b and a draw perpendiculars to these lines; the points B and C, where they intersect the center lines of the levers, are the extremities of these levers. Join B and C by the link $B\ C$, and the point O, where the center line of this link cuts the line of motion $S\ S$, is the position of the guided point O on the link $B\ C$.

CAMS.

1463. A **cam** is a turning or sliding piece, which, by the shape of its curved edge, or a groove in its surface, imparts a variable or intermittent motion to a roller, lever, rod, or other moving part.

1464. General Case.—Fig. 365 represents the general case for a **plate cam.** The cam C is supposed to turn in a right-handed direction about the axis B, and to transmit a variable motion through the roller A, and the lever L, to the rod R. The lever swings on the axis 0_0, the roller moving up or down on the arc 0 to 6, as the cam revolves. The roller is held in contact with the cam by its own weight and that of the lever and the rod.

Suppose the location of the cam shaft B and the rod R to be known, and that the cam is to revolve uniformly right-handed and impart motion to the rod, so that during the first half of every revolution it will move uniformly downwards, during the next quarter turn it will remain stationary, and during the last quarter it will return to its former position with a uniform motion.

1465. In order to determine the outline of the cam upon which the *circumference* of the roller bears, it is necessary to find an outline which will give the *center* of the roller the required motion. Then, by placing the point of the

compasses at different places on this outline, and striking arcs inside of it, with radii equal to the radius of the roller, the curve for the actual cam can be drawn, the curves being tangent to the arcs, and parallel to the first outline.

In this case, the roller *A* moves in an arc directly over the center of *B*. Knowing the distance the rod *R* is to move, we must so choose the point O_0 and the throw of the cam, that is, the distance that *A* is to move, that (see figure) the movement of *R* will be to the movement of *A* as *r* is to *a*.

Now, with O_0 as a center and a radius equal to *a*, describe

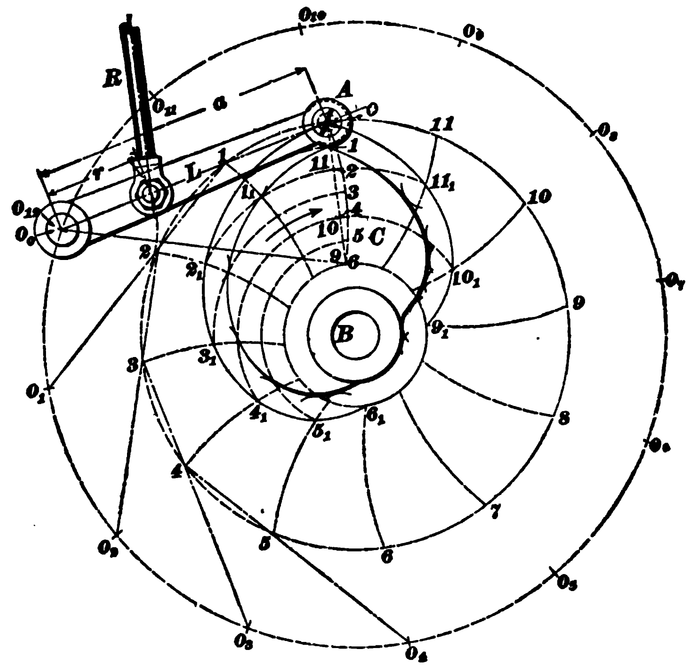


FIG. 365.

the arc *0-3-6*, in which the center of the roller is to move, and mark the highest and lowest points *0* and *6* of the roller. The lower point should not be near enough the shaft to allow the roller to strike the hub of the cam.

1466. It evidently makes no difference with the relative motions of the cam and roller whether the cam turns

right-handed, and the lever remains with its axis at O_6, or whether the cam is assumed to be stationary, and the lever and roller move *left-handed* in a circle about the center B. This latter process will be adopted.

With B as the center, draw a circle through O_6 and space it into a number of equal parts, say 12, and number the divisions around to the left. Now, assume the lever to move around the axis B in a left-handed direction. It will take positions O_1-*1*, O_2-*2*, O_3-*3*, etc. Hence, using these several points on the outer circle as centers, and with radii equal to a, the length of the lever, describe a series of arcs corresponding to the original arc *0-3-6*. Number these arcs *1*, *2*, *3*, etc., to correspond with the numbers on the outer circle.

During the first half-turn of the cam, or, what is the same thing, while the lever is moving from its position at O_6 to O_6 on the outer circle, the center of the roller must move uniformly from its outer to its inner position. Hence, draw the chord of the original arc *0-3-6*, and divide it into six equal parts, numbering them towards the center as shown. Then, with B as a center, describe an arc through point *1*, intersecting arc No. *1* in point 1_1. Now, sweep arcs through the other points, getting 2_1, 3_1, 4_1, etc., which are all points in the curve of the cam outline, for the center of the roller. From O_6 to O_9 (on the original circle), the center of the roller remains at a constant distance from B; hence, *6* and *9* must be connected by a circular arc. From O_9 to O_{12} the points are found as before by dividing the chord *0-6* into three equal parts and numbering them as shown, the numbers running outwards.

The final steps are to draw the cam outline for the center of the roller through points 1_1, 2_1, 3_1, 4_1, etc.; then, draw the outline for the cam itself parallel to it, as explained at first. This is easily done by setting the dividers to describe a circle whose radius shall be the same as that of the roller A. Then, with various points on the curve 0-1_1-2_1-3_1——11_1 as centers (the more, the better), describe short arcs as shown. By aid of the irregular curve, draw a curve which shall be tangent to the series of short arcs; it will be the required

outline of the cam, and will be parallel to the curve $0\text{-}1_1\text{-}2_1\text{-}3_1$——$11_1$.

1467. The question sometimes arises in designing cams of this nature, whether it is the chord *0-6* or the arc *0-6* that should be divided to give the roller the proper outward and inward motions. For all practical purposes either way is sufficiently exact, but *neither* is quite correct, though it is better to space the chord. The *exact* way would be to draw the rod R and the roller in the different positions desired, and then design the cam to meet the roller at these points.

1468. The cam shown in Fig. 366 differs in principle from the preceding one only in that the roller moves in a straight line, *passing to one side* of the center B of the shaft. Let it be required to design a cam of this nature to revolve right-handed, and which shall cause the roller A and rod R to rise with a uniform motion to a distance h during two-thirds of a revolution. When the roller reaches its highest point, it is to drop at once to its original position, and to remain there during the remainder of the revolution. Assume the distance from the center B to the center line of R to be equal to r.

With r as a radius, describe a circle about B, as shown. The center line of the rod will be tangent to this circle in all positions. With the same center and a radius equal to $B\ A_0$, A_0 being the extreme outward position of the roller, describe the outside circle A_0 *4-8* A_0. Divide this circle into some convenient number of equal parts, the number depending upon the fraction of a revolution required for the different periods of motion. Since the roller is to rise during two-thirds of a revolution, we may use 12 divisions as before, thus giving $\frac{2}{3} \times 12 = 8$ whole divisions for the first period.

Now, proceeding as before, by assuming the rod to move about the cam to the left, its positions when at the points of division A_1 A_2, etc., will be represented by drawing lines

through these points, and tangent to the inner circle, whose radius is r. A and A_0 are the two extreme positions of the roller. Divide the line $A\ A_0$ into eight equal parts, numbering them from the inside outwards, since the first movement of the roller is outwards. With B as a center, draw concentric arcs through these points, intersecting the tangents at 1_1, 2_1, 3_1, etc. At point 8 on the outer circle, the roller drops along the line $8\text{-}8_1$, the point 8_1 being determined by drawing an arc about B with a radius $B\ A$. From

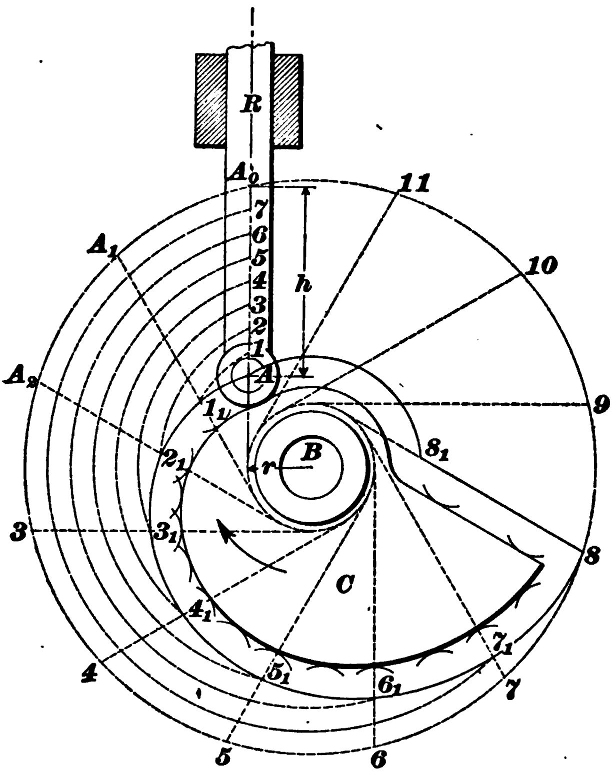


FIG. 366.

point 8 back to A the rod is at rest. The true cam outline is now to be found, as was done in the last example, by striking small arcs from points on the curve $A\ 1_1\text{-}2_1\text{-}3_1\text{-}4_1\text{-}5_1\text{-}6_1\text{-}7_1\text{-}8\text{-}8_1$

as centers. This cam can revolve in only one direction when operating the rod *R*.

1469. Harmonic-Motion Cams.—If a cam is required to give a rapid motion between two points, without regard to the *kind* of motion, its surface should be laid out so as to gradually accelerate the roller at the start, and to gradually retard it at the end of its motion, in order that the movement may be as smooth and free from shocks as possible. For this reason cams are frequently designed to produce harmonic motion—that is, a uniform motion of the cam produces a motion of the roller like that of the slotted cross-head in Fig. 350. The diagram in Fig. 367 shows how this latter motion is plotted. Let *A C* be the stroke line of

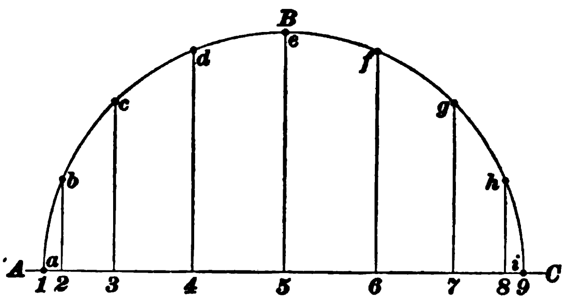


FIG. 367.

the cross-head, and *A B C* the crank-pin circle, which is divided into a number of equal spaces by the points *a*, *b*, *c*, *d*, etc. Dropping perpendiculars from these points, we obtain points *1*, *2*, *3*, *4*, etc., on the stroke line. The spaces between the latter points represent the distances traversed by the cross-head while the crank-pin moves through the equal spaces *a b*, *b c*, *c d*, etc. It will be seen that the distances increase from points *1* to *5*, and decrease from *5* to *9*.

1470. To apply the motion to the two cams previously taken up, we should simply have to lay off the distances *1-2*, *2-3*, *3-4*, etc., on the chord *0-6*, in Fig. 365, or the line $A_6\ A$, in Fig. 366, in place of the equal spaces used in these figures.

Fig. 368, which represents the left side of the first cam considered, laid out in this way, shows a convenient method of spacing. Upon the chord *0-6*, as a diameter, draw the semi-circle *0 d 6*; divide it into a suitable number of equal parts and project these divisions by straight lines to the chord *0-6*. Through the points of intersection *1*, *2*, *3*, *4*, etc., and with *B* as a center, describe the arcs $1\text{-}1_1$, $2\text{-}2_1$, $3\text{-}3_1$, etc., and complete the cam outline, as before.

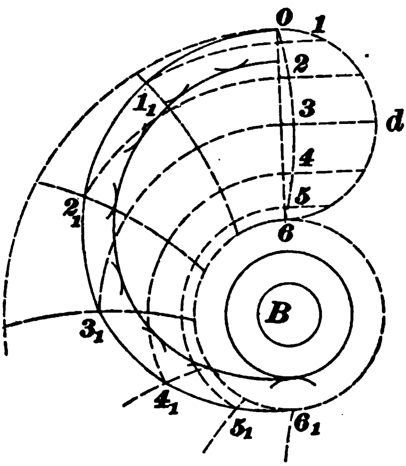


FIG. 368.

1471. Positive-Motion Cams.—The cams thus far considered can drive the roller in one direction only, making a spring or weight necessary to keep the two in contact. If the cam plate should extend beyond the roller, however, and a groove should be cut in it for the roller to run in, the motion of the roller would be *positive* in both directions.

1472. The word **positive,** when applied to a mechanism, has a different meaning from any heretofore given to it. A mechanism so constructed that nothing short of actual breakage of some one of its parts can keep it from working properly when motion is imparted to one of the links which operates it is called a **positive mechanism,** or a **positive gear,** when speaking of valve gears, and the motion produced is called a **positive motion.** Those mechanisms which depend for their operation upon the raising or lowering of a weight (i. e., upon gravity), or the action of a spring, are termed **non-positive** or **force-closed mechanisms.** Non-positive mechanisms, although extensively used, are of a lower order of mechanical excellence than positive mechanisms, and, other things being

equal, a positive motion should be chosen when designing a mechanism, since a non-positive one will refuse to work if the weight or the part operated by the spring should get "caught."

1473. Sometimes a positive motion is secured with a plate cam by causing it to revolve between two rollers rigidly connected, as illustrated in Fig. 369. The rollers A and E turn upon pins in one end of the rod R. The rod is slotted between the rollers, so that the cam shaft may

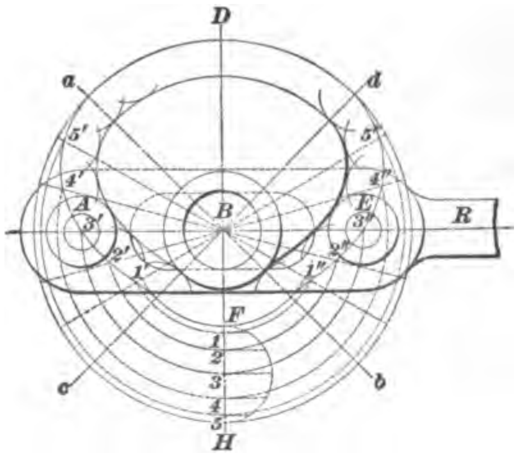


FIG. 369

pass through it and still allow the rod to move to the right and left. The center line of motion of the rollers passes through the center B of the shaft, whatever the position of the cam or rod.

1474. Let it be required that during one-quarter of a revolution of the cam the rod shall move to the right or left, according to the position of the cam at the start; that during the next quarter the rod shall be held stationary; that during the third quarter it shall move to its original position, where it is to remain for the rest of the revolution.

Draw the line $D\ H$, and mark the point B. Lay off a distance $B\ F$, such that when the center of one of the rollers is

at F the hub of the cam will not interfere with the roller. Lay off the distance $F H$ beyond F, equal to the required movement of the rod. With B as a center and a radius equal to $B H$, describe a circle. Divide the circle into four quadrants by the lines a b and c d, making angles of 45° with $D H$, and with center B and radius B F strike an arc as shown. That part of the arc included in the lower quadrant, and that part of the outer circle included in the upper quadrant, form two parts of the required outline, and the distance between the centers of the rollers must be equal to $D F$.

Now, for the two side quadrants, it is evident that the distance between any two diametrically opposite points on the outline must be equal. Thus, the distance from $1'$ to $5''$, from $2'$ to $4''$, from $3'$ to $3''$, must all be equal to the distance $D F$. These points are obtained by laying off points on $F H$ and drawing arcs through them, proceeding exactly as with the other cams; *but, in order to have the curves correct, point 1 must be as far from F as 5 is from H; point 2 as far from F as 4 is from H, etc.* The harmonic motion curve fulfils this condition, and was used in this case. The other points of construction should be understood from what has gone before.

1475. A third kind of positive-motion cam consists of a cylinder having a groove on the surface, which imparts motion to the roller in a plane parallel to the axis of the cam.

Suppose that during one-half a revolution of the cylinder, in Fig. 370, the arm is to vibrate to the left and back once, as indicated, and that during the other half revolution it is not to move. Let the motion of the roller be harmonic.

The problem consists in finding the center line of the groove, from which, by striking arcs, the sides against which the roller bears can be determined. To lay out the curves, assume the surface of the cylinder to be unwrapped, or developed, as represented by the figure a b d c, which represents only a little more than one-half of the length of the

surface, in order to save room. Draw a line $M N$ through the center of this strip, of a length equal to the circumference of the cylinder, and divide it into a number of equal parts, as indicated by O_0 O_1, O_1 O_2, etc. Also, draw two lines, $S T$ and $L P$, parallel to $M N$, and at a distance from it equal to one-half the desired stroke of the roller. Now, with

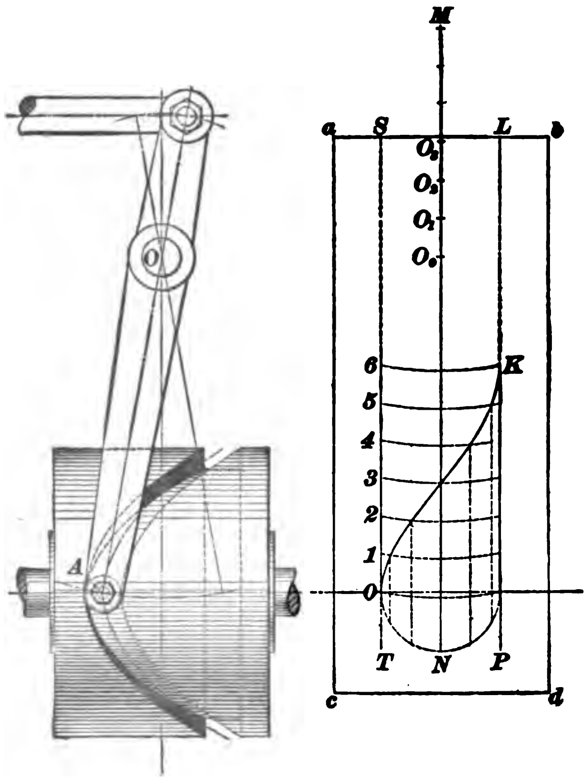


FIG. 870.

a radius $O A$ (left-hand figure), and centers O_0, O_1, O_2, etc., strike arcs *0*, *1*, *2*, *3*, etc., and plot the curve by the aid of these arcs. The line $O K L$ is the center line curve for one-half of the cam, the other half being like the first. The outline can easily be transferred to the cylinder itself.

BELTING.

1476. Belts running over pulleys form a convenient means for transmitting power, but they are not suited to transmit a precise velocity ratio, owing to their tendency to stretch and slip on the pulleys. For driving machinery, however, this freedom to stretch and slip is an advantage, since it prevents shocks that are liable to occur when a machine is thrown suddenly into gear, or when there is a sudden fluctuation in the load.

1477. Velocity Ratio.—Let four pulleys be connected by belt, as shown in Fig. 371. Let D_1 and D_2 be the diameters of the drivers, F_1 and F_2 of the followers, N_1 and N_2

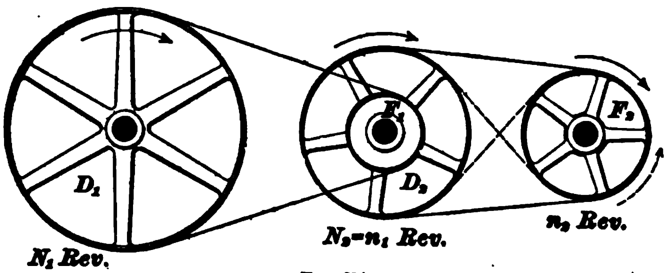


FIG. 371.

the number of revolutions per minute of the drivers, and n_1 and n_2 of the followers. The two middle pulleys are keyed to the same shaft and revolve together.

Consider first the pulleys whose diameters are D_1 and F_1. Assuming that there is no slipping or stretching of the belt, the circumferential speeds of the pulleys will be the same as the velocity of the belts passing over them. Hence, $D_1 \times 3.1416 \times N_1$, the circumferential speed of the first driver, $= F_1 \times 3.1416 \times n_1$, the circumferential speed of the first follower. Canceling 3.1416 from both sides of the equation, we have $D_1 N_1 = F_1 n_1$, or, dividing by D_1 and n_1,

$$\frac{N_1}{n_1} = \frac{F_1}{D_1}. \qquad (136.)$$

That is, *the speeds or numbers of revolutions of two connected pulleys are inversely proportional to their diameters.*

1478. A short way of applying this principle is by the following rule for two pulleys:

Rule.—*Multiply together the number of revolutions and diameter of one pulley, and divide by the given number of revolutions, or given diameter, of the other pulley. The result will be the required diameter or number of revolutions.*

EXAMPLE.—A pulley 30 inches in diameter, making 210 revolutions per minute, drives a second pulley 14 inches in diameter. How many revolutions per minute does the latter pulley make?

SOLUTION.—$30 \times 210 = 6{,}300$, and $6{,}300 \div 14 = 450$ revolutions. Ans.

EXAMPLE.—The driving pulley of a machine is one foot in diameter and must make 750 revolutions in 5 minutes. What size pulley should be used on the driving shaft, if its speed is 143 revolutions per minute?

SOLUTION.—In all examples of this kind the speeds and diameters must be reduced to the same units. 750 rev. in 5 min. $= 750 \div 5 = 150$ rev. per min.; one foot $= 12$ in. Hence, $12 \times 150 = 1{,}800$, and $1{,}800 \div 143 = 12.6$ in., nearly. Ans.

1479. From the equation $D_1 N_1 = F_1 n_1$, derived above, we obtain, by dividing by F_1, $n_1 = \frac{D_1 N_1}{F_1}$. In like manner, taking the other two pulleys in Fig. 371, we obtain $N_2 = \frac{F_2 n_2}{D_2}$. But the two middle pulleys revolve together, so that the values of N_2 and n_1 are equal, and may be placed equal to each other; thus, $\frac{D_1 N_1}{F_1} = \frac{F_2 n_2}{D_2}$, or, multiplying by F_1 and D_2, $N_1 D_1 D_2 = n_2 F_1 F_2$. **(137.)**

1480. That is, *the speed of the first pulley, multiplied by the diameter of each of the drivers, equals the speed of the last pulley, multiplied by the diameter of each follower.*

This formula is in most cases convenient to apply as it stands.

EXAMPLE.—Referring to Fig. 371, let the diameters of the drivers be 32 in., the diameter of the first follower be one foot, and of the second follower 15 inches. If the first shaft has a speed of 60 revolutions per minute, what is the speed of the last shaft?

SOLUTION.—Substituting in formula **137**, $60 \times 32 \times 32 = n_2 \times 12 \times 15$, or dividing by 12×15, $n_2 = \frac{60 \times 32 \times 32}{12 \times 15} = 341\frac{1}{3}$ rev. per min. Ans.

EXAMPLE.—An emery grinder is to be set up to run at 1,200 revolutions per minute. The countershaft (corresponding to the middle shaft in Fig. 371) has pulleys 20 and 8 inches in diameter. If the pulley on the grinder is 6 inches in diameter, what size pulley must be used on the main line shaft, its speed being 180 revolutions per minute?

SOLUTION.—Substituting in formula **137**, $180 \times D_1 \times 20 = 1{,}200 \times 8 \times 6$, or $D_1 = \frac{1{,}200 \times 8 \times 6}{180 \times 20} = 16$ inches. Ans.

1481. When the speeds of the first and last shafts are given, and the diameters of all the pulleys are to be found, the following method is convenient:

Rule.—*Divide the higher speed by the lower. If two pulleys are to be used, this will be the ratio of their diameters.*

If four pulleys are required, find two numbers whose product equals the above quotient. One of these numbers will be the ratio of the diameters of one pair of pulleys, and the other number of the other pair.

EXAMPLE.—It is required to run a machine 1,600 revolutions per minute, the driving shaft making 320 revolutions per minute. What size pulleys are required, (*a*) when two pulleys are used; (*b*) when four pulleys are used?

SOLUTION.—(*a*) $1{,}600 \div 320 = 5$. The two pulleys must, therefore, be in the ratio of 5 to 1, the driving pulley being 5 times as large as the driven pulley, since the latter has the greater speed. We will assume diameters of 30 and 6 inches.

(*b*) $2\frac{1}{2} \times 2 = 5$. One pair of pulleys must be in the ratio of $2\frac{1}{2}$ to 1, and the other pair of 2 to 1. We will assume diameters of 25 and 10 inches for one pair, and of 12 and 6 inches for the other pair.

1482. Direction of Rotation.—It will be noticed, by referring to Fig. 371, that the pulleys are connected by **open belts** where indicated by full lines, and by **crossed belts** where indicated by dotted lines. *Pulleys connected by open belts turn in the same direction, and those connected by crossed belts in opposite directions.*

EXAMPLES FOR PRACTICE.

1. A driving pulley is 54 inches in diameter, and a driven pulley which runs at 112 revolutions per minute is $2\frac{1}{2}$ feet in diameter. What is the speed of the driving shaft? Ans. 62.22 rev. per min.

2. The fly-wheel of an engine running at 180 revolutions per minute is 8 ft. 5 in. in diameter. What should be the diameter of the pulley

which it drives, if the required speed of the latter is 600 revolutions per minute? Ans. $80\frac{5}{16}$ in., nearly.

3. In Fig. 371, let the diameters of the two drivers be 48 and 25 inches, and of the two followers 16 and 12 inches. If the last pulley's shaft rotates 800 times in 5 minutes, what is the speed per minute of the first shaft? Ans. 25.6 rev. per min.

4. If the first pulley in Fig. 371 turns right-handed, and is connected with the second by a crossed belt, and the third with the fourth by an open belt, in what direction would the last pulley turn? In what direction would it turn if two crossed belts were used?

5. A machine is to be belted through a countershaft, so as to run at 1,200 revolutions per minute, the speed of the driving shaft being 120 revolutions per minute. Find three ratios that could be used for each pair of pulleys.

Ans. $\begin{cases} 5 : 1 \text{ and } 2 : 1. \\ 4 : 1 \text{ and } 2\frac{1}{2} : 1. \\ 3\frac{1}{3} : 1 \text{ and } 3 : 1. \end{cases}$

6. An emery grinder is to be set to run at 1,400 revolutions per minute. The countershaft has pulleys 30 and 8 inches in diameter. The pulley on the grinder is 7 inches in diameter. What size pulley should be used on the main line shaft, its speed being 185 revolutions per minute? Ans. $14\frac{1}{2}$ inches.

POWER TRANSMISSION BY BELT.

1483. The Effective Pull.—In Fig. 372, let D and F be two pulleys connected by a belt, D being the driver and F the follower. To avoid undue slipping, the belt must be drawn tight. This will produce a tension in the upper and lower parts which we call T_2 and T_1, respectively.

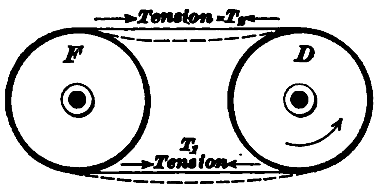


FIG. 372.

Suppose the two pulleys to be stationary and that the belt is put on with a certain tension. Then, T_1 will equal T_2. But if the pulley D should be turned in the direction of the curved arrow it would tend to stretch the lower part of the belt, increasing its tension still more, while the tension of the upper part would be diminished an equal amount. This would go on until the difference of the tensions was sufficient to start pulley F.

This difference (T_1—T_2) is the pull that does the work in transmitting power, and is called the effective pull. In any

case, therefore, the number of foot-pounds of work transmitted by a belt must equal the effective pull of the belt in pounds times the number of feet passed through; or, taking a minute as the unit of time, *the power transmitted in foot-pounds per minute = the effective pull × the velocity in feet per minute.*

1484. To show clearly how the effective pull enters into calculations of power transmissions, two examples will be solved.

EXAMPLE.—The diameter of the driving pulley D is 36 inches. It makes 150 revolutions per minute and carries a belt transmitting 6 horsepower. What is the effective pull of the belt?

SOLUTION.—Velocity of the belt in feet per minute =

$$\frac{150 \times 36 \times 3.1416}{12} = 1,413.72.$$

This, multiplied by the effective pull in pounds must equal the foot-pounds of work done per minute, or $6 \times 33,000$. Hence, letting P = the effective pull, and equating these expressions, we have

$$P \times 1,413.72 = 6 \times 33,000 = 198,000,$$

or $P = \dfrac{198,000}{1,413.72} = 140.06$ lb., nearly.

EXAMPLE.—A pulley 4 feet in diameter is driven at 100 revolutions per minute, and transmits power to another pulley by means of a belt without slip. If the tension on the driving side of the belt is 400 pounds, and on the slack side is 100 pounds, what is the horsepower transmitted?

SOLUTION.—$P = (T_1 - T_2) = 400 - 100 = 300$ lb. Horsepower transmitted $\times$ 33,000 = foot-pounds of work done per minute; or

$$\text{H.P.} \times 33,000 = 300 \times 100 \times 4 \times 3.1416, \text{ whence}$$

$$\text{H. P.} = \frac{300 \times 100 \times 4 \times 3.1416}{33,000} = 11.424. \text{ Ans.}$$

1485. To Determine the Width of Belt.—A belt should be wide enough so that it will bear safely and for a reasonable length of time the greatest tension that will be put upon it. This will be the tension T_1 of the driving side of the belt. As belts are usually laced, or fastened with metallic fasteners, both of which require holes to be punched in the ends, it is customary to use the breaking strength through the lace holes, divided by a suitable factor of safety, as the greatest allowable tension. The average breaking strength for single leather belts, through the lace

holes, is 200 pounds per inch of width. This divided by three, which is a suitable factor of safety for belting, gives $66\frac{2}{3}$ pounds. Thus, in the last example, the tension of the driving side of the belt was assumed to be 400 pounds. Hence, using $66\frac{2}{3}$ pounds as the safe working stress per inch of width, a belt $\frac{400}{66\frac{2}{3}} = 6$ inches wide would be required.

This tension T_1, for any particular case, depends upon three things—viz., the effective pull of the belt, the coefficient of friction between the belt and pulley, and the size of the arc of contact of the belt on the smaller pulley. As the equations involving these quantities are somewhat complicated, Table 33 has been calculated. It will afford a convenient means for finding not only the width of belt for a given horsepower, but the horsepower for a given width as well. In the first column, the arc covered by the belt is stated in degrees, and in the second column in fractional and decimal parts of the circumference covered. The third column gives the greatest allowable values of $(T_1—T_2)$, or the effective pull, per inch of width, for single leather belts having any arc of contact. It was computed by assuming a value for T_1 of $66\frac{2}{3}$ pounds, and a coefficient of friction of .27. This latter has been found by experiment to be a fair value to use for leather belts running over cast iron pulleys, under conditions met with in practice.

TABLE 33.

Arc Covered by Belt.		Allowable Value of Effective Pull, or $(T_1 - T_2)$ per Inch of Width.
Degrees.	Fraction of Circumference.	
90	$\frac{1}{4} = .25$	23.0
$112\frac{1}{2}$	$\frac{5}{16} = .312$	27.4
120	$\frac{1}{3} = .333$	28.8
135	$\frac{3}{8} = .375$	31.3
150	$\frac{5}{12} = .417$	33.8
$157\frac{1}{2}$	$\frac{7}{16} = .437$	34.9
180 or over.	$\frac{1}{2} = .50$	38.1

1486. To use the table in finding the width of a single leather belt required for transmitting a given horsepower, we have the following rule :

Rule.—*Compute the effective pull of the belt. Divide the result by the suitable effective pull per inch of width, as given in Table 33; the quotient will be the width of belt required, in inches.*

EXAMPLE.—What width of single belt is needed to transmit 20 horsepower with contact on the small pulley of $\frac{3}{8}$ of the circumference and a speed of 1,500 feet per minute?

SOLUTION.—First finding the effective pull, $P \times 1{,}500 = 20 \times 33{,}000$,

$$P = \frac{20 \times 33{,}000}{1{,}500} = 440.$$

Hence, $440 \div (T_1 - T_2)$, from the table, $= \frac{440}{31.8} = 14$ inches, nearly. A 14-inch belt would be used. Ans.

1487. To Determine the Horsepower that a Belt Will Transmit.—The process for a single belt must evidently be just the reverse of the preceding. It is as follows:

Rule.—*Multiply together a suitable value for the effective pull, taken from Table 33, the width of the belt in inches, and its velocity in feet per minute. The result divided by 33,000 will be the horsepower that the belt will transmit.*

EXAMPLE.—What horsepower will a one-inch belt transmit with a speed of 900 feet per minute and an arc of contact of 180°?

SOLUTION.—$T_1 - T_2$, from the table, $= 38.1$. $38.1 \times 1 \times 900 = 34{,}290$, which, divided by 33,000, gives 1.04 horsepower, nearly. Ans.

1488. General Rule for Belting.—From the last example, we see that *a single belt traveling 900 feet a minute will transmit one horsepower per inch of width when the arc of contact on the smaller pulley does not vary much from 180°.* This is used by many engineers as a general rule for belting, to be applied to all cases.

The following three formulas express the operations that would be performed in applying this rule:

Let H = horsepower to be transmitted;
W = width of belt in inches;
S = belt speed in feet per minute.

Then, $$H = \frac{WS}{900}. \quad \textbf{(138.)}$$

$$W = \frac{900H}{S}. \quad \textbf{(139.)}$$

$$S = \frac{900H}{W}. \quad \textbf{(140.)}$$

EXAMPLE.—Two pulleys, 48 inches in diameter, are to be connected by a single belt, and make 200 revolutions per minute. If 40 horsepower is to be transmitted, what must be the width of belt?

SOLUTION.—The belt speed $= \frac{200 \times 48 \times 3.1416}{12} = 2,513$ feet per minute, about. Applying formula **139**, $W = \frac{900 \times 40}{2,513} = 14.3$ inches. Ans.

A 14-inch belt might safely be used, since the rule gives a liberal width when the pulleys are of equal size.

EXAMPLE.—What size pulleys should be used for a 4-inch belt, which is to connect two shafts running at 400 revolutions per minute and transmit 14 horsepower? Both pulleys are of the same size.

SOLUTION.—By formula **140**, $S = \frac{900 \times 14}{4} = 3,150$ feet per minute. Since this speed = the circumference of the required pulley in feet $\times$ 400, we have circumference of pulley $= \frac{3,150 \times 12}{400} = 94.5$ inches; diameter $= \frac{94.5}{3.1416} = 30$ inches. Ans.

1489. Double Belts.—Double belts are made of two single belts cemented and riveted together their whole length, and are used where much power is to be transmitted. As the formulas for single belts are based upon the strength through the rivet holes, a double belt, which is twice as thick, should be able to transmit twice as much power as a single belt, and, in fact, more than this, where, as is quite common, the ends of the belt are glued instead of being laced.

Where double belts are used upon small pulleys, however, the contact with the pulley face is less perfect than it would be if a single belt were used, owing to the greater rigidity of the former. More work is also required to bend the belt as it runs over the pulley than in the case of the thinner and more pliable belt, and the centrifugal force tending to throw the belt from the pulley also increases with the thickness. Moreover, in practice, it is seldom that a double belt is put on with twice the tension of a single belt. For these reasons, the width of a double belt required to transmit a given horsepower is generally assumed to be seven-tenths the width of a single belt to transmit the same power. Upon this basis, formulas **138, 139,** and **140** become, for double belts,

$$H = \frac{WS}{630}. \qquad \textbf{(141.)}$$

$$W = \frac{630\,H}{S}. \qquad \textbf{(142.)}$$

$$S = \frac{630\,H}{W}. \qquad \textbf{(143.)}$$

EXAMPLES FOR PRACTICE.

1. If the effective pull on a belt per inch of width is 50 pounds, and the belt passes over a pulley 36 inches in diameter, which makes 160 revolutions per minute, how wide should the belt be to transmit 12 horsepower? Ans. $5\frac{1}{2}$ inches.

2. What width of single belting should be used to transmit 5 horsepower, when the belt speed is 2,000 feet per minute, and the arc of contact on the smaller pulley is 90°? Ans. $3\frac{1}{2}$ + inches.

3. Using the general rule, find the horsepower that a 16-inch single belt will transmit, the belt speed being 1,000 feet per minute. Ans. 17.8 H. P.

4. Calculating from Table 33, how much power could the above belt be depended upon to transmit if the arc of contact on the smaller pulley is $\frac{1}{4}$ of the circumference? Ans. 14 H. P., nearly.

5. Required the diameter of pulleys necessary to enable a 10-inch belt to transmit 9 horsepower at 125 revolutions per minute, both pulleys being of the same size. Ans. 2 ft., nearly.

6. How much power would the belt in example 3 transmit, if the belt were double? Ans. 25.4 H. P., nearly.

SPEED CONES.

1490. Speed cones are used for varying the velocity of a shaft or other rotating piece driven by a belt. Their method of operation will be clearly seen in Figs. 373 to 376.

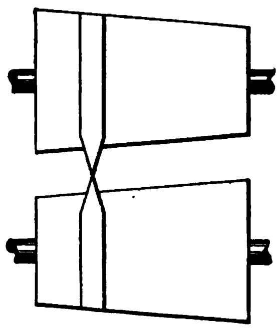


FIG. 373.

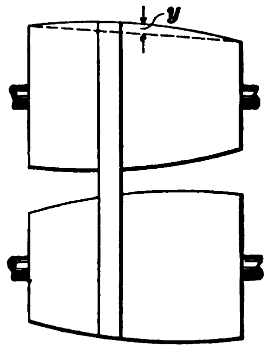


FIG. 374.

Figs. 373 and 374 show continuous cones and conoids, respectively, the former being suitable for crossed, and the latter for open belts, where the speed of the driver shaft can be raised gradually by shifting the belt. Figs. 375 and 376 show sets of stepped pulleys. As flat belts tend to climb

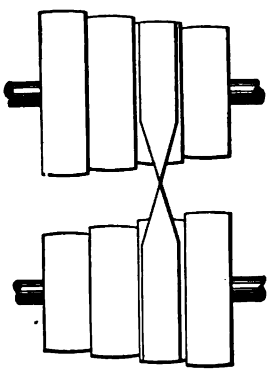


FIG. 375.

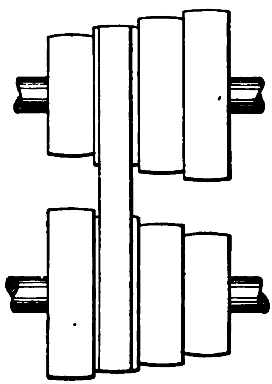


FIG. 376.

a conical pulley, continuous cones or conoids require special provision for keeping the belt in place. For this reason the stepped cones are generally used. Whenever

possible, it is desirable to have both pulleys alike, so that they can be cast from one pattern, and in what follows it will be assumed that this is to be done.

1491. Continuous Cones for Crossed Belts.—Let A and B, in Fig. 377, represent two speed cones of the same size, having the diameters of the large and small ends equal to D and d, respectively. A is the driver, and revolves at a constant number of revolutions N, while the speed of B is n_1 or n_2, according as the belt runs at the small or large end.

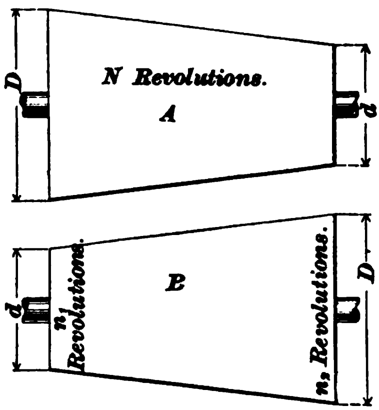


FIG. 377.

It is assumed that pulley B is to have a range of speeds between n_1 and n_2 revolutions, n_1 being the greater.

Since A and B are to be of the same size, it will be found that a certain relation must exist between N and n_1 and n_2 as follows:

From formula **136**,

$$ND = n_1 d, \text{ or } D = \frac{n_1 d}{N}, \qquad \textbf{(144.)}$$

$$\text{and } Nd = n_2 D, \text{ or } D = \frac{Nd}{n_2}. \qquad \textbf{(145.)}$$

Equating **144** and **145**, $\frac{n_1 d}{N} = \frac{Nd}{n_2}$, or

$$N = \sqrt{n_1 n_2}. \qquad \textbf{(146.)}$$

That is, N must equal the square root of the product of n_1 and n_2.

1492. Having determined N, the relation between E and d can be found. From **144** and **145** we have:

$$N = \frac{n_1 d}{D} \text{ and } N = \frac{n_2 D}{d}.$$

Equating, $\frac{n_1 d}{D} = \frac{n_2 D}{d}$, or $n_1 d^2 = n_2 D^2$.

Hence, $$D = d\sqrt{\frac{n_1}{n_2}}. \qquad \textbf{(147.)}$$

That is, the large diameter must equal the small diameter, multiplied by the square root of the quotient of n_1 *divided by* n_2.

1493. Speed cones, to be properly designed, should have their diameters at different points, so proportioned that the belt will always have the same length, when tightly drawn, whatever its position. Let d and D' represent the diameters of two pulleys, connected by a crossed belt, whose centers are O and C, respectively. (See Fig. 378.) The length of the belt is $2\,A\,B+$the length of the arcs from F to A and B to E. These arcs subtend equal angles,

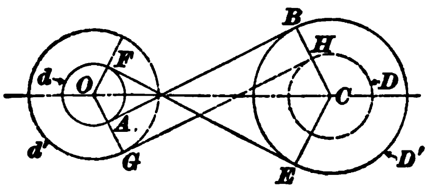


Fig. 378.

and supposing each to contain x degrees, the length of the belt $= 2\,A\,B + 3.1416\,(d + D')\frac{x}{360}$, there being 360 degrees in a circle. Now, draw a line $G\,H$ parallel to $A\,B$, and, about O and C as centers, describe dotted circles whose diameters are d' and D tangent to $G\,H$, representing two other pulleys. Then, for the length of the belt, we have $2\,G\,H\ (=2\,A\,B) + 3.1416\ (d' + D)\frac{x}{360}$. But $(d' + D) = (d + D')$, since what was taken from D' to make D was added to d to make d'. Hence, the length of the belt in each case is the same, and we have the rule that *for crossed belts the sum of the corresponding diameters of two speed cones should be the same at all points.*

From this it follows that if two cones not exactly alike are to be driven by cross belt, it is only necessary to see that the sum of the diameters remains the same.

EXAMPLE.—Two continuous speed cones are to be designed to give a range of speed between 100 and 700 revolutions per minute. They are both to be alike in all respects. What must be the speed of the driving shaft, and the large diameter of the cones, assuming the small diameter to be 4 inches?

SOLUTION.—From formula **146**, $N = \sqrt{n_1 n_2} = \sqrt{100 \times 700} = \sqrt{70,000} = 264.57 +$ revolutions per minute. Ans.

From formula **147**, $D = d\sqrt{\frac{n_1}{n_2}} = 4\sqrt{\frac{700}{100}} = 4 \times 2.645 = 10.58$ inches. Ans.

1494. When an open belt is used, the values of N, D, and d are calculated as above, but for the other diameters a different rule is required. In Fig. 379, which is similar to Fig. 378, the pulleys are connected by an open belt. The figure is drawn so that $D + d = D' + d'$, the circles D' and d' being made equal to represent the middle sections of two cones. It is evident that the belt over D and d is longer than the one over D' and d'. Hence, we see that the middle sections of speed cones for open belts must be larger proportionately than for crossed belts. This is indicated by y in Fig. 374. Calling this middle diameter M, and letting C be the distance in inches between the centers of the two shafts, upon which the cones are placed, it can be shown that

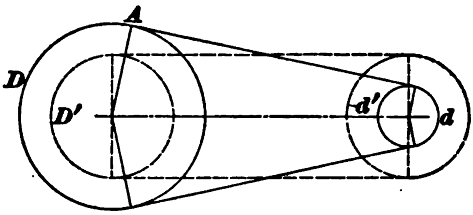


FIG. 379.

$$M = \frac{D + d}{2} + \frac{.08\,(D - d)^2}{C}. \qquad (148.)$$

As the proof is a long one it will not be given. Having thus determined the end and middle diameters, the curve of the conoid may be taken as the arc of a circle passing through the extremities of the three diameters.

EXAMPLE.—What should be the middle diameter for the speed cone of the last example, having end diameters of 4 and 10.58 inches, when an open belt is to be used? The distance between centers is 50 inches.

SOLUTION.—From formula **148**,

$$M = \frac{10.58 + 4}{2} + \frac{.08\,(10.58 - 4)^2}{50}$$
$$= 7.29 + .07 = 7.36 \text{ inches. Ans.}$$

1495. Stepped Pulleys.—When stepped pulleys, or cone pulleys, as they are more commonly called, are to be used, continuous cones should first be laid out as described, like *A B C D*, in Fig. 380. Then draw as many diameters, at equal distances apart, as there are to be steps in the cone plus 1, as *A D*, *E F*, etc. These will serve as center lines for the different steps, which are to be drawn through the intersections of the above diameters, with the outside lines *A B* and *D C*, in the manner indicated.

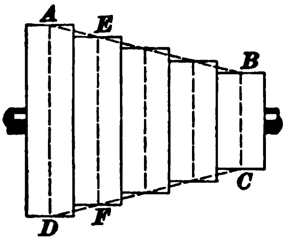


FIG. 380.

1496. It should be noted that, when speaking of a cone pulley as having a certain number of steps, the number of steps is one less than the number of pulleys on the cone. Thus, the cone, in Fig. 380, is a 4-step cone, and has five pulleys. Consequently, if a cone pulley (or cone) is spoken of as having five steps, there are six pulleys and six changes of speed.

If the distance between the axes of the pulleys to be connected by open belt be great, or if, as is sometimes the case, one of the axes be adjustable (the proper tension on the belt being obtained by the weight of the pulley on the adjustable axis), the diameters can be calculated as though the belt were crossed. Otherwise, when designed for open belts they should be laid out as before described.

EXAMPLES FOR PRACTICE.

1. Two continuous speed cones are required to give a range of speed between 100 and 600 revolutions per minute. Assuming the large diameters of the cones to be 14 inches, (*a*) what must be the small diameters, and (*b*) the speed of the driving shaft? Both cones are to be alike.

Ans. { (*a*) 5.71 inches. (*b*) 244.95 rev.

2. In the above example, if the speed of the driving shaft were 260 revolutions per minute, and the slowest speed of the driven cone 140 revolutions, (*a*) what would be the greatest speed of the driven cone? (*b*) What would be the ratio of the large and small diameters of the cones?

Ans. { (*a*) 482.86 rev. (*b*) 1.86 : 1.

8. What should be the middle diameter for the speed cones of example 1, when an open belt is used, the distance between centers being 80 inches? Ans. 10.04 inches.

THE CARE AND USE OF BELTING.

1497. Belts most commonly used are of leather, single and double. Canvas belts covered with rubber are sometimes used, especially in damp places, where the moisture would ruin the leather.

Leather belts are generally run with the hair, or grain, side next the pulley. This side is harder and more liable to crack than the flesh side. By running it on the inside the tendency is to cramp or compress it as it passes over the pulley, while, if it ran on the outside, the tendency would be for it to stretch and crack. Moreover, as the flesh side is the stronger side, the life of the belt will be longer if the wear comes upon the weaker or grain side.

1498. The lower side of a belt should be the driving side, the slack side running from the top of the driving pulley. The sag of the belt will then cause it to encompass a greater length of the circumference of both pulleys, as illustrated by the dotted lines in Fig. 372. Long belts, running in any direction other than the vertical, work better than short ones, as their weight holds them more firmly to their work.

It is bad practice to use rosin to prevent slipping. It gums the belt, causes it to crack, and prevents slipping for only a short time. If a belt properly cared for persists in slipping, a wider belt or larger pulleys should be used, the latter to increase the belt speed. Belts, to be kept soft and pliable, should be oiled with castor or neatsfoot oil. Mineral oils are not good for the purpose.

1499. Tightening or guide pulleys, whenever used to increase the length of contact between the belt and pulley or to tighten a belt, should be placed on the slack side, if possible. Thus placed, the extra friction of the guide pulley bearings and the wear and tear of the belt that would result from the greater tension of the driving side are avoided.

1500. Guiding Belts.—When belts are to be shifted from one pulley to another, or must be guided to prevent running off the pulley, the fork, or other device used for guiding, should be close to the driven pulley, and so placed as to guide the advancing side of the belt.

This principle is sometimes made use of where pulleys have flanges to keep the belt on the pulleys. Where constructed with straight flanges, as in Fig. 381, if the belt has any inclination to run on one side, its tendency is to crowd up against the flange as shown at *a*, *a*. When constructed as in Fig. 823,

FIG. 381.

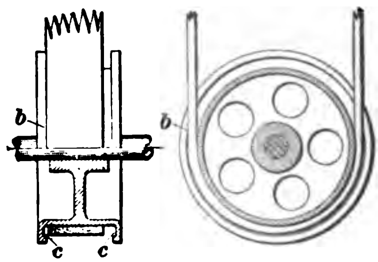


FIG. 382.

however, with the flanges grooved as at *c*, *c*, the advancing side of the belt will be guided at *b*, just as it reaches the pulley, by contact with the thick portion of the flange, and during the rest of the way will not touch the flange at all.

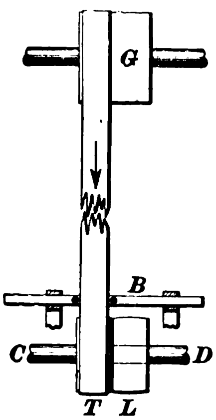


FIG. 383.

1501. In Fig. 383, is shown the arrangement for a belt shipper. *G* is the driving pulley and *T* and *L* are tight and loose pulleys on the driven shaft *C D*. *B* is the shipper, and can be moved parallel with *C D*. The acting surface, or *face*, of *G* is made straight to allow the belt to shift readily, and the faces of *T* and *L* are crowned, so that the belt will not tend to run off.

1502. The Climbing of Belts.—In Fig. 384, suppose the shafts *a*, *a* to be parallel, and the pulley *A* to be cone-shaped. The right-hand side of the belt will be pulled ahead more rapidly than the left-hand side,

because of the greater diameter and consequently greater speed of that part of the pulley. The belt will, therefore, leave its normal line at c, and climb to the "high" side of the pulley. This tendency is taken advantage of by crowning pulleys in the middle. Each side of the belt then tends to move towards the middle of the pulley; that is, the tendency is for the belt to stay on the pulley.

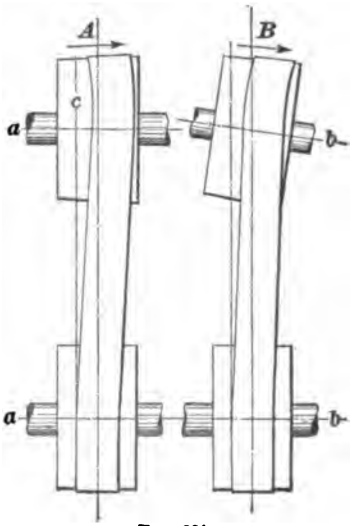


FIG. 384.

Suppose, on the other hand, the shafts b, b not to be parallel, the right-hand ends being nearer together. The belt will in that case pass spirally on the pulley B towards the "low" side.

1503. Belt Fastenings.—There are many good methods of fastening the ends of belts together, but lacing is generally used, as it is flexible like the belt itself, and runs noiselessly over the pulleys. The ends to be laced should be cut squarely across and the holes in each end for the lacings should be exactly opposite each other when the ends are brought together. Very narrow belts, or belts having only a small amount of power to transmit, usually have only one row of holes punched in each end, as in Fig. 385. A is the outside of the belt, and B the side running next the pulley. To lace, the lacing should be drawn half way through one of the middle holes, from the under side, as at *1*. The upper end should then be passed through *2*, under the belt and up

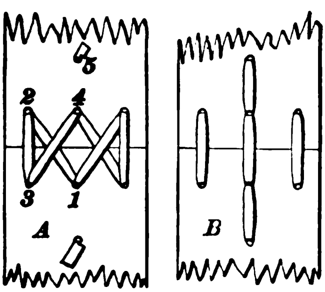


FIG. 385.

through *3*, back again through *2* and *3*, through *4* and up through *5*, where an incision is made in one side of the lacing, forming a barb that will prevent the end from pulling through. The other side of the belt is laced with the other end, it first passing up through *4*. Unless the belt is very narrow, the lacing of both sides should be carried on at once.

1504. Fig. 386 shows a method of lacing where double lace holes are used, *B* being the side to run next the pulley. The lacing for the left side is begun at *1*, and continues through *2*, *3*, *4*, *5*, *6*, *7*, *4*, *5*, etc. A 6-inch belt should have seven holes, four in the row nearest the end, and a 10-inch belt, nine holes. The edge of the holes should not be nearer than $\frac{3}{4}$ of an inch from the sides; and the holes should not be nearer than $\frac{7}{8}$ of an inch from the ends of the belt. The second row should be at least $1\frac{3}{4}$ inches from the end.

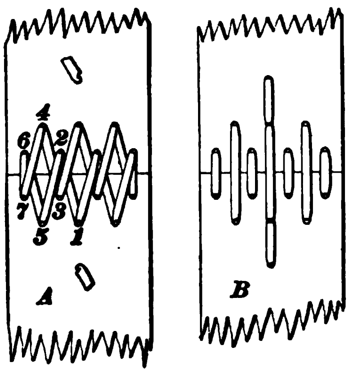


FIG. 386.

Another method is to begin the lacing at one side instead of in the middle. This method will give the rows of lacing on the under side of the belt the same thickness all the way across.

BELTS TO CONNECT NON-PARALLEL SHAFTS—GUIDE PULLEYS.

1505. It very frequently happens that one shaft must drive another at an angle with it. Sometimes this involves the use of guide pulleys, and occasionally guide pulleys must be used to connect parallel shafts, where the shafts are near together, or there is some obstruction in the way. In all these cases, however, we have the general principle that *the point at which the center of the belt is delivered from each pulley must lie in the middle plane of the other pulley.*

The middle plane of a pulley, it will be understood, is a plane through the center of the pulley, perpendicular to its axis.

Unless the shafting is to turn backwards at times, it is immaterial in what direction a belt *leaves* a pulley; but it must always be *delivered* into the plane of the pulley towards which it is running. If it is necessary for a belt to run backwards as well as forwards, it must *leave* in the plane of the pulley, also. This principle applies to the guide pulleys as well as to the main pulleys.

1506. Shafts at Right Angles.—One of the most frequent cases met with is that of two shafts at right angles, but not intersecting, and the common method of connecting them is by means of a **quarter-turn belt,** shown in Fig. 387. Here, *D* is the driver, revolving in the direction shown, and *F* is the follower. The point at which the belt is delivered from the pulley *D* lies in the middle plane of the pulley *F*, that is, in the line *b b*; also, the point at which the belt is delivered from the pulley *F* lies in the middle plane of the pulley *D*, or in the line *a a*. Thus arranged, the pulleys cannot run backwards, because *d*, the point of delivery of *F*, is not in the middle plane *a a*, and *e* is not in the middle plane *b b*.

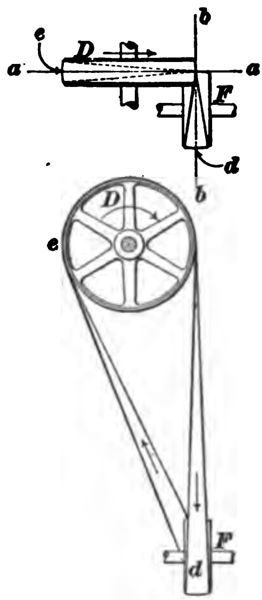


FIG. 387.

1507. The following simple method of locating the pulleys for a quarter-turn belt may be used in practice: Let *D*, Fig. 387, be the driving pulley, and *F* the follower or driven pulley, which drives a machine. Locate the pulley *D* and the machine so that the pulleys *D* and *F* will be as near the correct position as can be judged by the eye. Using a plumb-bob, drop a plumb-line from the center of the right-hand side of the pulley *D*, and move the machine until the center of the *back* side of the pulley *F* touches the

plumb-line. In case it should not be convenient to move the machine, shift the pulleys instead. If it be desired to run the belt in the opposite direction to that indicated by the arrow in Fig. 387, shift the machine carrying F to the left until the center of the *front* side of the pulley F touches a plumb line dropped from the center of the left-hand side of the pulley D; that is, from the point e.

1508. There is the objection to a quarter-turn belt that, when the angle at which the belt is drawn off the pulleys is large, the belt is strained, especially at the edges, and it does not hug the pulleys well. Small pulleys placed quite a distance apart, with narrow belts, give the best results, from which it follows that quarter-turn belts, like the foregoing, are not well suited to transmit much power. Fig. 388 shows how the arrangement can be improved by placing a guide pulley against the loose side of the belt. The driver D revolves in a left-handed direction, making $a\ b$ the driving or tight side of the belt. To determine the position of the guide pulley, select some point in the line $a\ b$, as d; draw lines $c\ d$ and $e\ d$; the middle plane of the guide pulley should then pass through the two lines. Looked at from a direction at right angles to pulley F, line $c\ d$ coincides with $a\ b$; looking at right angles to pulley D, line $e\ d$ coincides with $a\ b$.

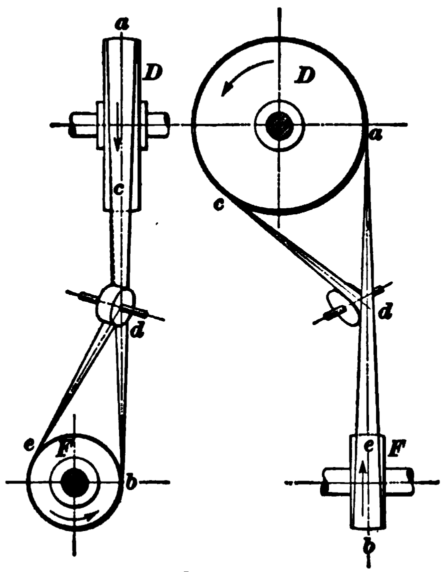


FIG. 388.

1509. A third form of belting for connecting two shafts at right angles consists of pulleys placed as in Fig. 389.

In general, it is to be preferred to the quarter-turn belts. D is the driver. The belt passes around the loose pulley L, and up again around a loose pulley on the driving shaft back

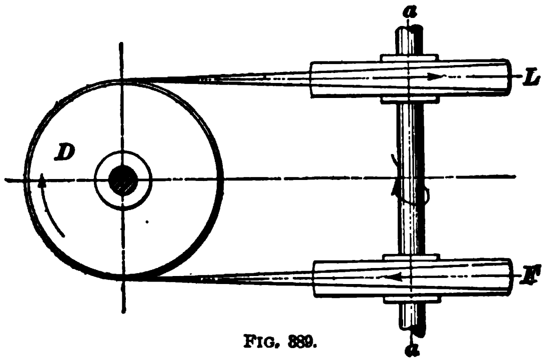


FIG. 889.

of D. It then goes down, around F, which is fast upon the shaft a a, and finally up again and around D. Since the loose pulleys revolve in a direction opposite to that of their shafts,

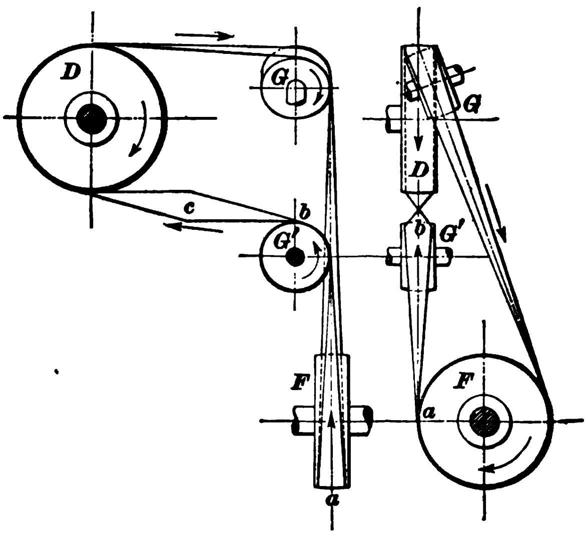


FIG. 890.

their hubs should be long. The two pulleys on each shaft must be of the same size. It is evident that either F or D can be the driver, and run in either direction.

It is to be observed that while a quarter-turn belt can be used with the shafts at an angle other than a right angle, the last arrangement cannot.

1510. In Fig. 390 is shown a method of connecting the shafts when it is not possible to put the follower F directly under D. The guide pulleys G, G' must be so placed that the belt will lead correctly from the point a into the middle plane of the guide pulley G', from b into the middle plane of D, and so on around. By twisting the belt at c, the same side will come in contact with all the pulleys; this is a desirable arrangement.

1511. We now come to a case that is different from the preceding, in that the shafts A and B, Fig. 391, would

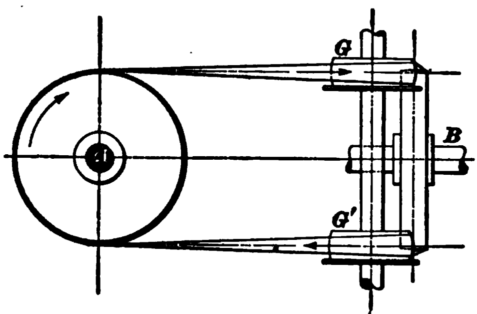


FIG. 391.

intersect, if long enough, as, for example, where shafting running on two sides of a room is to be connected. Guide pulleys, like those in the figure, termed **mule pulleys,** are used. As their planes are horizontal, means must be provided to prevent the belt from running off at the bottom. Sometimes this is done by simply crowning the pulley, and sometimes by putting flanges on the lower sides.

1512. Other Examples of Belt Transmission.—Guide pulleys are sometimes used to lengthen the belt between two shafts which are too close together to be

connected directly, or it may be that it is not possible to get two pulleys in the same plane. Fig. 392 shows an arrangement of this kind. The diameter of the guide pulleys should equal the distance between the planes of *D* and *F*. With the guide pulleys arranged as shown, the belt will run in both directions. It is more convenient, however, to place them on one shaft. In that case their axes would be on the line *O O'*. *G'* would have to be in the line *C K*, and *G* in the line

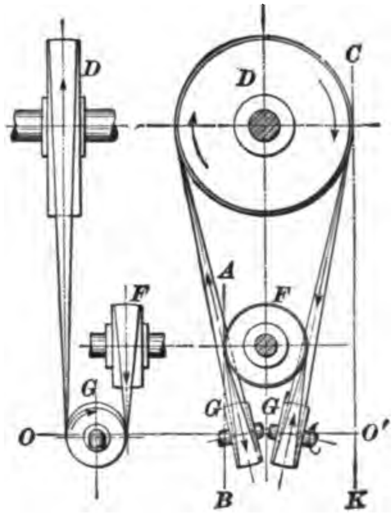


FIG. 392.

A B. Then the belt would be delivered from *D* into the middle plane of *G'*, and from *G* into the middle plane of *F*. The belt would run in only one direction, however.

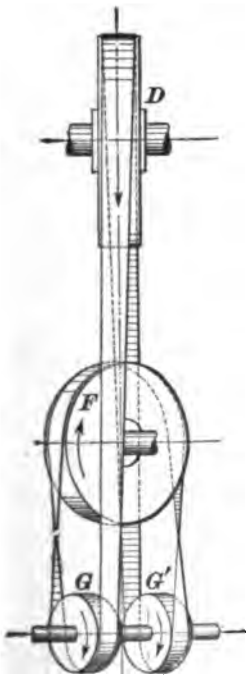


FIG. 393.

1513. A device for connecting two horizontal shafts making an angle with each other is given in Fig. 393. It can be used where a quarter-turn belt would not work successfully. The guide pulleys turn in the same direction, which is a convenience, because they can then be mounted on one shaft, turning in bearings at the ends, and the belt will run in either direction.

WHEELS IN TRAINS.

1514. The principles relating to the velocity ratio of pulleys connected by belting apply to any series of wheels arranged in a train. Where gears are used, however, the relative proportions of the wheels may be stated in terms of the numbers of teeth, instead of in terms of their diameters, if desired. Moreover, in any one train, it is only necessary that the proportions of each pair of wheels be stated in the same terms; different pairs may be given in different terms.

For example, take a train of four axes or shafts and six wheels, as in Fig. 394. Four of the wheels are gear-wheels,

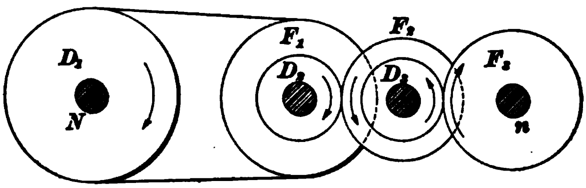


FIG. 394.

represented by their pitch lines, and two are pulleys connected by a belt. Let D_1, D_2, etc., denote the drivers, and F_1, F_2, etc., the followers, N the number of revolutions of D_1, and n the number of revolutions of F_3.

Suppose D_1 to be 40 inches in diameter and F_1 35 inches, D_2 to have 54 teeth, and F_2 60 teeth; D_3 to be 1 foot in diameter, and F_3 2 feet. What is the speed of F_3 if $N = 100$?

Placing the product of the drivers $\times$ the speed of the first shaft = the product of the followers $\times$ speed of the last shaft, $100 \times 40 \times 54 \times 1 = n \times 35 \times 60 \times 2$. Hence,

$$n = \frac{100 \times 40 \times 54 \times 1}{35 \times 60 \times 2} = 51\tfrac{3}{7} \text{ rev. per min.} \quad \text{Ans.}$$

It is evident that the arrangement of the drivers and followers is indifferent, and that they may be interchanged among themselves. It should be noticed, however, that if the diameter of one driver be given in inches, the diameter of its follower must be given in inches, also, etc.

1515. Direction of Rotation.—Axes connected by gear wheels rotate in opposite directions, as though con-

nected by a crossed belt. Hence, *in a train consisting solely of gear wheels, if the number of axes be odd, the first and last wheels will revolve in the same direction; if the number be even, they will revolve in the opposite direction.*

It is evident that a pinion working in an internal gear is an exception to this rule. It will turn in the same direction as the gear.

1516. Idle Wheels.—In the train, in Fig. 394, shaft No. 3 carried two gears, one being driven by gear D_2 and the other driving F_3. Sometimes, however, only one intermediate gear is used, serving both as a driver and a follower. Such a wheel is called an **idle wheel,** or *idler, and while it affects the relative direction of rotation of the wheels it is placed between, it does not affect their velocity ratio.*

The following examples will illustrate this, as well as show a few ways in which idle wheels are used. One method of arranging the change gears on the end of an engine lathe for changing the speed of the lead screw, which is used to feed the tool in screw cutting, is shown in Fig. 395. D_1, the driver, receives motion from the lathe spindle, and F_2, the follower, is on the lead screw. The middle wheel, which is an idle wheel, acts both as a driver and follower, or as D_2 and F_1.

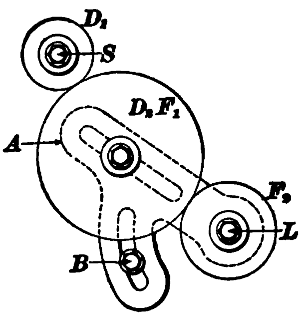


FIG. 395.

Letting N_1, represent the number of revolutions of D_1 and n_2 of F_2, we have, from formula **137,** $N_1 \times D_1 \times D_2 = n_2 \times F_1 \times F_2$. But, as D_2 and F_1 represent the same wheel, they have the same value, and $n_2 = \frac{N_1 D_1}{F_2}$. That is, the speed of F_2 is exactly the same as though no idle wheel was used.

1517. To change the speed of the lead screw, a different size wheel is put on in place of D_1, or F_2, or both. The idle wheel turns on a stud clamped in the slot in the arm A.

This stud can be moved in the slot to accommodate the different sizes of wheels on L, and to bring the wheel in contact with the gear on S, the arm is swung about L until in the right position, when it is clamped to the frame by the bolt B.

The way in which an idle wheel changes the direction of rotation is well shown in Fig. 396, which is a reversing mechanism, sometimes placed in the head stock of a lathe for reversing the feed. Here the idler I is in contact with D and F, making three axes, an odd number, so that D and F revolve in the same direction. Wheels I and I', however, are both pinioned on the plate P, which can swing about the axis of F, and are always in contact. Moreover, as plate P swings about its center, I must necessarily remain in contact with F. If the handle H be moved to the right, the plate will be revolved to the right, by means of the pin working in the slot in the lever L. The two idlers will take the dotted positions shown, and D will drive F through both of them. The number of axes will then be even, so that D and F will revolve in the opposite direction.

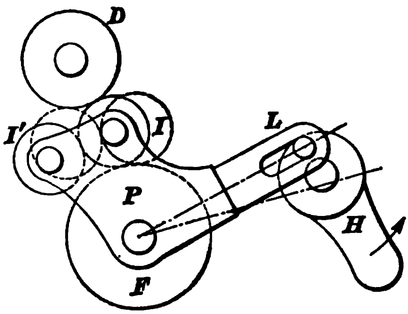


FIG. 396.

1518. Fig. 397 shows still another device in which an idle wheel is used, known as a **knee-joint.** If shaft A is fixed and shaft C is compelled by some arrangement, not shown, to move along a path, as $m\ n$, and at the same time it is desired to drive C from A, the connection can be made as

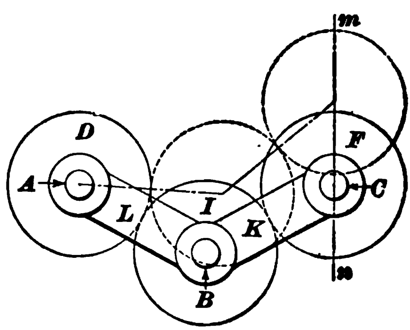


FIG. 397.

shown. Wheels D and F are fast to shafts A and C, and the idler I is supported by the links L and K, the ends of the links being loose upon shafts A, B, and C. Link L serves to keep D and I at the right distance from each other, and K serves the same purpose for I and F. The dotted lines show another position of the wheels. Among other applications of the knee-joint is one used to drive the rollers which deliver the ingot in a rolling mill to the rolls. These latter are ranged above one another in pairs, and the rollers have bearings in an iron frame, called the roll table, that is raised or lowered by hydraulic pressure, thus bringing the ingot in line with any pair of rollers. A familiar example of this train is seen in routing machines, pulleys and belts being used, however, in place of gears. The knee-joint is in reality an example of an epicyclic train, a description of which is to follow.

ENGINE LATHE TRAIN.

1519. Some of the best examples of wheels in trains are to be found in engine lathes. Fig. 398 represents the head-stock of an engine lathe, the spindle S turning in the bearings, as shown, and having a face-plate P, and a "center" on the right end for placing the work. The lead screw L, used in screw-cutting, is connected with the spindle by the train of gears on the left, which will be described later. The **back gears** F_1 and D_2, on the shaft $m\ n$, have been drawn above the spindle for convenience of illustration, instead of back of it, where they are really placed.

1520. Back Gear Train.—It is important to keep the cutting speed within limits that the tool will safely stand. For turning work of different diameters and material, therefore, the spindle must be driven at different speeds. This is accomplished by means of the cone pulley marked C, driven by a similar one on the countershaft by means of a belt, and by the back gears.

The gear F_2 is fast to the spindle and always turns with it. The cone C, however, is loose on the spindle, but can be made to turn with it by means of a lug, or catch, operated

by a nut under the rim of F_2. When the nut is moved out from the center, the lug engages with a slot on the large end of the cone. The cone will then revolve with the spindle, and as many changes of speed may be had as there

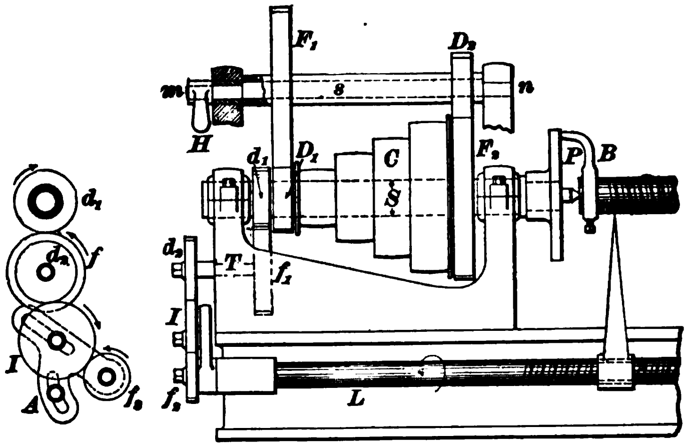


FIG. 398.

are pulleys on the cone. As ordinarily constructed, however, the cone alone does not give a range of speed great enough to include all classes of work, nor is the belt power sufficient for the larger work and heavier cuts. It makes the lathe more compact and satisfactory to construct the cone for the higher speeds and lighter work, and to obtain the speeds for the heavier work by means of back gears. Referring to the figure, it will be seen that the back gears are connected by the sleeve s, and so turn together, F_2 meshing with D_2, and D_1, which is loose upon the spindle but fastened to the cone, with F_1. To get the slower speeds, the nut mentioned before is moved in towards the center of F_2, disengaging the gear from the cone, which latter is now free to turn on the spindle. Hence, if the back gears are in mesh with the gears on the spindle, the belt will drive the spindle at a slower speed through the cone and the train $D_1 F_1 D_2 F_2$.

The back gears cannot remain in gear when the cone and gear F_2 are connected; otherwise, the lathe would not start,

or teeth would be broken out of the wheels. To provide for throwing them in and out of gear, as required, the rod on which the back gears and sleeve *s* revolve is provided with eccentric ends at *m* and *n*, fitting in bearings in the frame. By turning the rod part way around by means of the handle *H* the gears can be thrown either in or out of gear.

1521. A lathe is spoken of as running **back geared** when the back gears are in, and as being in single gear when they are out of gear. This same arrangement, or a modification of it, is used on upright drills, boring mills, milling machines, and other machine tools.

1522. Screw Cutting.—One of the chief uses of engine lathes is for screw-cutting. In Fig. 398, the screw-cutting mechanism is driven from d_1, which is fast to the spindle. This connects with the lead screw *L*, through f_1, d_2, the idler *I*, and f_2. To cut a screw-thread, the work is placed between the centers of the lathe and made to turn with the face plate by the dog *B*, clamped to the end of the work. The spindle runs towards the operator, or in a right-hand direction, as looked at from the outer end of the head-stock, and the carriage, tool post, and tool, all of which are here represented by the pointer, are moved by the lead screw, on shears parallel with the axis of the spindle.

When cutting a right-hand thread, the tool must move from right to left, and from left to right when cutting a left-hand thread. Hence, to cut a right-hand screw with the gearing as arranged in the figure, a left-hand lead screw must be used. To cut a left-hand screw another idle wheel should be inserted in the connecting train, arrangements for which are usually provided. To cut screws of different pitch a set of gears is always furnished, any of which may be placed on the stud *T*, or the end of the lead screw. The method of adjustment to accommodate the different size gears by means of the vibrating slotted arm *A* has already been explained.

1523. Suppose the lead screw to have six threads to the inch, or a pitch of $\frac{1}{6}$ inch, and let it be required to

cut a screw of six threads per inch. If the gearing is such that the lead screw turns once while the spindle makes one turn, it is evident that the tool will cut six threads to the inch. If it is required, to cut more than six threads to the inch, however, the lead screw must turn slower than the spindle; if less than six threads, it must turn faster than the spindle.

Now, the ratio between the speed of the spindle and the speed at which the lead screw must turn for cutting a certain number of threads per inch is simply the ratio between the number of turns made by the spindle and the number of turns made by the lead screw while the tool moves along one inch. This is evidently equal to the ratio of the number of threads per inch to be cut to the number of threads per inch on the lead screw. Thus, if 10 threads per inch are to be cut, and the lead screw has six threads per inch, the spindle must turn 10 times while the lead screw turns six times; if $4\frac{1}{2}$ threads are to be cut, it must turn $4\frac{1}{2}$ times to six turns of the lead screw.

The problem, then, is to determine what size gears should be placed on the stud and lead screw to give the latter the right speed for any case. Here, as heretofore, the principle must hold that the speed of the first driver, multiplied by each driver, equals the speed of the last follower, multiplied by each follower. From this and from what has gone before we have, therefore, that *the number of threads per inch of the screw to be cut, multiplied by the number of teeth of each driver, equals the number of threads per inch of the lead screw, multiplied by the number of teeth of each follower.*

It is a simple matter, therefore, to find the ratio between the gear on the stud and the gear on the lead screw, and from that to determine what gears will be suitable.

1524. The process to be followed will be made clear by the following:

EXAMPLE.—Let the number of teeth in the different wheels in Fig. 898 be as follows: d_1, 30; f_1, 60; I, idle wheel. Assuming L to have six threads per inch, what change gears should be used to cut 4 threads per inch?

SOLUTION.—By formula **137**, letting N and n be the number of revolutions of the spindle and lead screw, respectively, we have $N \times d_1 \times d_2 = n \times f_1 \times f_2$, or $4 \times 30 \times d_2 = 6 \times 60 \times f_2$. Hence, $d_2 = \frac{6 \times 60 \times f_2}{4 \times 30} = 3 f_2$.

That is, the number of teeth of the gear on the stud ($= d_2$) must equal the number of teeth of the gear on the lead screw, multiplied by three. In other words, the gear on the stud must have three times as many teeth as the gear on the lead screw.

In like manner we have

$$\text{for 5 threads, } d_2 = \frac{12}{5} f_2;$$

$$\text{for 6 threads, } d_2 = 2 f_2;$$

$$\text{for 7 threads, } d_2 = \frac{12}{7} f_2;$$

etc., etc., etc.

For four threads we might use wheels of 84 and 28 teeth; for 5 threads 84 and 35, etc. In this way, a table might be calculated and arranged in columns, as below:

Threads per Inch.	Gear on Stud.	Gear on Lead Screw.
4	84	28
5	84	35
6	84	42
7	84	49
etc.	etc.	etc.

Such reference tables are always provided with lathes, and the gears are generally so chosen that the smallest number possible will have to be furnished with the lathe to cut the range of threads desired. Thus, in the above table, the gears were chosen so that one with 84 teeth would serve for cutting several different threads.

1525. Many lathes adapted for screw-cutting are not provided with the stud T, the gear d_2 being keyed directly to the end of the spindle S.

In such a case, if it is desired to find whether the lathe will cut a certain thread not marked on the plate which is usually attached to the head-stock, write the number of threads to be cut as the numerator of a fraction and the number of threads on the lead screw as the denominator; multiply both numerator and denominator by some number, and ascertain whether any of the gears in stock have the number of teeth corresponding to the results obtained; if not, multiply again by some other number, etc. Thus, suppose the lead screw has 8 threads per inch and it is desired to cut a thread which shall have $7\frac{1}{2}$ threads per inch. Forming the fraction gives $\frac{7\frac{1}{2}}{8}$. Multiplying both numerator and denominator by 6 gives $\frac{45}{48}$. Should any of the gears have 45 and 48 teeth, the 45-tooth gear should be put on the lead screw and the 48-tooth gear on the spindle.

1526. Again, when the lathe has a stud, the gears d_1 and f_1 are never changed, and the required change gears may be found by multiplying the fraction formed as directed above by a second fraction whose numerator shall be the number of teeth in d_1 and denominator the number of teeth in f_1, and then proceeding as before.

Thus, suppose it is desired to cut a thread having 19 threads per inch, the lead screw having 8 threads; the gear d_1, 24 teeth, and the stud pinion f_1, 48 teeth. Then, $\frac{19}{8} \times \frac{24}{48} = \frac{19}{16}$; and $\frac{19}{16} \times \frac{4}{4} = \frac{76}{64}$; hence, a gear having 76 teeth placed upon the lead screw, and one having 64 teeth placed on the stud, will cut a screw having 19 threads per inch.

REVERSING MECHANISMS.

1527. It is frequently required to design machinery, parts of which must have a reciprocating motion alternately in opposite directions. The various crank motions are sometimes employed for this purpose, but in many cases their

use is inadmissible, as they produce a variable motion, and for other reasons are inconvenient. It is also often necessary that machinery be constructed to run in either direction, at will. All such cases require the use of some reversing mechanism other than those previously described. A few additional examples will now be explained.

1528. Mangle Gearing.—Figs. 399 and 400 show the principle of mangle gearing, so named from its use in

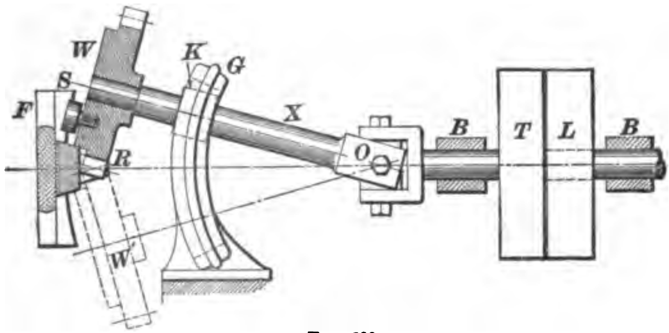


FIG. 399.

mangles for pressing clothes. Its principal use, at the present time, is for actuating the tables of printing presses. In Fig. 399 is a rack *R*, shown in cross-section, and so constructed that the gear-wheel *W* can run in mesh with it, either on top in position *W*, or underneath, as at *W'*. The rack is fastened to a frame *F*, also in section, attached directly to the table of the press. The object is to drive the table back and forth in a direction to and from the reader. The method will be understood by reference to Fig. 400, which is the rack and gear of Fig. 399, looked at from the left. As the frame *F* would obstruct the view of the other parts, it is omitted. A longitudinal section of the rack is shown.

Suppose the gear *W* to be on top of the rack, and to turn uniformly in the direction indicated. The rack will be driven to the right with a velocity equal to that of the pitch circle of the gear, until the roll *A* comes under the gear. One tooth of the gear is here omitted, leaving a large space which is rounded to fit the roll. From this point the gear

will gradually drop down in the line *m n*. At position *2* it will have made a quarter-turn, and the rack will have moved a distance *r* (= radius of the gear) to the right. At position *3* the rack will be at its former place, and further rotation of the gear will drive it to the left at a uniform speed. The motion of the rack is harmonic during the

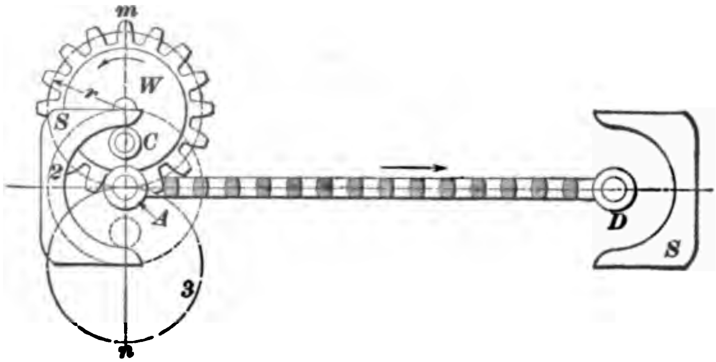


FIG. 400.

reversal. To place it under full control of the gear at the points of reversal, "shoes" *S*, *S* are fastened securely to the frame *F*. The inner curves of these are concentric with *A* and *D*, respectively, and serve as surfaces for the roll *C* on the gear to roll upon.

It is evident that the length *A D* of the rack must be equal to the circumference of the gear, or some multiple of the circumference, so that the wide space will come in contact with the rolls at each end. If, however, two spaces are cut diametrically opposite to each other, length *A D* may be some multiple of the half circumference. The total length of the stroke of the table is *A D*, plus the diameter of gear.

1529. Referring again to Fig. 399, the driving mechanism consists of tight and loose pulleys *T* and *L* on the shaft running in bearings *B*, *B*. *O* is a universal joint which allows the gear to rise and fall. *K* is a square block loose on the gear shaft *X*. It slides in a slotted guide *G*. and constrains the gear to move in a vertical straight line at the points of reversal .

The advantages of this gearing are that, neglecting the irregular motion of the universal joints, a uniform reciprocating motion of any length of stroke may be given the table with a gradual reversal. The disadvantage is that, where once constrained, the stroke cannot be adjusted.

1530. Clutch Gearing.—A reversing mechanism often used, especially when the reversal must take place automatically, is shown in Fig. 401. It consists of three

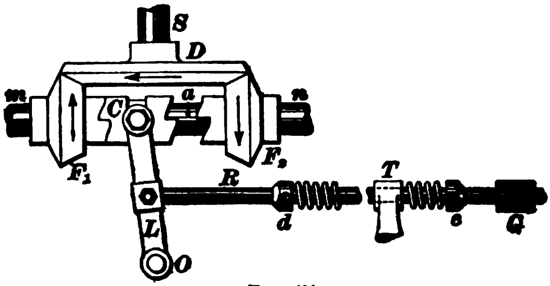


FIG. 401.

bevel gears, of which D is the driver, fast on the driving shaft S. F_1 and F_2 are continually in contact with D, and are loose upon shaft $m\ n$. A sleeve C can move endwise upon $m\ n$, but is compelled to turn with it by a key a provided in the shaft. When in the position shown, the notches clutch with F_1, and $m\,n$ turns; when in mid-position, $m\ n$ will not turn, and when thrown to the right, the direction of rotation will be reversed.

1531. If the reversal is to be automatic, some provision must be made to insure that after C has been pushed out of contact on one side, it will be thrown in contact with the other gear. One way of doing this is shown in the figure. The lever L is pivoted at O, and at the other end is forked to embrace C, a roller on each prong running in the groove shown. On the rod R, which is pivoted to L at one end and slides in a guide G at the other end, are two collars d and e, held in place by set screws. Helical springs are also placed on R against the inside of the collars. Now, suppose the tappet T, which is free to slide on R, be given a motion to

the right through mechanism connected with F_1, but not shown in the cut. When it reaches the spring, it will compress it until the pressure of the spring on *e* is sufficient to overcome the resistance of the clutch. Further movement of *T* will move *C* to the right until free of F_1, when the energy stored in the spring will suddenly throw *C* in contact with F_2. The time at which reversal occurs can be adjusted by changing the position of *d* and *e*.

1532. Sometimes it is desirable to have a slow motion in one direction with a "quick return." Figs. 402 and 403 show two methods that may be used for this purpose. In the first, the driver *D* is made cup shaped so as to allow a smaller driver *d* to be placed inside. For the slow motion we have *d* driving F_2, and, for the quick return, *D* driving F_1. In Fig. 403, *S* is the driving shaft, giving a slow and powerful motion through the worm gearing, shown clearly in the end view. The quick-return motion is through the gears D_1, F_1, D_2, and F_2.

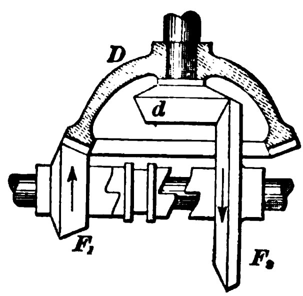


FIG. 402.

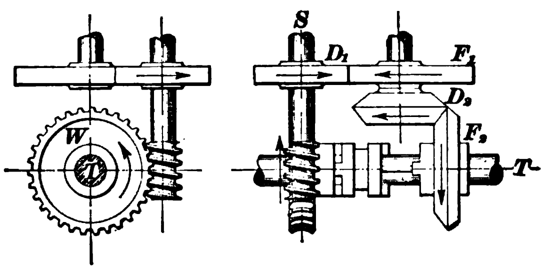


FIG. 403.

1533. It is evident that these mechanisms are not suitable for quick-running or heavy machinery. For such use, two belts are generally employed in place of the gears, one open and one crossed. Often these belts are made to shift alternately from a tight to a loose pulley, while in other cases they are arranged to drive two pulleys on the same

shaft in opposite directions, either of the pulleys being thrown in or out by means of a friction clutch, as, for example, in lathe countershafts.

1534. Shifting-Belt Mechanism.—Of the former class, Fig. 404 affords an illustration, as applied to a planer operating on metal. *H* is a section of the table, which is driven

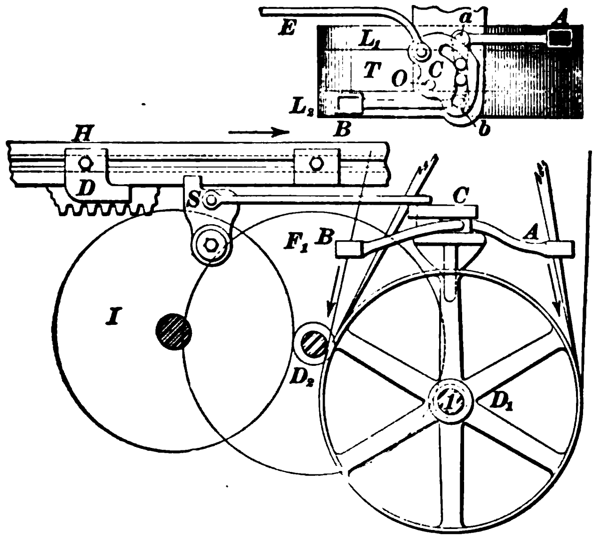


FIG. 404.

forwards and backwards by the rack and gearing shown. The work to be planed is clamped to the table, and during the forward, or cutting, stroke a stationary tool removes the metal. As no cutting takes place on the return stroke, the table is made to run back from two to five times as fast as forward, the latter speed being about 18 feet per minute for work on cast iron. There are three pulleys on shaft *1*. *T* is the tight or driving pulley, and L_1 and L_2 are both loose (see top view). There are two belt shifters *A* and *B*, in the shape of bell-cranks, pivoted at *a* and *b*, respectively. A crossed belt, moving in the direction of the arrow, runs from a small pulley on the countershaft, and is guided by the shifter *A*. This belt drives the table during the cutting

stroke. The other belt is "open," runs over a large pulley on the countershaft, and is guided by the shifter B.

The short arms of the shifters carry small rollers working in a slot in a cam plate C, which is pivoted at O. Each end of this slot is concentric about O, but one end has a greater radius than the other. As shown, the two shifters are in mid-position, both belts being on the loose pulleys. Suppose, however, the rod E be pulled to the left. Shifter B will not move, because its roller will continue to be in the same end of the slot, which is concentric about O; but the roller on A will be pulled to the left by passing from the end of the slot, which is of larger radius, to the end of the smaller radius. The crossed belt will, therefore, be shifted to the tight pulley, which will cause the table to run forward until the dog D strikes the rocker S, throwing the cam in the other direction. The effect of this will be to first shift the crossed belt from the tight to the loose pulley, and then to shift the open belt to the tight pulley; one motion follows the other, and thus decreases the wear and tear of the belts. The length of the stroke of the table can be varied by changing the position of the dogs, which are bolted to a T slot on the edge of the table.

The table is driven as follows: The belt pulleys drive gear D_1, which is keyed to the same shaft; D_1 drives F_1, which in turn drives D_2, keyed to the same shaft. D_2 drives gear I, which drives the table by means of a rack underneath it. The circumferential speeds of I and D_2 are evidently the same, and the speed of the table is the same as the circumferential speed of the gear D_2. The velocity ratio of the gearing is generally expressed in terms of the number of feet traveled by the belt to one foot passed over by the table.

1535. Suppose the belt pulleys to be 24 inches in diameter, and to make N revolutions while the table travels one foot. Let the diameter of D_1 be 3 inches; of F_1, 26 inches, and of D_2, 4 inches. The circumference of D_2 is $12\frac{1}{2}$ inches, nearly, so that for every foot traveled by the table, D_2

(and F_1) will turn $\frac{12}{12\frac{1}{2}} = \frac{24}{25}$ times. To find the number of turns made by the belt pulleys for each foot passed over by the table, we have, therefore,

$$N \times 3 = \frac{24}{25} \times 26, \text{ or}$$

$$N = \frac{24 \times 26}{75} = 8.32.$$

This multiplied by the circumference of the pulley $= 8.32 \times 2 \times 3.1416 = 52\frac{1}{4} +$ feet. That is, the planer is geared to run $52\frac{1}{4}$ to 1.

EXAMPLES FOR PRACTICE.

1. In a wheel train, a pulley A drives a pulley B by open belt; B is on the same shaft with a gear C, which drives a gear D; a gear E is on the same shaft with D and meshes with a gear F. Suppose A to be 30″ in diameter and B 26″, and let C have 90 teeth; D, 80 teeth; E, 70 teeth, and F, 60 teeth. How many revolutions will F make in five minutes, if A runs at 45 revolutions per minute?
Ans. 340.74 revolutions.

2. An engine lathe belt is on the third speed, the diameters of the steps upon which the belt is running being 9″ on the countershaft and 4″ on the lathe spindle. If the back gears are "in," how many turns will the countershaft make for one of the lathe spindle, supposing gears F_1 and F_2 (Fig. 398) to have 64 teeth each, and D_1 and D_2, 24 teeth each?
Ans. 3.16 revolutions.

3. In Fig. 398, suppose the stud T to make 3 turns while the lathe spindle makes 4. What change gears could be used to cut 10 threads per inch, the lead screw having 6 threads per inch?
Ans. The gears should be in the ratio of 4 : 5, the latter being the lead screw gear.

4. In Fig. 404, let the diameter of the pulleys be 30″; of D_1, 4″; of D_2, 4″, and of F_1, 22″. What is the ratio of belt to cutting speed?
Ans. 41.4 + : 1.

DIFFERENTIAL GEARING.

1536. Heretofore, in the discussion of wheel trains, it has been assumed that the bearings of the various wheels remained fixed. Occasionally, however, cases are met with in practice where the wheels not only turn about their own axes, but in which one or more of them revolves bodily about

some other axis, thus having a compound motion of rotation and translation. Such combinations are known as **differential gearing** or **epicyclic trains.** Their action is somewhat confusing at first, but can be made to appear simple by applying the principle of successive movements.

1537. The Two-Wheel Train.—I.—In Fig. 405 are shown two gears D and F united by an arm A. Suppose the number of teeth in D to be represented by m, and the number in F, by n; then, when the arm A is fixed, and D is revolved once about its axis right-handed, F will be revolved left-handed about its axis $\frac{m}{n}$ times. Denoting right-hand rotation by $+$ and left-hand rotation by $-$, gear D turns $+1$ times and gear F, $-\frac{m}{n}$ times. This is the ordinary case of transmission of motion by two gears.

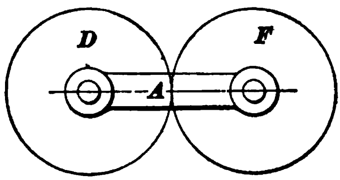


FIG. 405.

1538. II.—Suppose, however, that gear D is fixed in position so that it cannot turn, and the arm A is given a $+$ rotation about the axis of D, the gear F then partakes of two rotations, $+1$ about the axis of D and $+\frac{m}{n}$ about its own axis. In order to see this more clearly, imagine the two gears to be replaced by two friction wheels whose diameters are in ratio $\frac{m}{n}$; then, it is perfectly clear that, in order to rotate the arm, the wheel F must roll on D in the *same* direction, and that the number of times it turns on its axis is $\frac{m}{n}$. The only difference in the motions, in so far as the rotation of F on its own axis is concerned in these two cases, is that it has a negative rotation in the first case and a positive one in the second case. If the two gears are equal, one revolution of the arm will cause F to turn once on its own axis. This may also be considered in the following manner:

(*a*) Let the wheels be locked, or wedged together, so that neither can turn relatively to the other, and let the whole combination, *D*, *F*, and *A*, be turned as one body about the axis of *D* once R. H. (right-handed). The arm *A* has now been turned around once R. H., just as was desired; but, in doing so, *D* has been turned once R. H., when, according to the conditions, it should have remained stationary.

(*b*) Hence, the next step is to unlock the wheels, hold the arm stationary and turn *D* back one turn L. H. (left-handed) to where it should be. This will cause *F* to turn $\frac{m}{n}$ times R. H. After that is done, each part of the combination will have been through just the same relative motion that it would have had if the conditions had been carried out directly.

The results of the two steps upon *D*, *F*, and *A* can be tabulated thus, + indicating R. H. rotation and − indicating L. H. rotation:

	D	*F*	*A*
(*a*) Wheels locked	$+1$	$+1$	$+1$
(*b*) Arm stationary	-1	$+\frac{m}{n}$	0
(*c*)	0	$1+\frac{m}{n}$	$+1$

The algebraic sum of these in the third horizontal row gives the total motion of each part. It will be seen from this that if *D* and *F* have the same number of teeth, *F* will make two revolutions to one of the arms, one being about its own axis, and the other about the axis of *D*.

EXAMPLE.—Referring to Fig. 405, suppose *D* has 50 teeth, *F*, 20 teeth, and *A* to be turned 10 times L. H. How many turns will *F* make, and in what direction?

SOLUTION.—	*D*	*F*	*A*
Wheels locked	-10	-10	-10
Arm stationary	$+10$	$-10 \times \frac{50}{20}$	0
	0	$-(10+25)$	-10

F makes 35 turns L. H.

1539. III.—Suppose that the shaft on which F turns were the crank-shaft of a steam engine, and that the gear D were keyed to the end of the connecting-rod, the arm A being loose on both shafts. Then, one stroke of the piston would carry D to a position diametrically opposite, and the return stroke would bring it back to its first position; in other words, D would pass entirely around F, but without turning on its own axis, because of being keyed to the connecting-rod. The result will be that F will turn twice for one revolution of the arm when the gears D and F are of the same size. Although difficult to explain in a simple manner, a little thought will convince the student of the truth of this statement, and if he can obtain a couple of gears and try the experiment the result will amply reward him for the trouble. The result may be arrived at very simply, however, by means of the method of analysis used above. Suppose the gears to be locked and the whole combination, including the connecting-rod, to be revolved once right-handed. Denoting the connecting-rod by C, and remembering that the connecting-rod as a whole does not revolve, we must now return it to its original position by giving it a left-hand rotation (the arm A being fixed). This causes D to turn once left-handed and F, $\frac{m}{n}$ times right-handed, assuming that D has m teeth and F has n teeth. Tabulating the results, we have

	C	D	F	A
Wheels locked	$+1$	$+1$	$+1$	$+1$
Arm stationary	-1	-1	$+\frac{m}{n}$	0
	0	0	$1+\frac{m}{n}$	$+1$

The algebraic sum shows that for one revolution of A, F turns $1+\frac{m}{n}$ times and that C and D do not turn at all, which is perfectly true, since, when D is keyed to C, it is no longer a separate link, but is a part of C. Consequently, if D and

F are of the same size, $m = n$ and F turns twice right-handed for one right-handed revolution of the arm A.

This motion was used by Watt to drive his engine, some one else having patented the crank motion. It is known as the **sun and planet motion**; the fixed gear being the sun and the revolving gear the planet.

1540. Higher-Wheel Trains.—In Fig. 406 is represented a three-wheel train, derived from the two-wheel train by inserting the idle wheel I. Here, as in previous cases, the number of turns made by the wheel F is entirely independent of the size or number of teeth in the idler.

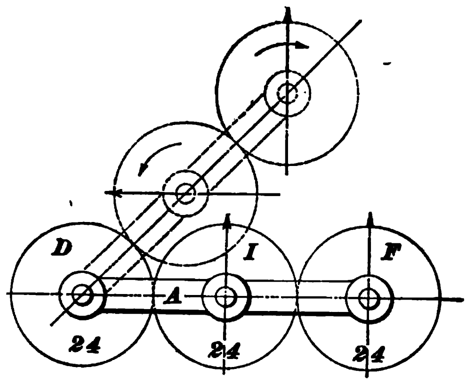


FIG. 406.

EXAMPLE.—Suppose all three wheels to have 24 teeth. If the arm makes -5 turns about the axis of D, how many turns will F make, and in which direction?

SOLUTION.—	D	I	F	A
Wheels locked	-5	-5	-5	-5
Arm stationary	$+5$	-5	$+5$	0
	0	-10	0	-5

The method of procedure is exactly the same as with the two-wheel train, and it will be noticed that the action of I in this train is the same as that of F in the two-wheel train. F, in the three-wheel train, however, does not turn at all. The straight arrows in the figure are supposed to be fastened to the wheels, so that the action of the train can be seen by noticing the directions in which the arrows point in the dotted positions.

1541. The statement that F does not turn at all is not literally true, since it turns once on its own axis and once on the axis of D. What the statement really implies is this:

The gear F cannot impart motion to an annular gear (supposing it to mesh with one) keyed to the shaft of D; that is, if an annular gear, keyed to the axis of D, meshes with gear F, and is caused to revolve owing to the rotation of the arm A, it will receive no motion from the gear F owing to the rotation of F on its own axis. It will, in fact, be deprived of a part of the rotation it should receive from the arm of A owing to this positive rotation of F, the amount of which is represented by $\frac{m}{p}$, m being the number of teeth in each of the three gears and p the number of teeth in the annular gear. Suppose, however, that the three gears D, I, and F were to be replaced by the two gears D and F, of Fig. 405, both gears being free to turn on their axes. Then, representing the number of teeth in the annular gear by p, one + revolution of the arm will cause the annular gear (which we will call L) to rotate $1+\frac{\frac{m}{n}\times n}{p}=1+\frac{m}{p}$ times. For the annular gear and gear F, both turn right-handed (D being fixed) and F turns $\frac{m}{n}$ times; the number of teeth in L which come in contact with corresponding teeth in F is $\frac{m}{n}\times n=m$; hence, L will make a part of a revolution represented by $\frac{m}{p}$ due to the turning of F on its axis, and one revolution due to the rotation of the arm A, the whole movement being represented by $1+\frac{m}{p}$. Using our method of analysis and applying to the first case, we have, assuming all three gears attached to A to be of the same size, and remembering that an annular gear always turns in the same direction as its pinion,

	D	I	F	L	A
Wheels locked	$+1$	$+1$	$+1$	$+1$	$+1$
Arm stationary	-1	$+1$	-1	$-\frac{m}{p}$	0
	0	$+2$	0	$1-\frac{m}{p}$	$+1$

For the second case, with two gears,

	D	F	L	A
Wheels locked	$+1$	$+1$	$+1$	1
Arm stationary	-1	$+\frac{m}{n}$	$+\dfrac{\frac{m}{n} \times n}{p}$	0
	0	$+1+\frac{m}{n}$	$+1+\frac{m}{p}$	$+1$

It should not be imagined that the gear F imparts a backward motion to L in the first case above, for it does not; its rotation simply prevents the arm from imparting to L its entire motion, which it would do if F were keyed to the arm.

1542. **A four-wheel train,** one of the wheels being an annular wheel, is shown in Fig. 407. D is fixed, the other wheels all turning. A revolves about the axis of F_1. D_1 and F are on the same spindle, F rolling inside of D.

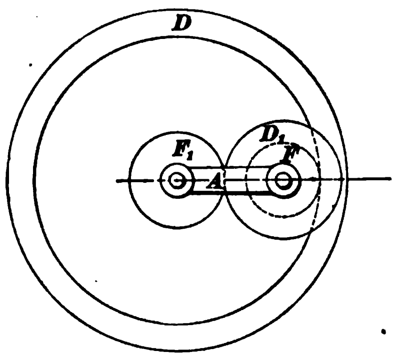


FIG. 407.

In working out a train of this kind, first consider all the wheels, including D, to be locked together and to turn with the arm, as in previous cases. Then, make the arm stationary, and turn D back to the position it should occupy.

EXAMPLE.—Let D have 100 teeth; F, 20; D_1, 45, and F_1, 35. A makes + 10 turns. Required the number of turns made by the pinions.

SOLUTION.—	D	F and D_1	F_1	A
Wheels locked	$+10$	$+10$	$+10$	$+10$
Arm stationary	-10	-50	$+\left(\frac{45}{35} \times 50\right)$	0
	0	-40	$+74\frac{2}{7}$	$+10$

1543. Differential Bevel Train.—A **differential bevel train** is shown in Fig. 408. It consists of three miter wheels, two of which, E and C, are on the shaft S, the

third revolving on the arm A, which is also on S. It is assumed that none of the wheels can slide endways upon their shafts. This train has certain peculiar as well as very useful properties.

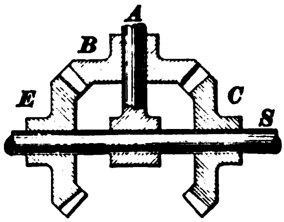


FIG. 408.

Suppose A to be held rigidly in one position, and E and C to be loose on the shaft. Then, if C be turned once one way, E will turn *once* in the *opposite* direction, the three wheels forming a simple train of gears, B being an idle wheel.

Now, suppose C to be held in one position and A to be turned once about the shaft S. E will then revolve *twice* in the *same* direction as the arm. That it will do so can be clearly seen by applying the principle of relative motions, as follows:

	A	C	E
Wheels locked	$+1$	$+1$	$+1$
Arm stationary	0	-1	$+1$
	$+1$	0	$+2$

From the above it will be seen that if we consider C the driver and E as fixed, A will revolve one-half as fast as C.

If we assume that the arm and all the wheels are free to turn, and if C is rotated one way at a uniform speed and E the opposite way at the same speed, B will revolve upon A, but A will remain still. If, however, C is rotated faster than E, the arm will move in the direction of C through one-half the angle gained; if C turns slower than E, the arm will move correspondingly the opposite way.

It will thus be seen that a bevel train is capable of a variety of combinations that can be applied in a useful way. A few practical applications of this and the other differential trains will now be given.

EXAMPLES FOR PRACTICE.

1. In Fig. 406, let D have 25 teeth; I, 30 teeth, and F, 50 teeth. If D is stationary and the arm makes 5 turns L. H., how many turns will F make, and in what direction? Ans. $2\frac{1}{2}$ turns L. H.

2. In Fig. 407, suppose D to be 40" in diameter; F, 10"; D_1, 14", and F_1, 16". If D remains stationary, how many turns will F_1 make to each turn of the arm? Ans. $4\frac{1}{4}$ turns.

3. In Fig. 408, suppose E to make 1 turn R. H. (looked at from the right) and A, two turns L. H. How many turns will C make, and in which direction? Ans. 5 turns L. H.

1544. Differential "Back-Gears."—Upright drills for metal work are sometimes provided with arrangements for increasing the range of the speeds and driving power that are different from the back-gears explained under the engine-lathe train. Fig. 409 shows one arrangement for this purpose. S is the shaft, and C the cone pulley which is loose on the shaft. D is a casting, also loose on the shaft, having

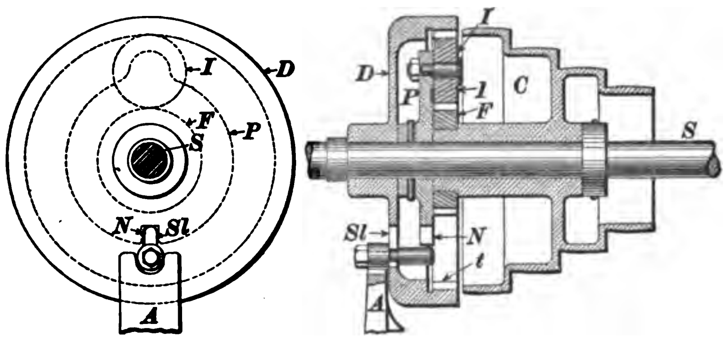


FIG. 409.

teeth on the inside, thus forming an annular gear. A plate P, carrying the small gear or pinion I, is fast to the shaft. On the left-hand end of the pulley hub is another gear F, which is fast to the hub of the cone.

The action is as follows: A pin, on which there is a collar and nut, is clamped in the slot $S\,l$ in D. The pin projects through D so that when it is placed in the inner end of the slot it will engage with a corresponding slot in the plate P. When it is lowered, however, the pin disengages with the plate, but the collar can be made to fall in a slot in the arm A, which is a part of the frame. In the former position D, P, and F are locked together, so that the shaft must turn with the cone. In the latter position D is locked with the

frame and cannot turn, while F revolves with the cone. The plate P, therefore, revolves with the shaft at a reduced speed, the arrangement being similar to that in Fig. 407.

1545. Differential Motion.—Perhaps the most important application of the differential bevel train is to be found in spinning machinery, where it is necessary to wind the partly twisted fiber, or *roving*, upon bobbins. As each successive layer is wound on the bobbin, the latter becomes correspondingly larger and must revolve at a reduced speed; otherwise, the roving, which is delivered at a constant speed, would be broken.

1546. In Fig. 410, B is the bobbin at the top of which the flyer f is suspended. The roving passes through a hole c in the center of the upper part of the flyer, then down

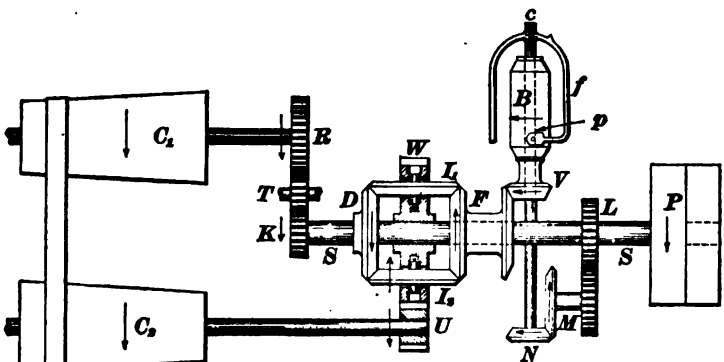


FIG. 410.

through the hollow arm f of the flyer and through a hole in the presser p to the bobbin. This gives a uniform twist to the roving. The flyer moves up and down the length of the bobbin winding on the roving in helical layers.

The machine is driven by a belt through the pulley P and the shaft $S S$. The first gear L on the shaft drives the flyer at a constant velocity through M and N. The next gear that is fast to the shaft is the miter wheel D. This forms part of a differential bevel train, the other wheel on the shaft being F, which is loose. To improve the running qualities, there are two intermediate wheels, I_1, I_2, instead of one, as in Fig.

408. A gear W, shown in section and loose upon the shaft, serves the purpose of the arm used in Fig. 408. Its action is exactly the same as the arm, the wheel being used simply as a means for causing it to revolve about the shaft. The object of the epicyclic train is to drive the bobbin, by means of F and V, at a *varying* velocity suited to the varying diameter of the bobbin.

This is brought about by the cone pulleys C_1 and C_2. The former is driven by the main shaft through the gears K, T, and R, and the latter drives W through gear U. Now we have D rotated uniformly in one direction, which turns F the opposite way. W has a motion opposite to that of D, the number of revolutions that it makes depending upon the position of the belt on the cones. As the belt is moved automatically to the left, W turns slower, F turns slower, and hence the bobbin turns slower as the roving is wound on. The gearing is such that the bobbin revolves in the same direction as the flyer and just enough faster to take the roving as it is delivered.

1547. Hartford Water-Wheel Governor.—Another application of the bevel train is to be found in the water-wheel governor, in Fig. 411. W_1 and W_2 are two wide-faced pulleys driven by the belt which passes over the guide pulleys P_1 and P_2. W_1 is a frustum of a cone, with its diameter at the middle, the same as that of W_2, which is an ordinary pulley. These pulleys give motion to the double

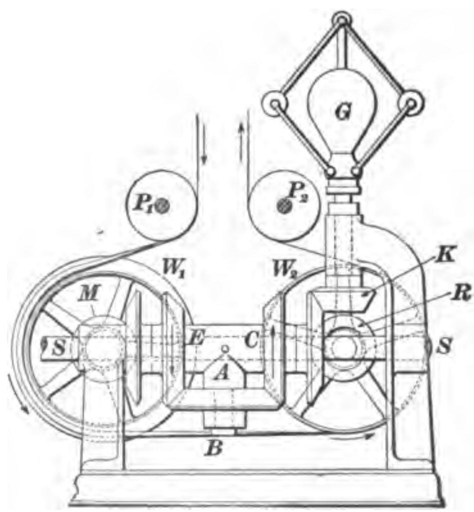


FIG. 411.

bevel wheels E and C, both of which are loose upon the shaft,

through the bevel gears M and R, respectively. The inner parts of E and C gear with the miter gear B on the arm A, the latter being keyed to the shaft. The outer part of C drives the governor G through the gear K. The work of the governor is to shift the belt on the cone by means of a fork (not shown) near pulley P_1.

The action is as follows: When the water-wheel is running at the proper speed, the belt is at the middle of the cone and E and C turn equally in opposite directions. Hence, A does not move. Should the wheel run too fast, however, the governor balls would rise, since the speed of W_2 must always be proportional to that of the water-wheel. This at once causes the belt to be shifted to the small end of the cone, increasing the speed of E. This, in turn, makes A, and hence the shaft, rotate in the direction of E, and an arrangement at the left-hand end of the shaft S S (not shown) lowers the water-wheel gate. If the wheel should slow down, the reverse operation would take place.

GEAR WHEELS.

1548. Let two wheels with parallel axes be held in firm

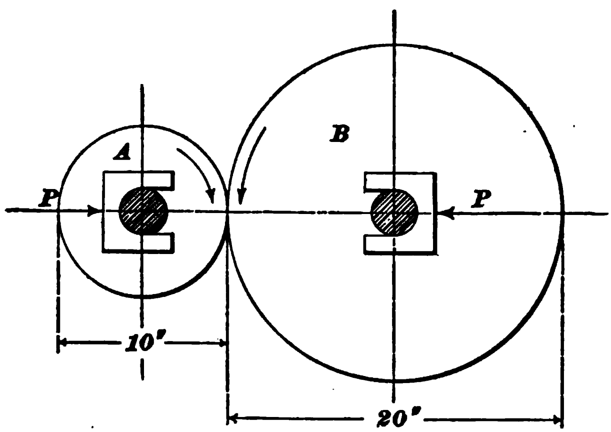


FIG. 412.

rolling contact by pressure upon their axes, as in Fig. 412.

If one be turned in either direction, and there is no slipping, the other will rotate in the opposite direction with a circumferential, or surface, velocity equal to that of the first, as though connected by a crossed belt, and the numbers of their revolutions will be inversely proportional to their diameters. Assuming wheel A to be 10" in diameter and B 20", B would make $\frac{10}{20} = \frac{1}{2}$ as many revolutions as A.

1549. Should slipping occur, however, B would make less than one-half as many revolutions as A, if A were the driver. To obviate slipping, suppose that pieces like a, a, Fig. 413, are fastened at equal distances on the peripheries of A and B, and that corresponding grooves like b, b are cut. Then, the projections, or teeth, on one wheel will run be-

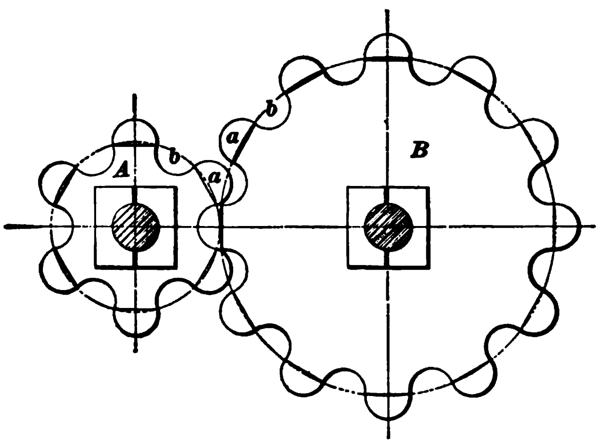


FIG. 413.

tween the teeth on the other, and B will necessarily revolve one-half as often as A. It is important to notice, however, that although the *number of revolutions* of the wheels in the latter case will be in the ratio of 2 to 1, the speed from *tooth to tooth* might vary somewhat from this ratio, unless the teeth were of a shape that would give a constant velocity ratio. This would result in an uneven motion that would be undesirable, even though the variation was very slight.

1550. The object, then, in designing the teeth of gear-wheels should be to so shape them that the motion transmitted will be exactly the same as with a corresponding pair of wheels, or cylinders, without teeth, and running in contact without slipping.

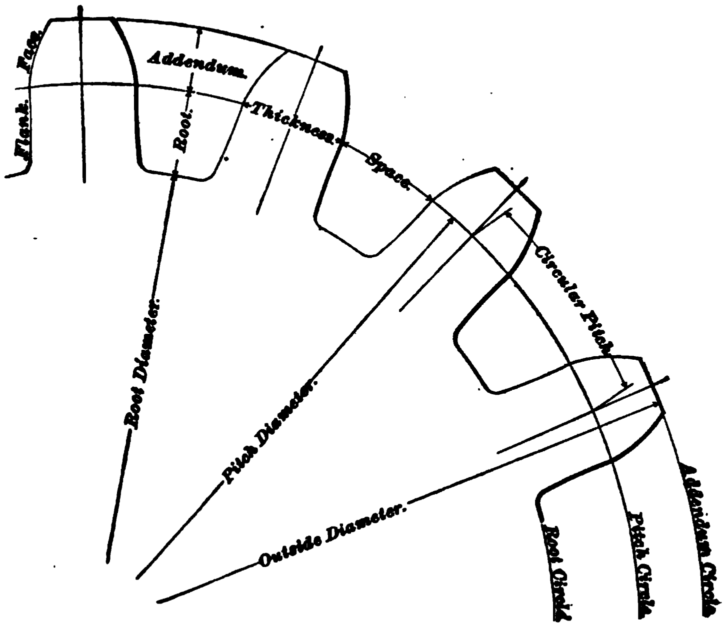


FIG. 414.

Such cylinders are called **pitch cylinders,** and are always represented on the drawing of a gear-wheel by a line called the **pitch circle.** (See Fig. 414.) The pitch circle is also called the **pitch line.**

1551. The diameter of the pitch circle is called the **pitch diameter.** When the word "diameter" is applied to gears, it is always understood to mean the **pitch diameter,** unless otherwise specially stated, as "outside diameter" or "diameter at the root."

1552. Circular and Diametral Pitch.—The distance from a point on one tooth to a corresponding point on the next tooth, *measured along the pitch circle*, is the **circular pitch.** It is obtained by dividing the circumference (pitch circle) by the number of teeth, and is used in laying out the teeth of large gears, and also when calculating their strength.

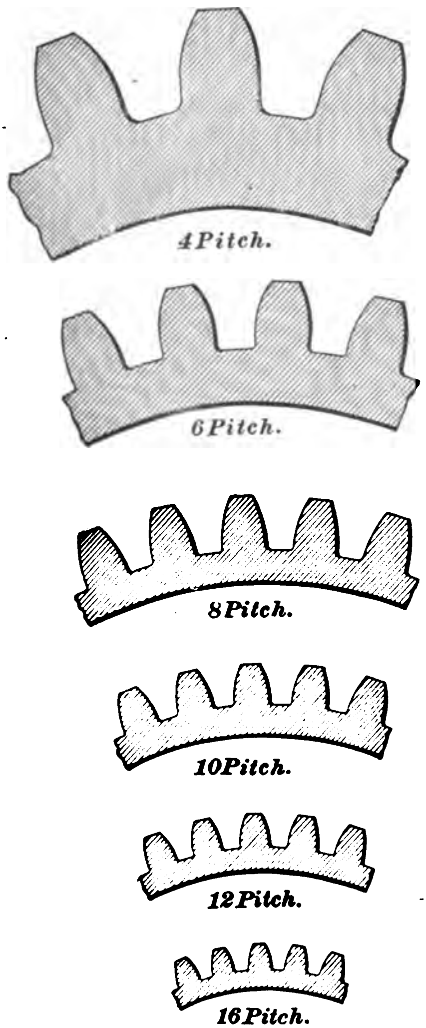


FIG. 415.

It would be very convenient to have the circular pitch expressed in manageable numbers like 1 inch, $\frac{3}{4}$ inch, etc.; but as the circumference of a gear is 3.1416 times its diameter, this requires awkward numbers for the diameters. Thus, a wheel of 40 teeth, 1 inch pitch, would have a circumference on pitch circle of 40 inches and a diameter of 12.732 inches. Of the two, it is more convenient in the great majority of cases to have the diameters expressed in numbers that can be easily handled. In order, however, to have the pitch in a convenient form also, a new pitch has been devised, *expressed in terms of the diameter* and called the **diametral pitch.**

1553. The diametral pitch is not a measurement like the circular pitch, but a *ratio.* *It is the ratio of the number of teeth in the gear to the number of inches in the diameter; or, it is the number of teeth on the circumference of the gear for one inch diameter of the pitch circle.* It is obtained by dividing the number of teeth by the diameter.

A gear, for example, has 60 teeth and is 10 inches in diameter. The diametral pitch is the ratio of 60 to 10 $= \frac{60}{10} = 6$, and the gear would be called a 6-pitch gear. From the definition, it follows that teeth of any particular diametral pitch are of the same size, and have the same width on the pitch line, whatever the diameter of the gear. Thus, if a 12-inch gear had 48 teeth, it would be 4 pitch. A 24-inch gear to have teeth of the same size would have twice 48 or 96 teeth, and $96 \div 24 = 4$, the same diametral pitch as before.

Fig. 415 shows the sizes of teeth of various diametral pitches.

Diametral pitch has also been defined as the number of teeth in a gear of one inch diameter, which amounts to the same as the definitions above.

Using for illustration a wheel 10 inches in diameter with 60 teeth, we have

$$\text{circular pitch} = \frac{\text{circumference}}{\text{No. of teeth}} = \frac{10 \times 3.1416}{60} = .524 \text{ inch.}$$

$$\text{Diametral pitch} = \frac{\text{No. of teeth}}{\text{diameter}} = \frac{60}{10} = 6.$$

1554. Other Definitions.—The other necessary definitions applying to the parts of a gear can be readily understood from Fig. 414. The thickness of the tooth and width of the space are measured on the pitch circle. A tooth is composed of two parts, the **addendum,** or outside of the pitch circle, and the **root,** which is inside.

A line through the outside end of the addendum is called the **addendum circle,** or **addendum line,** and one through the inside part of the root is called the **root circle,** or **root line.** The amount by which the width of the space is

greater than the thickness of the tooth is called the **back lash,** or **clearance.**

1555. Proportions for Gear Teeth.—With gears of large size, and often with cast gears of all sizes, the circular pitch system is used. In these cases, it is usual to have the addendum, whole depth, and thickness of the tooth conform to arbitrary rules based upon the circular pitch. None of these rules can be considered absolute, however. Machine-moulded gears require less clearance and back lash than hand-moulded, and very large gears should have less, proportionately, than smaller ones. The following table of proportions that have been used successfully will serve as an aid in deciding upon suitable dimensions. Column 1 is for ordinary cast gears, and column 2 is for very large gears having cut teeth. C stands for circular pitch.

TABLE 34.

	1	2
Addendum	$0.30C$	$0.30C$
Root	$0.40C$	$0.35C$
Whole Depth	$0.70C$	$0.65C$
Thickness of Tooth	$0.48C$	$0.495C$
Width of Space	$0.52C$	$0.505C$

The gears most often met with are the cut gears of small and medium size, like those, for example, on machine tools, which are almost invariably diametral pitch gears. The teeth are cut from the solid with standard milling cutters, proportioned with the diametral pitch as a basis. The system is also coming into very general use for cast gearing. In all diametral pitch gears, the addendum is made equal to 1 divided by the diametral pitch, and the working depth twice the addendum. The end clearance is usually taken equal to $\frac{1}{8}$ of the addendum for cut gears, though the Brown & Sharpe Mfg. Co. use $\frac{1}{10}$ of the thickness of the tooth on the pitch line as the clearance. The side clearance, or back

lash, is made just enough to give a good working fit, and seldom exceeds $\frac{1}{40}$ of the pitch.

Using the above proportions, a 4-pitch gear would have the addendum $= 1 \div 4$, or $\frac{1}{4}$ of an inch; the working depth would be $2 \times \frac{1}{4} = \frac{1}{2}$ inch, and the clearance $\frac{1}{8} \times \frac{1}{4} = \frac{1}{32}$ inch. The whole length of the tooth would be $\frac{1}{2} + \frac{1}{32} = \frac{17}{32}$ inch. The thickness of the tooth would be $\frac{1}{2}$ the circular pitch, nearly. In a 10-pitch wheel the addendum would be $\frac{1}{10}$ inch and the length of the tooth $\frac{17}{80}$ inch; in a $2\frac{1}{2}$ pitch it would be $1 \div 2\frac{1}{2} = \frac{2}{5}$ inch and the length $\frac{17}{20}$ inch.

1556. Sizing Gear Blanks.—It is quite as important to be able to solve problems involving the diameter, number of teeth, and circular and diametral pitch of gear-wheels, as to be able to lay out the correct tooth curves. Several rules and examples will be given covering cases likely to be met with.

For convenience, these symbols will be used:

$P =$ diametral pitch;
$D =$ diameter of pitch circle;
$O\,D =$ outside diameter;
$C =$ circular pitch;
$N =$ number of teeth;
$A =$ distance between centers of two wheels;
$V =$ velocity—i. e., revolutions per minute.

When two wheels run together, small letters, like the above, will be used for the smaller wheel, the large letters applying to the larger wheel.

The product of the circular pitch of a gear and the diametral pitch is always the constant number 3.1416. *Hence, to change circular to diametral pitch, divide 3.1416 by the circular pitch; to change diametral to circular pitch, divide 3.1416 by the diametral pitch.* That is,

$$P = \frac{3.1416}{C}, \text{ and} \qquad \textbf{(149.)}$$

$$C = \frac{3.1416}{P}. \qquad \textbf{(150.)}$$

Example.—If the circular pitch is 2 inches, the diametral pitch, or $P, = \frac{3.1416}{2} = 1.571$ inches, nearly. If the diametral pitch is 4, the circular pitch, or $C, = \frac{3.1416}{4} = .7854$ inch.

1557. Table 35 gives in the first two columns values of circular pitch corresponding to common values of diametral pitch, and in the last two columns values of diametral pitch corresponding to circular pitch values.

TABLE 35.

Diametral Pitch.	Circular Pitch.	Circular Pitch.	Diametral Pitch.
2	1.571 inches.	2 inches.	1.571
$2\frac{1}{4}$	1.396 "	$1\frac{7}{8}$ "	1.676
$2\frac{1}{2}$	1.257 "	$1\frac{3}{4}$ "	1.795
$2\frac{3}{4}$	1.142 "	$1\frac{5}{8}$ "	1.933
3	1.047 "	$1\frac{1}{2}$ "	2.094
$3\frac{1}{2}$	.898 "	$1\frac{7}{16}$ "	2.185
4	.785 "	$1\frac{3}{8}$ "	2.285
5	.628 "	$1\frac{5}{16}$ "	2.394
6	.524 "	$1\frac{1}{4}$ "	2.513
7	.449 "	$1\frac{3}{16}$ "	2.646
8	.393 "	$1\frac{1}{8}$ "	2.793
9	.349 "	$1\frac{1}{16}$ "	2.957
10	.314 "	1 "	3.142
11	.286 "	$\frac{15}{16}$ "	3.351
12	.262 "	$\frac{7}{8}$ "	3.590
14	.224 "	$\frac{13}{16}$ "	3.867
16	.196 "	$\frac{3}{4}$ "	4.189
18	.175 "	$\frac{11}{16}$ "	4.570
20	.157 "	$\frac{5}{8}$ "	5.027

The product of the diameter and diametral pitch in any gear is equal to the number of teeth, or

$$N = D P. \qquad \textbf{(151.)}$$

Also, knowing the number of teeth and the diametral pitch, the diameter may be found from the same formula.

EXAMPLE.—(*a*) If a wheel is 30 inches in diameter and 3 pitch, how many teeth has it?

(*b*) Of what diameter is a $2\frac{1}{2}$ pitch gear having 20 teeth?

SOLUTION.—(*a*) $N = DP = 3 \times 30 = 90$ teeth. Ans.

(*b*) $D = \frac{N}{P} = \frac{20}{2\frac{1}{2}} = \frac{40}{5} = 8$ inches. Ans.

The outside diameter of a gear equals the pitch diameter, plus twice the addendum, or $OD = D + 2 \times \frac{1}{P}$. But $D = \frac{N}{P}$. Hence, to obtain the outside diameter, knowing the diametral pitch and number of teeth, we have

$$OD = \frac{N}{P} + 2 \times \frac{1}{P} = \frac{N+2}{P}. \qquad \textbf{(152.)}$$

EXAMPLE.—A wheel is to have 48 teeth, 6 pitch; to what diameter must the blank be turned?

SOLUTION.—By formula **152**, $OD = \frac{N+2}{P} = \frac{48+2}{6} = 8.333$ inches. Ans.

EXAMPLE.—A gear blank measures $10\frac{1}{2}$ inches in diameter and is to be cut 4 pitch. How many teeth should the gear cutter be set to space?

SOLUTION.—From formula **152**, $OD = \frac{N+2}{P}$, or $N = OD \times P - 2 = 10\frac{1}{2} \times 4 - 2 = 42 - 2 = 40$ teeth. Ans.

To find the diameter, the circular pitch and number of teeth being given, we have, from the definition of circular pitch,

$$D = \frac{C \times N}{3.1416}. \qquad \textbf{(153.)}$$

EXAMPLE.—What is the diameter of a gear-wheel which has 75 teeth and whose circular pitch is 1.675 inches?

SOLUTION.—$D = \frac{1.675 \times 75}{3.1416} = 40$ inches. Ans.

Having given the distance between the centers of two gears and their velocities, the formulas for their diameters may be derived as follows:

From formula **136**, $VD = vd$, or $D = \frac{vd}{V}$, but $A = \frac{D+d}{2}$, or $D = 2A - d$. Equating values of D,

$$\frac{vd}{V} = 2A - d,$$

$$\text{whence } vd = 2AV - dV,$$

$$\text{or } d(v + V) = 2AV,$$

$$\text{and } d = \frac{2AV}{V+v}, \qquad \textbf{(154.)}$$

where V = velocity of the large gear and v that of the small gear.

In like manner,

$$D = \frac{2Av}{v+V}. \qquad \textbf{(155.)}$$

EXAMPLE.—Given the distance between centers of two gears $= 5\frac{1}{4}$ inches. What must be their diameters so that the ratio of their speeds will be as 3 is to 1?

SOLUTION.—By formula **154**, $d = \frac{2 \times 5\frac{1}{4} \times 1}{1+3} = 2\frac{5}{8}$ inches.

By formula **155**, $D = \frac{2 \times 5\frac{1}{4} \times 3}{3+1} = 8\frac{1}{8}$ inches. Ans.

EXAMPLES FOR PRACTICE.

1. (*a*) How many teeth has a $2\frac{1}{2}$-pitch gear, 4 feet in diameter? (*b*) What is the circular pitch of this gear? Ans. $\begin{cases}(a)\ 120 \text{ teeth.} \\ (b)\ 1.257 \text{ inches.}\end{cases}$

2. What is the outside diameter of a gear blank from which a wheel is to be cut having 50 teeth 4-pitch? Ans. 13 inches.

3. The pitch diameter of a gear is 25 inches. What is its outside diameter, supposing it to be 6-pitch? Ans. 25.333 inches.

4. A gear blank measures 10.2 inches in diameter and is to be cut 10-pitch. How many teeth should the gear cutter be set to space? Ans. 100 teeth.

5. Given the distance between the centers of two gears = 20″. What must be their diameters so that the ratio of their speeds will be as 6 : 5? Ans. 18.181 inches and 21.818 inches.

1558. Law of Tooth Contact.—The **pitch point** of two gears is the point of contact C, in Fig. 416, of the pitch lines. It is the point at which the line of centers $O\,O'$

intersects the pitch circles. The **point of contact** is the point where two teeth touch each other. In order that two

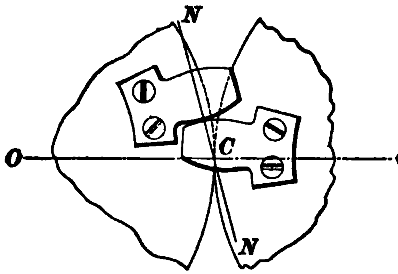


FIG. 416.

gear-wheels may have the same relative velocities at every point as their corresponding pitch cylinders, the tooth curves must be of such a shape that at the point of contact they will both be at right angles to a line $N\ N$, Fig. 416, passing through the pitch point and point of contact. This line is called the **common normal** to the tooth curves.

1559. The **path of contact** is the curve described by the point of contact during the entire action of a pair of teeth. This curve always passes through the pitch point.

1560. Angle and Arc of Action.—The angle through which a wheel turns from the time when one of its teeth comes in contact with a tooth of the other wheel until the point of contact has reached the line of centers is the **angle of approach**; the angle through which it turns from the instant the point of contact leaves the line of centers until the teeth are no longer in contact is the **angle of recess.** The sum of these two angles forms the **angle of action.** The arcs of the pitch circles which measure these angles are called the arcs of **approach, recess,** and **action,** respectively.

In order that one pair of teeth shall be in contact until the next pair begin to act, the arc of action must be at least equal to the pitch.

THE EPICYCLOIDAL SYSTEM.

1561. The Tooth Outline.—In Fig. 417, let O and O' be the centers of two pitch circles, in contact at the pitch point C; and let a smaller circle, whose center is at o, be tangent to both circles at C. Suppose the three centers to

be fixed, and the circles to move in rolling contact with each other in the direction of the arrows, the circle o carrying a marking point E. E will then describe a curve $E\ d$ on the plane of the circle O, and a curve $E\ e$ on the plane of the circle O'. $N\ N$, the common normal of these curves, will pass through the pitch point C, so they are suitable for tooth outlines.

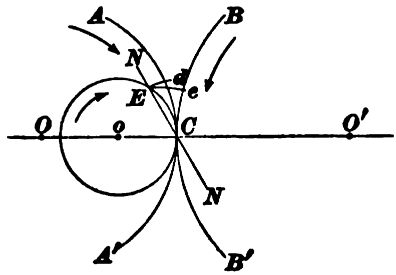


FIG. 417.

NOTE.—In mathematics, the words normal and perpendicular have the same meaning.

It will be observed that the relative motions of the circles are the same as though the small one rolled on the outside of O' and on the inside of O. $E\ e$ is, therefore, the epicycloid of $B\ B'$, and would answer for the face of a tooth of O', while $E\ d$ is the hypocycloid of $A\ A'$, and would serve as a flank for a tooth of O. In like manner, faces for O and flanks for O' could be generated by a circle inside of $B\ B'$.

Since the method of rolling up and using these curves was fully described in the subject of Mechanical Drawing, more will not be said here concerning the process.

1562. Interchangeable Wheels.—It is not necessary that the two generating circles used should be of the same diameter; provided that the flanks and faces which act on each other are generated by the same circle as in the previous case. It is customary, however, to use the same size circle for faces and flanks of both wheels, and where it is desired to make a set of gears, any two of which will run together, the same size circle must be used for all.

1563. In Figs. 418 to 420 are shown the effects of different sizes, describing circles upon the flanks of the teeth. In the first, the circle is half the pitch circle, and the flanks described are radial. In the second, with a smaller circle, the flanks curve away from the radius, giving a strong tooth, and in the third, with a larger circle, the flanks curve

inwards, giving a weak tooth, and one difficult to cut. It would seem, therefore, that a suitable diameter for the describing circle would be one-half the pitch diameter of

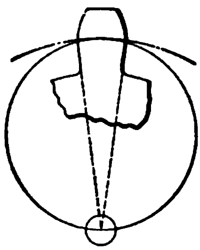


FIG. 418.

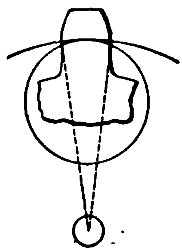


FIG. 419.

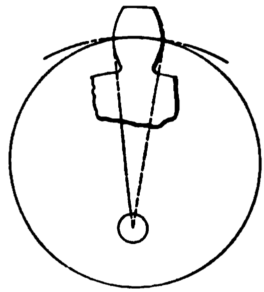


FIG. 420.

the smallest wheel of the set, or one-half the diameter of a 12-tooth pinion, which, by common consent, is taken as the smallest wheel of any set.

It has been found, however, that a circle of five-eighths the diameter of the pitch circle will give flanks nearly parallel, so that teeth described with this circle can be cut with a milling cutter. For this reason, some gear cutters are made to cut teeth based upon a describing circle of five-eighths the diameter of a 12-tooth pinion, or one-half the diameter of a 15-tooth pinion.

It is more common practice to take the describing circle equal to one-half the diameter of a 12-tooth than of a 15-tooth pinion, and this is the size used in this Course.

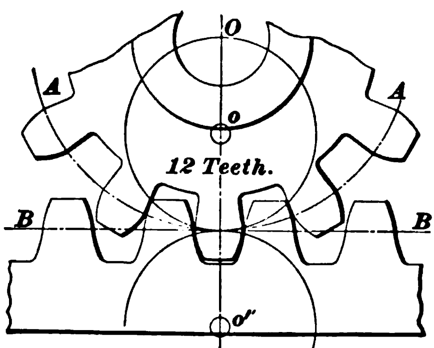


FIG. 421.

1564. Rack and Wheels.—A rack may be considered as a wheel having an infinite diameter. The pitch line of a rack is, therefore, a straight line, and for every revolution of the wheel the rack will travel a distance

equal to the circumference of the wheel. The construction of the tooth is shown in Fig. 421, describing circles for the rack teeth rolling on the line *B B*, and forming cycloids. If both circles are of the same diameter, and are the same as used for generating the tooth curves for an interchangeable series of wheels, the rack will evidently mesh with any of the wheels.

1565. Annular, or internal, gears are those having teeth cut on the inside of the rim. The width of space of an internal gear is the same as the tooth of a spur gear. Two describing circles are used as before, and, if they are of equal diameter, the gear will interchange with spur wheels for which the same describing circle was used.

In Fig. 422 is represented an internal gear with pitch circle *A A*, inside of which is the pinion with pitch circle *B B*. The generating circle *O*, rolling inside of *B B*, will

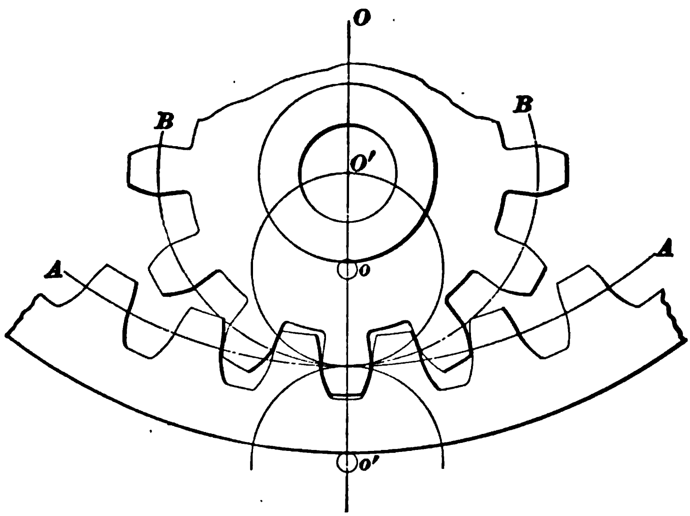


FIG. 422.

describe the flanks of the pinion, and rolling inside of *A A*, faces for the annular wheel. Similarly, the corresponding faces and flanks will be described by *o'*. The only special rule to be observed in regard to epicycloidal internal gears

is that *the difference between the diameters of the pitch circles must be at least as great as the sum of the diameters of the describing circles.*

This is illustrated by Fig. 423. A is the pitch circle of an internal gear, and B of the pinion. Then, for correct action, the difference $(D-d)$ of the diameters must be at least as great as c, the sum of the diameters of the describing circles. To take a limiting case, suppose A to have 36 teeth and B 24 teeth. A wheel with a diameter equal to $D-d$, as shown dotted at E, would, therefore, have 36, minus 24, teeth, or 12 teeth. In the 12-tooth interchangeable system, this latter would be the smaller wheel of the series, and the describing circles would be half its diameter. From this it follows that, if $D-d$, the diameter of E, is not to be exceeded by the sum c of the diameters of the describing circles, B is the largest wheel that can be used with A. *Hence, when the interchangeable system is used, the number of teeth in the two wheels must differ by at least the number in the smallest wheel of the set.* If we wished, for example, to have 18 and 24 teeth, and the gears were to interchange, describing circles, half the diameter of a 6-tooth pinion, would be used, this being taken as the smallest wheel.

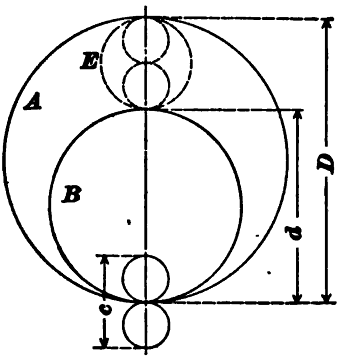


FIG. 423.

THE INVOLUTE SYSTEM.

1566. Let O and O' be the centers of two cylinders that are a short distance apart, and $D\,E$ a cord that has been wrapped several times around them in opposite directions, as shown in Fig. 424. If the circle $D\,D'$ be turned in the direction of the full arrow, the cord $D\,E$ will cause the cylinder $E\,E'$ to turn also in the opposite direction, as shown by the full arrow, and points on the cord $D\,E$ will describe

portions of the involute curve. In order to better comprehend this, imagine a piece of paper to be attached to the bottom of each cylinder, as shown, and that the width of each piece is the same as the distances between the cylinders. Now, suppose that a pencil be attached to the cord at E, the point of tangency of the line $D\ E$ with the cylinder $E\ E'$, in such a manner that it can trace a line on the piece of paper attached to the cylinder $E E'$, if a proper motion be given to the cord $D\ E$. Turn the cylinder $D\ D'$ in the direction of the arrow, i. e., rotate it left-handed. The point E will travel towards the cylinder $D D'$ in the straight line $E\ D$, and gradually diverge from the cylinder $E\ E'$. During this movement, the pencil attached at E will trace the involute curve m, shown dotted on the piece of paper. In the same manner, if the pencil be attached at D, and the cylinder $E E'$ be rotated in the direction of the dotted arrow, the dotted involute m' will be traced on the piece of paper attached to the cylinder $D\ D'$. Suppose that part of the pieces of paper to the right of the curve m and to the left of the curve m' be removed and that $E\ E'$ be rotated until the curve m takes the position $e\ d$; also, that $D\ D'$ be rotated until m' takes the position $g\ h$, the two curves being tangent to each other at k on the line $D\ E$. The cord $D\ E$ will be at right angles to both curves m and m' at the point of contact. The curves will, therefore, be suitable tooth curves, according to the law of tooth contact, if $E\ D$

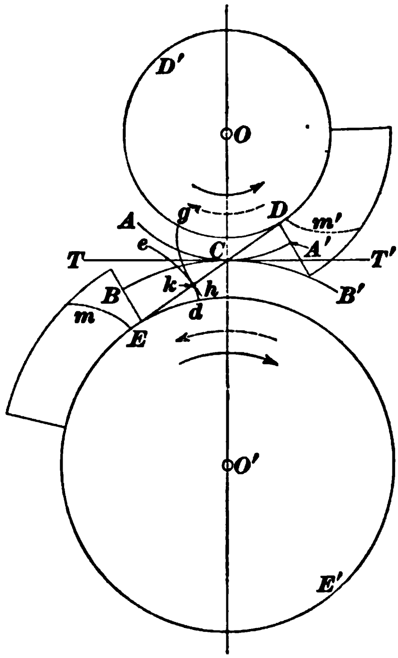


FIG. 424.

always passes through the pitch point. To make it do this, it is simply necessary to connect the two centers by the line *O O'*, and through its point of intersection *C* with *E D* to draw the pitch circles *A A'* and *B B'*. Two gears, therefore, with pitch circles *A A'* and *B B'*, and with involute teeth formed from circles the size of *D D'* and *E E'*, will have the same relative velocity as two pitch cylinders with radii *O C* and *O' C* in rolling contact. Such gears are sometimes called **single-curve gears**, because a single involute curve serves for both face and flank.

If the centers *O* and *O'* should now be moved apart so that the pitch circles do not touch, the relative velocities of *D D'* and *E E'* would evidently remain unchanged, since they are connected by the cord *E D*. The curves described by point *k* would also be the same, because they would still be involutes of the same circles. From this, it follows that the distance between the centers of involute gears may be varied without disturbing their relative velocity or the action of the teeth—a property peculiar to the involute system.

1567. Standard Gears.—In Fig. 424, let a line *T T'* be drawn at right angles to *O O'*. Then, the angle *D C T'*, made by *E D* with *T T'*, is called the **angle of obliquity,** and the circles *D D'* and *E E'*, from which the curves are derived, and which are tangent to *E D*, are called the **base circles.** In standard interchangeable gears, based upon the diametral pitch, the angle of obliquity is taken at 15°, which brings the distance between the base circle and the pitch circle at about $\frac{1}{60}$ the pitch diameter. It would be well to use these values for all gears.

1568. Fig. 425 shows two standard gears, *A A* and *B B* being the pitch circles, and *D D'* and *E E'* the base circles. *T T* was drawn through *C* at right angles with *O O'*, and *N N* was drawn through *C*, making an angle of 15° with *T T*. The path of contact is along the straight line *N N*, and the distance along *N N* from a point on one tooth to a corresponding point on the next is called the

normal pitch. The parts of the teeth above the base circle are involutes, and the flanks below the base circle are radial.

1569. Interference takes place when a part of one tooth crowds another at some point during the action, so that the gears will not run. To determine whether any pair of involute gears will work well together, draw lines *O D* and *O′ E*, Fig. 425, perpendicular to the line of action *N N*. So long as the intersections of the addendum circles (shown dotted) and the line of action fall between points *E* and *D*, as at *e*, there will be no interference. If they fall outside, as at *d*, both wheels will interfere, while, if the

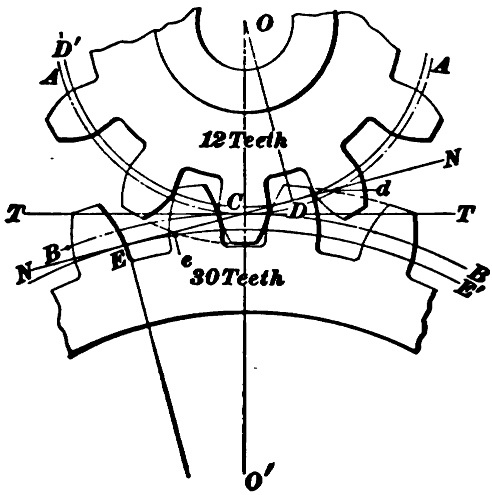


FIG. 425.

addendum circle of only one gear cuts the line of action outside, the teeth of that gear will interfere. Interference can be avoided by slightly rounding the ends of the teeth on the larger wheels, the amount to be determined by drawing the teeth in different positions. In the interchangeable system, all the gears are made to run with the smallest one of the set by giving epicyclodial points to the interfering teeth, so that they will work smoothly with the radial flanks.

1570. The **smallest wheel** in the interchangeable series has 12 teeth, the same number as in the epicycloidal

system. The reason for this is that it is the smallest wheel having a contact of the parts of the teeth which are true involute curves during an arc of action, equal to the circular pitch. Wheels of ten teeth will run together, however, although the action is not correct.

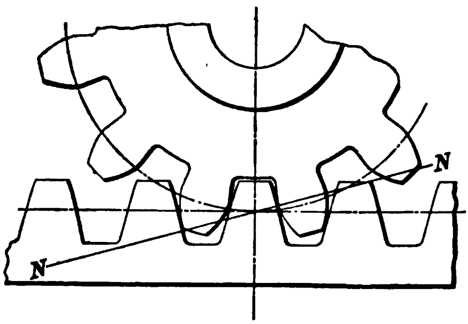


FIG. 426.

1571. The **involute rack** (see Fig. 426) has the sides of the teeth at an angle of 15° with the pitch line, or perpendicular to the line of action $N N$. The ends of the teeth should be rounded to run with the 12-tooth pinion.

1572. Internal Gears.—The construction is shown in Fig. 427. The obliquity (= 15°) is $T C N$, and the base

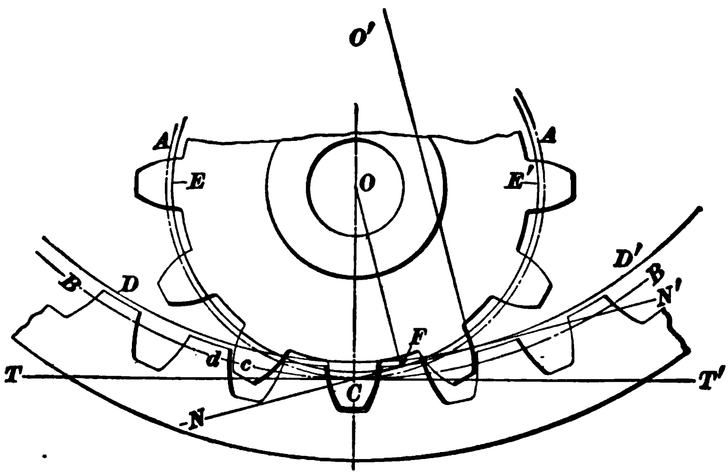


FIG. 427.

circles E E' and D D' are drawn tangent to the line of action N N', about the centers O and O', respectively. The addendum circle for the internal gear should be drawn

through F, the intersection of the path of contact $N N'$ with the perpendicular $O F$ drawn from the center of the pinion. The wheel will then be nearly, or quite, without faces, and the teeth of the pinion, to correspond, may be without flanks. If the two wheels are nearly of the same size, points c and d will interfere, which can be avoided by rounding the corners.

1573. In General.—Formerly, the epicycloidal system was used almost exclusively, but of late the involute is rapidly gaining in favor. Its distinctive features are, the adjustability of the centers of the wheel and the great strength of the tooth. The chief objection that has been raised against involute teeth is the obliquity of action, causing increased pressure upon the bearings. Where the obliquity does not exceed 15°, however, this objection is not a serious one.

BEVEL GEARS.

1574. In drawings of spur gears, the tooth curves in the epicycloidal system are obtained by causing the generating circles to roll upon the pitch circles. The tooth curves, however, represent curved surfaces perpendicular to the plane of the paper and the pitch circles, and generating circles represent the ends of cylindrical surfaces which are in rolling contact. It may be assumed, therefore, that tooth surfaces are generated directly by generating *cylinders* rolling upon pitch *cylinders.* In bevel gearing, the pitch surfaces are cones, which, when in rolling contact, have their apexes at a common point, and it may be assumed that the tooth surfaces are generated by generating *cones*, rolling upon the pitch cones.

In spur gearing, the teeth of two wheels bear along straight lines, which are perpendicular to the plane of the paper. In bevel gearing, the teeth are in contact along straight lines, *but these lines are perpendicular* to the surface of a sphere, and all of them pass through the center of the sphere, which is at the point where the apexes of the two pitch cones meet. That this is the case will now be explained.

1575. In Fig. 428, let $C\ O\ B$ represent a pitch cone, the part $C\ D\ E\ B$ being the pitch surface of a bevel gear, and let $A\ O\ C$ be the generating cone. If we suppose the generating cone to describe the tooth surface $m\ n\ o\ p$ by rolling upon the pitch cone, the line $n\ o$, representing the outer edge of the tooth, will lie upon the surface of a sphere whose radius is $O\ n$. For the point n, which describes this line, is always at a fixed distance from the center O; hence, every point in the line $n\ o$ is equally distant from O, and as, in a spherical surface, every point is equally distant from a point within called the center, it follows that $n\ o$ must lie upon a spherical surface.

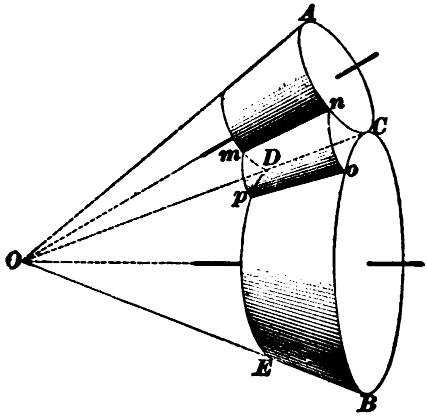


FIG. 428.

1576. To be theoretically exact, therefore, the tooth curves for bevel gears should be traced upon the surface of a sphere, as shown in Fig. 429. This method is not a practical one, however, and would have no advantage over what is known as **Tredgold's approximation,** which is much simpler and is universally used.

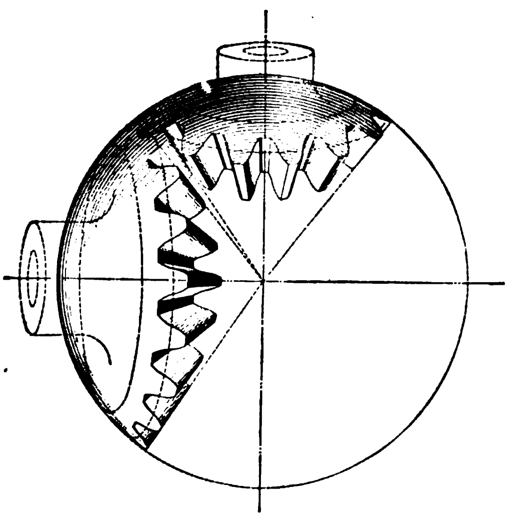


FIG. 429.

By this method, the tooth curves are drawn on cones tangent to the spheres at the pitch lines of the gears, as shown in Fig. 430. The process is simply to develop or unwrap the surfaces of the cones, the unwrapped surfaces being represented by $A\ B\ C$ and $C\ D\ E$ in the figure. The length of the arc $A\ B\ C$ is equal to the length of the pitch circle $A'\ C$, and the arc $C\ D\ E$ is equal to the pitch circle $C\ E'$. The gear teeth are then drawn upon the unwrapped surfaces, precisely

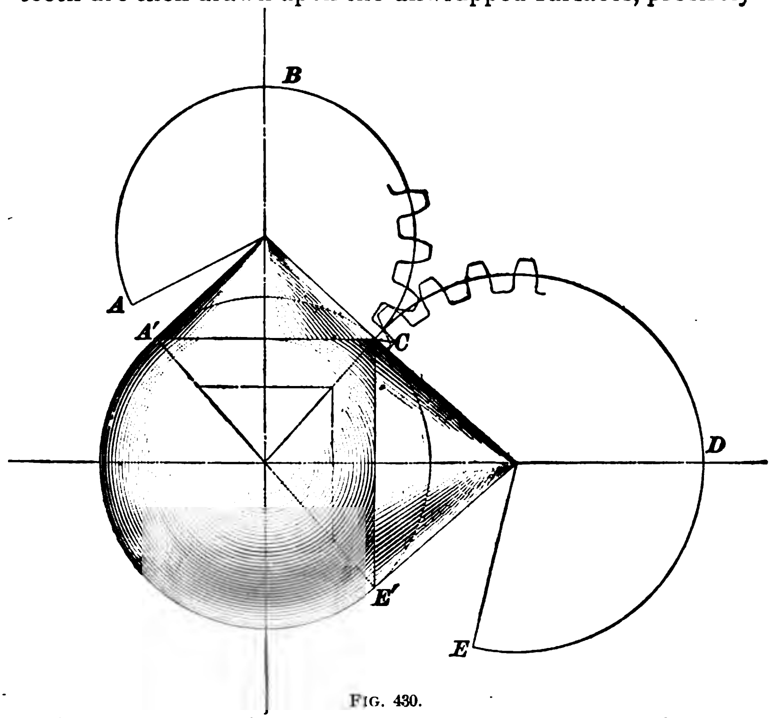


FIG. 430.

as for spur gears of the same pitch and diameter. This process is fully described in the subject of Mechanical Drawing.

The teeth, as laid out by Tredgold's method, will vary somewhat from the shape of the spherical teeth, though usually the variation is slight. The actual error of the system, however, is less than this difference, for though the tooth curves on each gear may not be the same as those on the sphere, the amount of their divergence from perfect curves to transmit a uniform motion will be of no practical importance.

WORM GEARING.

1577. A **worm** is a screw made to mesh with a wheel called a **worm-wheel,** the two forming **worm gearing,** or **screw gearing.** Worm gearing possesses the following characteristics:

I.—The velocity ratio depends upon the pitch of the screw, i. e., the distance the screw advances in one revolution, and not upon the diameters of the pitch cylinders. *If the worm is single-threaded, it must make as many turns as there are teeth in the wheel for every revolution of the latter; if double-threaded, it will make one-half as many turns.*

II.—The direction in which the worm-wheel turns depends upon whether the worm has a right-hand or left-hand thread.

III.—The end thrust of the screw causes the motion of the wheel.

1578. Form of Teeth.—Fig. 431 illustrates a worm and wheel. It will be noticed that in the longitudinal section, taken through the worm, the threads appear to be like involute rack teeth. The worm is usually made in a screw-cutting lathe, and as it is easier to turn the threads with straight sides, it is better that they should be of the involute form. Involute teeth should then be used on the wheel of a pitch to correspond with the threads on the worm.

1579. Pitch.—The circular-pitch system is almost universally used for worm gearing, because lathes are seldom provided with the correct change gears for cutting diametral pitches. It is not so inconvenient, however, in the case of worm gearing as with the spur gearing. If the diameter of the worm-wheel should come in awkward figures, the diameter of the worm can be made such that the distance between centers will be any desired dimension. The circular pitch of the gear must equal the pitch of the worm.

1580. Close-Fitting Worm and Wheel.—To make a close-fitting wheel, a worm is made of tool steel and then fluted and hardened similar to a tap. It is almost a duplicate

of the worm to be used, being of a slightly larger diameter to allow for clearance. This cutter, or **hob,** is placed in mesh with the worm-wheel, on the face of which notches have been cut deep enough to receive the points of the teeth

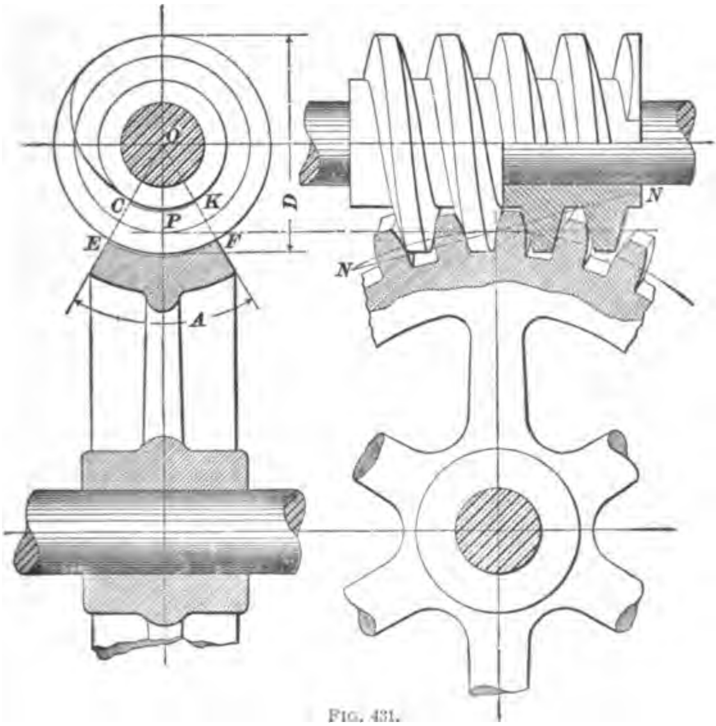


FIG. 431.

of the hob. The hob is then made to drive the wheel, and is dropped deeper into it at each revolution of the latter until the teeth are finished.

Fig. 431 represents a close-fitting worm and wheel. The pitch circles of each are in contact at *P*. The outside diameter *D* of the worm may be made four or five times the circular pitch. The arcs *C K* and *E F* are drawn about *O* and limit the addendum and root of the wheel teeth, the distance between them being the whole depth of the teeth. End clearance is allowed the same as for spur gearing. The

angle A is generally taken at either 60° or 90°. The whole diameter of the wheel blank can be obtained by measuring the drawing.

The object of hobbing a wheel is to get more bearing surface of the teeth upon the worm thread, making the outline of the teeth something like the thread of a nut.

1581. Worm-Wheels Like Spur Gears.—When worm-wheels are not to be hobbed, there is little to be gained by making the face of the wheel concave to fit the worm. It is better to construct the blanks like a spur-wheel blank. The teeth can then be cut in a straight path diagonally across the face of the blank to fit the angle of the worm thread.

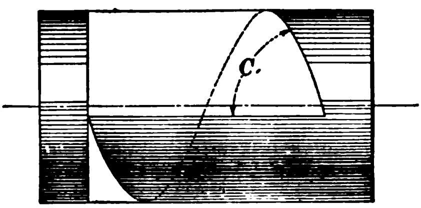


FIG. 432.

This angle may be obtained as follows: Fig. 432 represents a right-angled triangle cut out of paper and wrapped around a cylinder. The hypotenuse of the triangle forms the thread and, as the base is parallel with the axis of the cylinder, the angle of the worm thread is the angle C between the hypotenuse and the base of the triangle. *Hence, the tangent of the angle of the thread = the circumference of the cylinder, divided by the pitch.*

In Fig. 343 is shown a graphical method. The lines a a, b b, etc., are the development of the screw. Angle C' = angle C is the proper angle for the teeth of the worm-wheel. The distance $P' = P$, parallel to the axis of the screw, is the pitch.

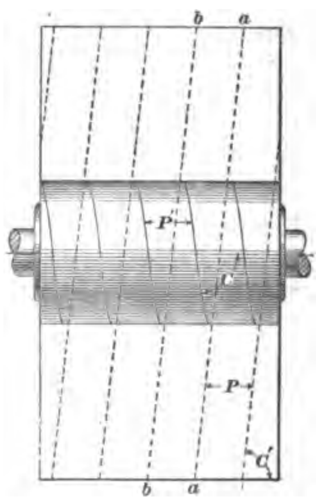


FIG. 433.

1582. Interference.—The same rules apply here that were given under the involute system. When the worm is to work with a wheel having a smaller number of teeth than it was designed for, interference will occur. It can be avoided by rounding the tops of the threads of the worm, but it is easier to make the wheel blank somewhat larger than is called for by the number of teeth. The teeth will then have very short flanks, the action being almost entirely upon the faces, where interference cannot occur.

RATCHET WHEELS.

1583. Fig. 434 represents a ratchet wheel *A* turning upon the pin *O*. *C* is a vibrating lever carrying the **pawl, click,** or **catch** *B*, which acts upon the teeth of the wheel. As the arm moves back, or right-handed, the click lifts and slides over the points of the teeth; when it returns, the click drops against a tooth and carries the wheel with it.

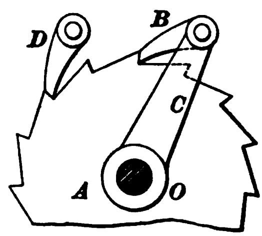


FIG. 434.

In case it should be desired to prevent the wheel from moving backwards when the click is moving backwards, a fixed pawl, similar to *D*, would be made to bear on the wheel and drop behind each tooth as it passed under. Here, *D* would allow a left-hand, but prevent a right-hand rotation of *A*.

1584. In Fig. 434, in order that the arm may produce motion in the wheel, its vibration must be at least sufficient to cause the latter to advance one tooth; but by arranging several clicks in the same lever it becomes possible to give a motion to the wheel corresponding to less than one tooth for each vibration of the arm. Fig. 435 shows such a construction, *B*, *B'*, and *B''* being proportioned so that they come into action alternately. Thus, when the wheel *A* has moved back an amount corresponding to one-third of a tooth, the click *B'* will be in contact with tooth

b', and, if the arm should then move the wheel forwards a distance of at least one-third a tooth and then return to its former position, click B'' would fall behind b'', ready to turn the wheel. Thus arranged, a slight motion is obtained and comparatively large teeth may be used. A neater construction than the above would be to put all the clicks on one pin side by side, in which case a wide wheel would be necessary.

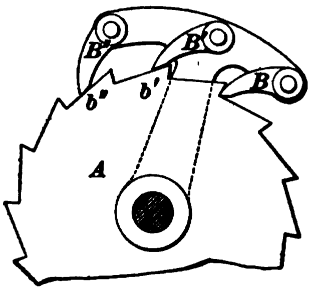


FIG. 435.

1585. Reversible Clicks.—In feed mechanisms, such as are used on shapers and planers operating on metal, and which must be driven in either direction, an arrangement like that in Fig. 436 is used. Wheel A has radial teeth, and the click, which is made symmetrical, can occupy either position B or B'. In order that the click may be held firmly against the ratchet wheel, its axis is provided with a small triangular piece, shown dotted, against which is a flat end-presser, always urged upwards by a spring (also shown dotted). Whichever position B may be in, it will be held against the wheel.

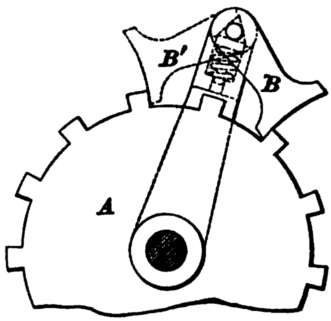


FIG. 436.

1586. Adjusting the Motion.—Ratchet wheels are largely used on machines requiring a "feed." In all such cases they must be so arranged that the feed can be easily adjusted. This is often done by changing the swing of the lever C, Fig. 434, which is usually connected by a rod with a vibrating lever, having a definite angular movement at the proper time for the feed to occur. This lever is generally provided with a **T** slot, in which the pivot for the rod can be adjusted by means of a screw and nut. By varying the

distance of the pivot from the center of motion, either one way or the other, the swing of the arm *C* can be regulated and the feed made to occur in either direction desired.

1587. Another method of adjusting the motion is shown in Fig. 437. The wheel turns upon a stationary shaft or stud *O*, and the end of the shaft is turned to a smaller diameter than the rest and is threaded, thus forming a shoulder against which an adjustable shield *S* can be clamped by the nut *n*. Back of the wheel is the arm, also loose on the shaft, carrying the click *B*, which latter should be of a thickness equal to that of the wheel, plus that of the shield. The teeth of the wheel may be made of a shape suitable to gear with another wheel to which the feed motion will then be imparted, or another wheel back of and attached to the one shown could be used for the purpose.

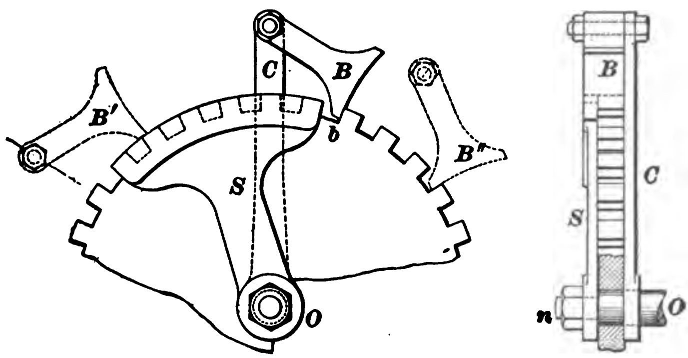


FIG. 437.

The extreme left-hand position of *B* is shown at *B'*. Here the click rides on the shield and does not come in contact with the teeth of the wheel. When the click comes to the right-hand edge of the shield, however, it will drop into contact at *b*; and, if *B''* is the extreme right-hand position, the wheel will be turned through a space corresponding to three teeth. If the shield should be turned to the right, a smaller number of teeth would be moved each time; if it should be turned to the left, a feed of four or more teeth, up to the full capacity of the stroke, could be obtained; while, with

the shield in its mid-position, it would carry the click during the whole swing of the arm and there would be no feed.

1588. Ratchet and Screw.—Where ratchets are employed in the feed motions of machine tools, they are made to operate a screw, which in turn drives the "head" carrying the tool.

EXAMPLE.—A ratchet having 80 teeth is attached to the end of a screw having six threads per inch. If the click is set to move the ratchet three teeth for every stroke of the arm, how much "feed" would the tool have, supposing it to be moved directly by the screw?

SOLUTION.—One turn of the screw would move the tool $\frac{1}{6}$ inch. But for each stroke, the ratchet and, hence, the screw, moves $\frac{3}{80}$ of a turn, and the tool would travel $\frac{3}{80} \times \frac{1}{6} = \frac{1}{160} = .00625$ inch. Ans.

APPLIED MECHANICS.

(CONTINUED.)

DYNAMOMETERS.

1589. **Dynamometers** are instruments for measuring power. They are divided into two main classes—**absorption dynamometers** and **transmission dynamometers.**

1590. The most common form of **absorption dynamometers** is the **Prony brake,** which consists simply of a friction brake designed to absorb in friction and measure the work done by a motor or the power given out by a shaft.

1591. A **transmission dynamometer** is used to measure the power required to drive a machine or do other work; then, to determine the power required to run the shafting in a mill, a transmission dynamometer would be interposed between the shafting and the source of power, and by suitable belt connections the shafting would be driven *through* the dynamometer, from which the power could be determined.

PRONY BRAKE.

1592. Fig. 438 represents a simple and common form of Prony brake. It consists of two wooden blocks A and B that are clamped together upon a pulley P, by the bolts and thumb-nuts c, c. The same bolts clamp on arm L to the upper block, from which a scale pan, bearing a known weight W, is suspended. The distance R, from the center of the

pulley to the perpendicular through the point from which the scale pan is suspended is also known. The counterweight w should be so adjusted as to just balance the extra length of L on the right, and the weight of the scale pan.

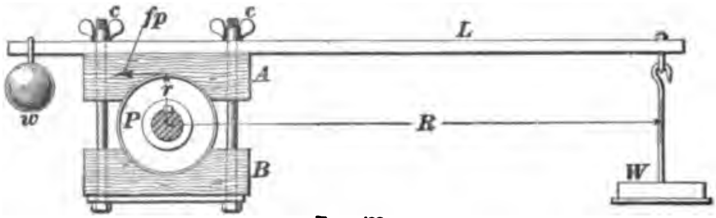


FIG. 438.

Suppose the pulley to revolve left-handed and the bolts c, c to be tightened until, with a weight W in the scale pan, the lever L will remain stationary in a horizontal position. *The foot-pounds of work absorbed by the brake can then be found by multiplying the weight W by the circumference of a circle whose radius is R (in feet) and by the number of revolutions of the pulley.*

It is important to note that neither the diameter of the pulley nor the pressure with which the blocks clamp the pulley enter into the calculations at all. For, letting p represent the pressure and f the coefficient of friction between the blocks and the pulley, the force at the face of the pulley tending to resist its rotation will be $f p$. The force tending to keep the lever L from turning, however, is W, and as the bolts are adjusted so that L remains constantly in a horizontal position, the moments of these two forces about the center of the pulley are equal, or $f p r = W R$.

1593. Now, the work done at the face of the pulley is equal to the force exerted $\times$ number of feet passed through, or, calling N the number of revolutions per minute, $2 \pi r \times f p \times N$. This, it will be seen, is the first member of the above equation multiplied by $2 \pi N$. Multiplying the second member by $2 \pi N$, also, to keep both members equal, we obtain as another expression for the work absorbed, $2 \pi R \times W \times N$. This is the formula used in calculations.

Hence, letting H. P. = number of horsepower absorbed;
R = length in feet of lever arm about center of shaft;
W = weight in scale pan;
N = number of revolutions per minute.

$$\text{H. P.} = \frac{2 \times 3.1416 \times R \times W \times N}{33,000}.$$

This may be reduced to

$$\text{H. P.} = .0001904\ R\,W\,N. \qquad \textbf{(156.)}$$

EXAMPLE.—A brake with an arm R, 6 ft. long, was placed on the fly-wheel of an engine. If the engine ran at 200 revolutions per minute, what power did it develop when the brake balanced with 14 pounds in the scale pan?

SOLUTION.—Applying formula **156**,
H. P. = .0001904 × 6 × 14 × 200 = 3.198 horsepower. Ans.

1594. Brakes are often constructed of a metal band which extends entirely around the pulley, the rubbing surface being formed of blocks of wood fitted to the inside of the band. A weight arm is attached to one side of the pulley and the friction is varied by means of a bolt and nut used to connect the two ends of the band.

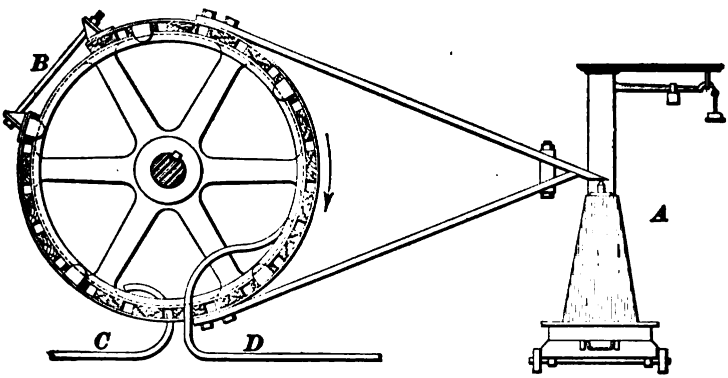


FIG. 439.

Instead of hanging weights in the scale pan the friction may be weighed on a platform scale, as shown in Fig. 439. In this case, the direction of rotation of both pulley and arm is the same.

It is essential that these brakes should be well lubricated, and for all except small powers means must be provided for conducting away the heat generated by friction. If there are internal flanges on the brake wheel, water can be run on to the inside of the rim, the flanges serving to retain the water at the sides and centrifugal force to keep it in contact with the rim. A funnel-shaped scoop can be used to remove

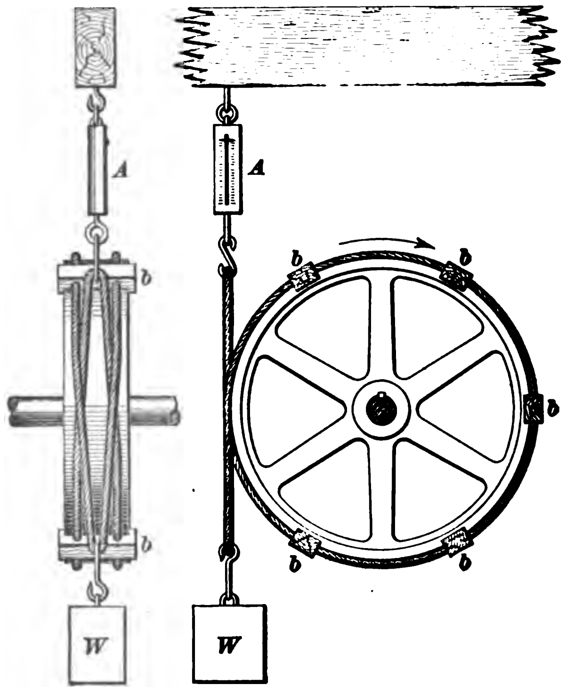


FIG. 440.

the water. It should be attached to a pipe and placed so as to scoop out the water, which should flow continuously. This arrangement is shown in Fig. 439.

1595. A rope brake, like that in Fig. 440, will give good results. The figure shows the construction so clearly that no description is necessary. To obtain the brake load, subtract the brake pull as registered by the spring balance from

the weight. In this case, the lever arm is r, equal to the radius of the pulley + $\frac{1}{2}$ the diameter of the rope. If this radius be given in inches, formula **156** becomes

$$\text{H. P.} = .00001586\, r\, W N \qquad \textbf{(157.)}$$

TRANSMISSION DYNAMOMETERS.

1596. There are several forms of transmission dynamometers regularly manufactured, but, as rules for their use always accompany the machines, only one form will be described here.

The side and end elevations of the **differential transmission dynamometer** are shown in Fig. 441. W_1 is a

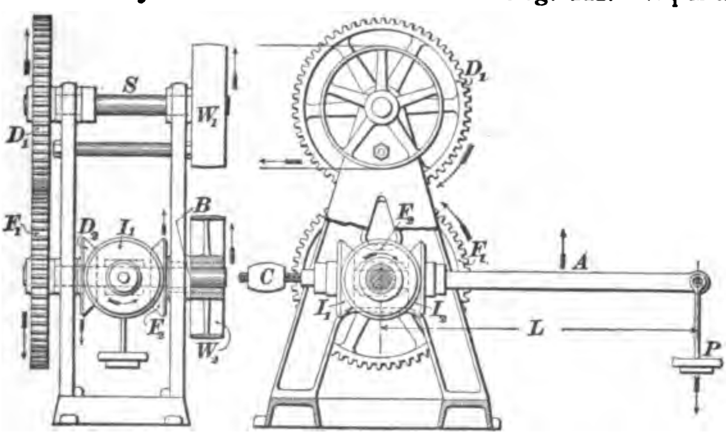


FIG. 441.

pulley to be belted up with the pulley on the line shaft, or other source of power, that has been driving the machine to be tested. W_2 (shown only in the end view), placed for convenience in the same plane with W_1, is of the same size as W_1, and is to be belted to the driving pulley on the machine. Connection between W_1 and W_2 is made by means of the shaft S, running in bearings on the frame, the two gears D_1, F_1, of equal diameters, and the differential gearing shown.

Of this latter, D_2 is keyed to the same shaft as F_1. F_2 is loose on the shaft and has a long hub reaching through the bearing B, and through the hub of the pulley W_2 to which

it is fastened. D_2 and F_2 are connected by the two miter gears I_1 and I_2 in the usual way, having their bearings on the arm A, and free to turn about the lower shaft. There is a scale pan at the right-hand end of the arm hung from a knife-edge, and it is clear that if a weight P be put in the pan sufficient to hold it down I_1 and I_2 will act simply as idlers, and F_2 will turn in the opposite direction and with the same speed as D_2. Hence, W_1 will turn in the same direction as W_2, and with the same speed.

A counterweight C is provided for balancing the arm when the dynamometer is at rest and there are no weights in the scale pan. To find the amount of power transmitted, the length L of the arm, the weight P in the scale pan, and the number of revolutions must be known. The arm is generally of such a length that the circumference which the knife-edge would describe if the arm revolved about the shaft would be some even number of feet, say ten.

From what has been stated before regarding bevel trains, it is evident that, if pulley W_1 is turned and F_2 is at the same time held fast so that it cannot turn, the arm will make one-half the number of revolutions made by W_1. Twice as great a weight will, therefore, be required in the scale pan to keep the arm stationary as would be necessary if the arm made the same number of revolutions. Hence, applying the principle of the Prony brake, and supposing that, in the circumference of the circle whose radius is L, the length of the arm is 10 feet, *two pounds in the scale pan will correspond to $1 \times 10 = 10$ foot-pounds of power transmitted per revolution of the shaft.*

EXAMPLES FOR PRACTICE.

(1) Given, lever arm of a Prony brake = 4.5 ft.; weight in scale pan, 2 lb. 4½ oz.; rev. per min. of pulley, 160. Required the power absorbed. Ans. .313 H. P.

(2) A rope brake is used on a pulley 36" in diameter. The diameter of the rope is ⅝"; revolutions of pulley per minute, 200. How many horsepower are absorbed, the weight being 210 lb. and the balance reading being 5 lb.? Ans. 11.9 H. P.

(3) An actual test with a rope brake showed a mean brake horsepower of 15.23. The mean number of revolutions of the wheel per

minute was 205, the weight used was 157 lb., and the mean back pull on the balance was 4 lb. What was the length of the lever arm?

Ans. 30.6 inches.

VALVE GEARS.

PLAIN SLIDE-VALVE.

1597. It is assumed that the student has studied carefully the discussion of valves and their relation to the action of the steam in the cylinder, as given in Art. **1232,** etc. We shall now treat more especially of the kinematics of valves and valve motions.

1598. Definitions.—For the purpose of review a few definitions will be repeated here in a brief form. They will also be convenient for reference.

The four principal points or events during one stroke of an engine are:

I.—The point of **admission** of steam to the cylinder.

II.—The point of **cut-off** of the steam from the cylinder.

III.—The point of **release** where the steam begins to be exhausted from the cylinder.

IV.—The point of **compression** where exhaust closes.

1599. Fig. 442 gives a sectional view of a plain slide or

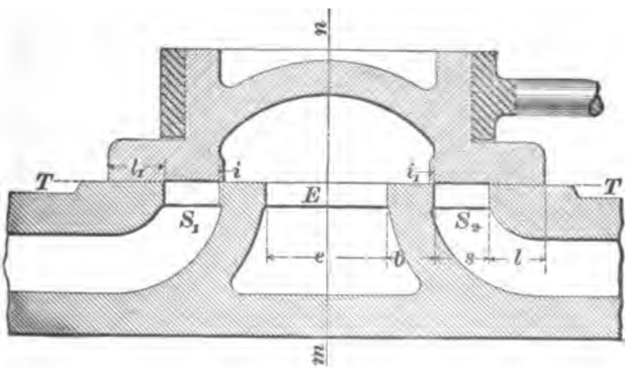


FIG. 442.

D valve. The under surface of the valve, called the **valve face,** slides over the **valve seat** *T T* on the cylinder. In

the cylinder are three **ports.** Two, S_1 and S_2, communicate with passages leading to the ends of the cylinder and are called **steam ports;** the third, E, leads to the atmosphere or condenser, and is termed the **exhaust port.**

In Fig. 442, the valve is in *mid-position* with its center n in line with the center m of the exhaust port E.

1600. The **lap** is the amount by which the edge of the valve overlaps the adjoining edge of the steam port when the valve is in mid-position. It is called the **outside** or **inside lap,** according as we refer to the outside or inside of the valve, l or i and l_1 or i_1 in the figure.

1601. **Lead** is the amount of the opening of the steam port at the beginning of the piston's stroke. The **lead angle** is the angle made by the center line of the crank with the center line of motion of the engine at the point of admission.

1602. The **displacement of the valve** is the distance that the center of the valve has moved from its mid-position.

1603. The **travel of the valve** is the total distance the valve moves one way. The **stroke** of an engine and **travel** of a valve are relatively similar terms.

1604. The valve is moved by an eccentric whose **throw** is equal to the diameter of the circle described by the center of the eccentric as it turns with the shaft. The radius of this circle is known as the **eccentricity.** The throw of the eccentric and travel of the valve are the same, if there is no intervening rocker which increases or decreases the action of the eccentric.

1605. The **angle of advance** of the eccentric is the angle by which the center line of the eccentric stands away from a line at right angles to the center line of the crank. The angle of advance is sometimes called *angular advance.* It is equal to the angle due to the lap + the angle due to the lead.

1606. Direction of Rotation.—The study of the slide-valve is essentially a study of the relative motions of the piston of the engine and the valve. The first thing to understand is the direction in which the crank will turn. This depends upon the way the eccentric is connected with the valve and upon the location of the angle of advance.

The eccentric may be connected with the valve in three ways: First, the eccentric rod may act directly upon the valve spindle; second, it may act through a rocker pivoted at one end, of the nature of a bell-crank; third, it may act through a reversing rocker pivoted near the center. In the first two instances, the valve will move with the eccentric, and the connection may be said to be direct. *In these cases, the eccentric will always be in advance of the crank, in the direction in which the crank is to turn, by an angle equal to 90° + the angle of advance.* That is, the crank will follow the eccentric.

The action of a reversing rocker is simply to cause the valve to move in a direction opposite to that in which the eccentric is moving. *Hence, when a reversing rocker is used, the eccentric will be behind the crank by an angle equal to 90°—the angle of advance.* That is, the crank will lead, and the eccentric will take positions exactly opposite to those in the previous case.

In what follows, the eccentric rod will be assumed to act directly on the valve spindle, unless stated to the contrary.

1607. Displacement of the Valve.—The next thing to consider is the position of the valve for any given position of the piston. Suppose at first that the crank and valve are actuated by slotted cross-heads like the one in Fig. 350, Art. **1442**, instead of by the usual connecting-rod and eccentric. Then, the displacement of the valve for any given piston position can be found as follows:

Draw the outer semicircle *A B C* (Fig. 443) on one side of the stroke line *A C*, with a radius *O R*, equal to the length of the main crank. This will represent the path of the crank-pin *R* during one stroke. From the same center, draw the

inner semicircle with a radius $O\,r$, equal to the length of the valve crank. Now, suppose the piston to have moved along its stroke $A\,C$ a distance of $A\,n$. The crank will then be in the position $O\,R$, R being perpendicularly above n. If we let a equal the angle of advance, the valve crank will

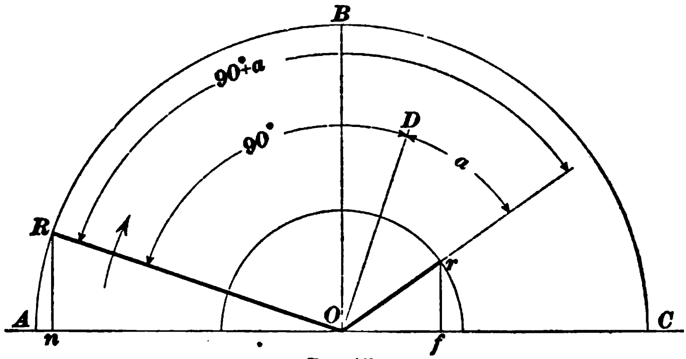


FIG. 443.

be at $O\,r$, ahead of the main crank by the angle $r\,o\,R$, equal to $90 + a$, and it is evident that the displacement will then be equal to $O\,f$. By valve crank is meant an imaginary crank, supposed to replace the eccentric, which has a radius equal to the radius of the eccentric.

1608. A more convenient method of finding the displacement is shown in Fig. 444. It forms the basis of a

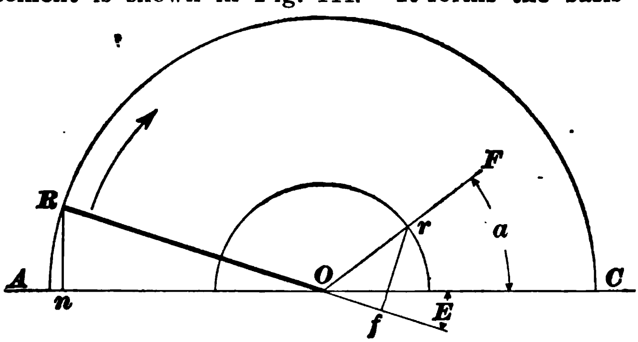


FIG. 444.

valve diagram, which will be explained later, and by which a slide-valve can be correctly proportioned, and examples involving lead, lap, angular advance, etc., can be solved.

Let the crank-pin paths be drawn as before on one side of the stroke line $A\ C$. From O, draw the line $O\ F$, making an angle a, with $O\ C$ equal to the angle of advance. Then, the length of a perpendicular drawn from r, the point of intersection of $O\ F$ and the inner circle, to the center line of the main crank, in whatever position it may be, will represent the displacement of the valve. In this case, the crank position is $O\ R$, and $r\ f$, perpendicular to the center line produced beyond O, is the displacement.

1609. Effect of Connecting-Rod.—Now, suppose that an engine has the crank and valve moved by a connecting-rod and eccentric, as in practice. It has previously been shown that in a crank motion, the longer the connecting-rod compared with the crank the more nearly the motion will approach that given by a slotted cross-head. Since in most engines the eccentric rod is very long compared with the eccentricity, the relative positions of the valve and cranks can be determined with sufficient accuracy by the foregoing method. The connecting-rod, however, is ordinarily only from four to six times the length of the crank. Hence, if it be required to accurately find the relative positions of the *crank* and *piston*, and hence the *valve* and *piston*, the effect of the connecting-rod should be taken into account.

1610. EXAMPLE.—Given, the length of the stroke of an engine, the travel of the valve, the angle of advance, and the length of the connecting-rod. What are the valve displacements at half stroke each way?

SOLUTION.—Describe the crank circle (Fig. 445) $A_1\ R_1\ C_1\ R_2$, with a radius equal to half the stroke. About the same center O describe the

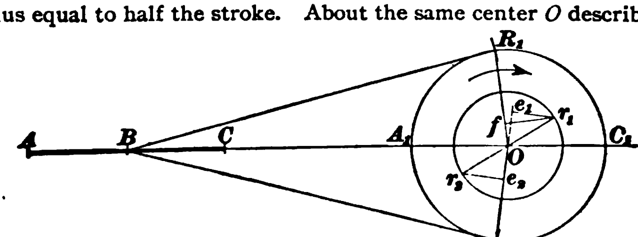


FIG. 445.

inner circle $r_1\ r_2$, which we will call the eccentric circle, with a radius equal to half the valve travel, or eccentricity. Draw the line of motion

$A\,C_1$, and on it lay off $A\,A_1$, $B\,O$, and $C\,C_1$, each equal to the length of the connecting-rod, giving the stroke $A\,C$ and the mid-position B of the piston. With B as a center and a radius equal to $B\,O$, cut the crank circle at R_1 and R_2, which will give the crank positions $O\,R_1$ and $O\,R_2$, corresponding to the mid-positions of the piston.

From O, draw $O\,r_1$, making an angle with $O\,C_1$ equal to the angle of advance of the eccentric, and intersecting the eccentric circle at r_1. Then, the perpendicular $r_1 f$, from r_1 to the crank position $O\,R_1$, is the displacement of the valve for that crank position. Likewise $r_1\,e_1$ drawn perpendicular to $O\,R_2$ extended is the displacement for crank position $O\,R_2$, the construction being precisely as in Fig. 444.

It often makes the diagram clearer to lay off the angular advance below the line of motion also, as shown at $O\,r_2$. *The upper point r_1, from which to measure the displacement, is then used for crank positions from $O\,r_2$ to $O\,r_1$ and the lower point r_2 for positions from $O\,r_1$ to $O\,r_2$.* In this case, the valve displacement for crank position $O\,R_2$ is $r_2\,e_2$, which it will be seen is the same as $r_1\,e_1$.

1611. Port Opening.—Suppose the valve in Fig. 442 to move to the left. Admission of steam through port S_2 will take place when the valve has moved a distance l equal to the lap, and the port opening will increase until the valve reaches the end of its travel, when *maximum port opening* will occur. This is not necessarily equal to the width of the port, as it is sometimes made less and sometimes greater. The amount that the port is open at any time, however, is evidently equal to the displacement from mid-position to the left minus the lap.

The movement of the valve to the left also opens port S_1 to the exhaust, the amount that it is open being equal to the displacement of the valve to the left minus the inside lap i. In like manner, the opening of S_1 and S_2 to steam and exhaust, respectively, is governed by the laps l_1 and i_1 and the amount of the displacement of the valve to the right.

1612. Diagram for Plain Slide-Valve.—Since, as we have seen, the port opening is equal to the displacement minus the lap, it can always be determined from the displacement diagram previously explained, provided the lap is known. Moreover, as the points of admission, cut-off, compression, and release occur when the port openings to steam

and exhaust are zero, the crank and piston positions for these points can easily be found.

On the following pages are a series of valve diagrams, a sectional view of a slide-valve and ports being placed under each one. Each sectional view is drawn to the scale of the diagram above it and shows the piston and valve positions corresponding to the diagram. In these diagrams, the distance the valve has moved from mid-position was found by the method already explained, and the port opening and, hence, the points of cut-off, compression, etc., by taking account of the laps. As a matter of convenience, the effect of the connecting-rod has been neglected.

1613. In Fig. 446, let $A\ C$ represent the stroke, $A\ B\ C$ the crank circle, and $a\ b\ c$ the eccentric circle. The piston is

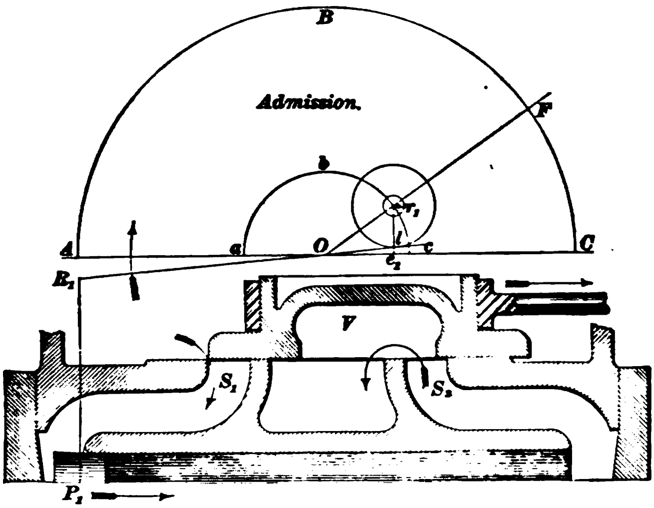


FIG. 446.

supposed to move from left to right, and angle $F\ O\ C$ is the angle of advance.

If the crank position should be $O\ A$, the displacement of the valve would be $r_1\ e_1$, the perpendicular from r_1 to $O\ A$ extended; and if $r_1\ l$, equal to the lap, is laid off on $r_1\ e_1$, $l\ e_1$ will be the port opening, since the port opening equals the

displacement minus the lap. As the port opening at a dead point is the lead, $l\,e_1$ will equal the lead.

1614. Now, with $r_1\,l$ as a radius, a circle is described about r_1, which we will call the lap circle. Also, an inside lap circle is described about the same center with a radius equal to the inside lap. Then, if the crank is in position $O\,R_1$, so that, when extended, its center line will be tangent to the outside lap circle, the displacement of the valve will be equal to the outside lap, and the valve will be at the point of admission. The sectional view of Fig. 446 shows the valve V in this position, with steam just entering the cylinder through port S_1. In the meantime, steam is being exhausted from the other end of the cylinder through port S_2. The center of the piston is at P_1.

1615. Fig. 447 shows the crank position at $O\,R_2$, the piston being in the corresponding position at P_2. $O\,R_2$ is

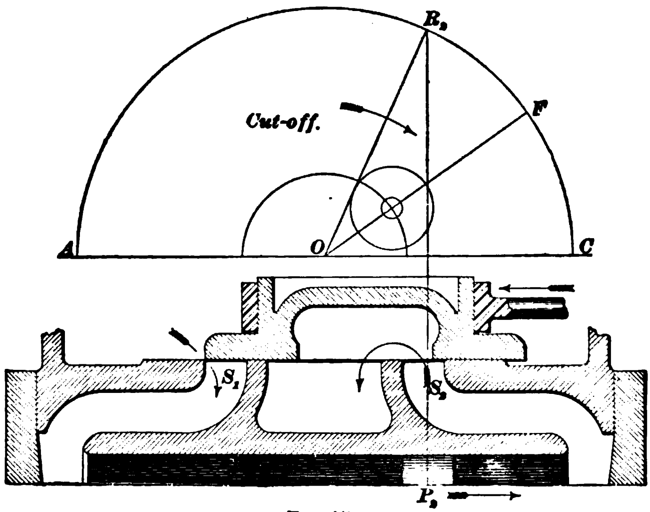


FIG. 447.

tangent to the lap circle, the displacement is again equal to the lap, and steam is cut off from the left end of the cylinder, but continues to exhaust from the right end of the cylinder. The valve is in the same position as before when

it was just opening S_1, but now it is moving in the opposite direction and is just closing the port.

1616. The next event to take place is the closing of

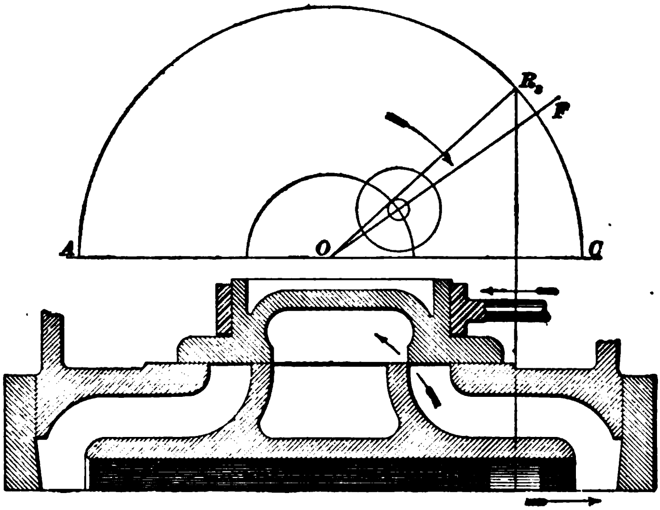


FIG. 448.

port S_2 to the exhaust at the point of compression. As the piston moves to the right and nears the end of its stroke,

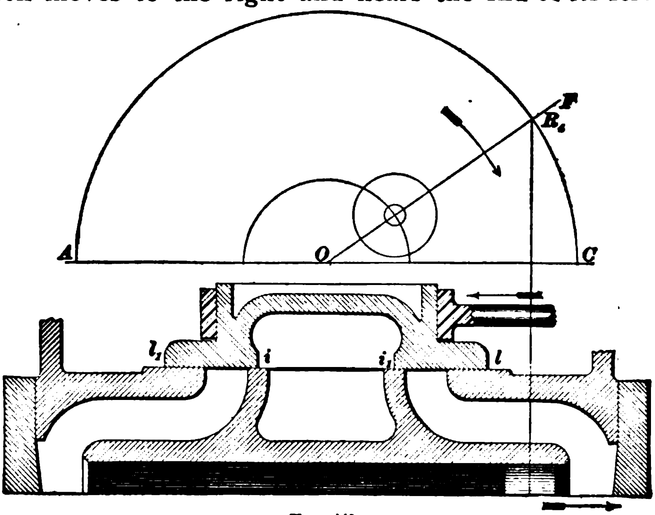


FIG. 449.

the crank reaches position $O\ R_3$, Fig. 448, tangent to the inside lap circle. The displacement is, therefore, equal to the inside lap, the valve is closed, and the steam enclosed in the right end of the cylinder will be compressed during the remainder of the stroke.

1617. In Fig. 449, the crank has reached the line of the angle of advance. The displacement is zero, bringing the valve in mid position as shown. Heretofore, the valve has been displaced to the right of the center line $m\ n$ (Fig. 442) of the exhaust port, and the acting edges of the valve have been l_1 and i_1. Now, the valve is to be displaced to the left, edges i and l are to act, so the plan before referred to of laying off the angle of advance below $A\ C$ has been adopted.*

1618. In Fig. 450, this has been done, and with r_2, the intersection of the angle of advance line $O\ F_2$, and the

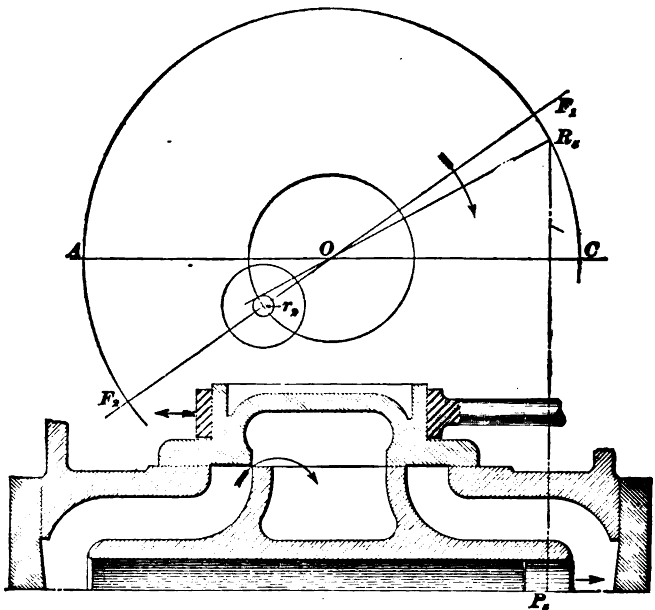


FIG. 450.

* Evidently where the laps on each end are the same it is not necessary to do this, but it is clearer and more consistent to do so.

eccentric circle as a center, the two lap circles were drawn corresponding to laps l and i, which in this case are equal to the other laps. Now, suppose the crank line produced to be $O R_5$, tangent to the inside lap circle. The valve will be at the point of release, and the steam, which has been expanding in the left end of the cylinder, will discharge. The piston is at P_5, very near the end of the stroke, and steam will shortly be admitted to the right side of the piston. Then will follow cut-off, compression, and release, as before, only for the opposite ends of the cylinder.

1619. There is but one other new position to be considered; that is, maximum port opening. Fig. 451 shows

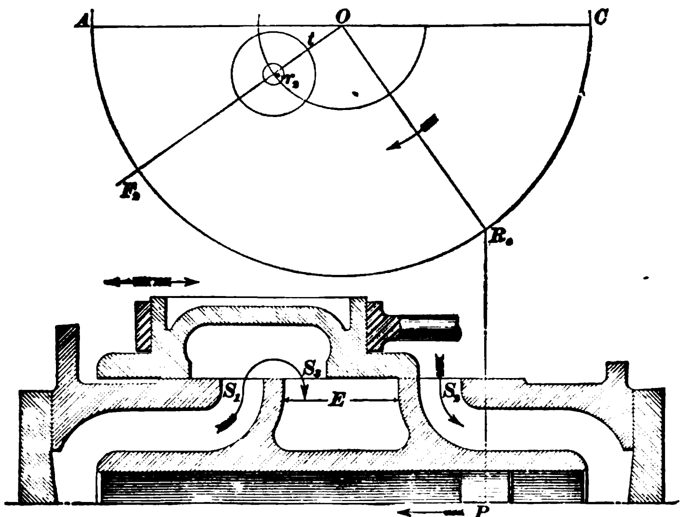


FIG. 451.

the crank at $O R_6$ at right angles to the angle of advance line $O F_2$, and the piston moving to the left on the return stroke. The valve displacement is the perpendicular distance from r_2 to $O R_6$, which is $O r_2$, the greatest it can possibly be, and the port opening is $O t$. From the sectional view, it will be seen that port S_2 is wide open to take steam, and port S_1 is wide open to the exhaust.

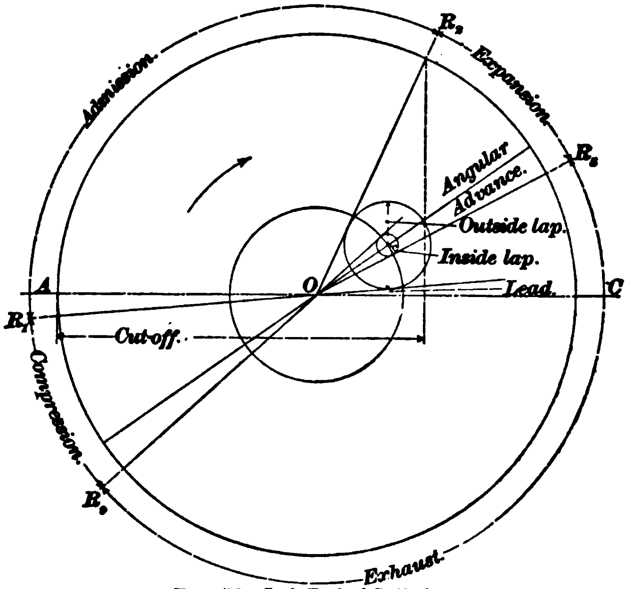


FIG. 452. Left End of Cylinder.

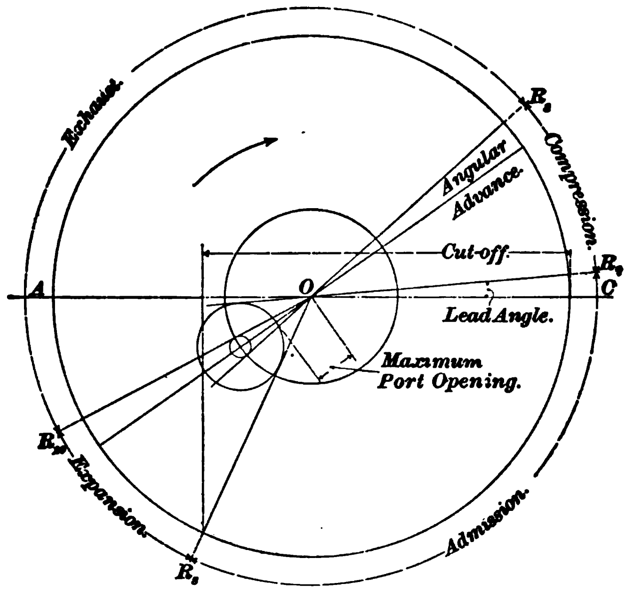


FIG. 453. Right End of Cylinder.

1620. Separate Diagram for Each End of the Cylinder.—Figures 452 and 453 show these various diagrams combined. To make them clearer, the events that take place in the left end of the cylinder during one revolution are represented in Fig. 452, and those that occur in the right end, in Fig. 453. In Fig. 452, admission begins at crank position $O\,R_1$, cut-off takes place at $O\,R_2$, release at $O\,R_5$, and compression begins at $O\,R_6$. In Fig. 453, for the other end of the cylinder, these four events occur at $O\,R_7$, $O\,R_8$, $O\,R_{10}$, and $O\,R_9$.

SLIDE-VALVE DESIGN AND PROBLEMS.

1621. In designing a slide valve, the sizes of the steam ports and the maximum port opening must be calculated and the width of the bridge determined. By **bridge** is meant the wall between the steam and exhaust ports. Then, when the lead and points of cut-off and release or compression have been decided upon, the laps, travel, and angle of advance can be found by means of the diagram, and the width of the exhaust port can be calculated. Sometimes the travel of the valve is fixed at first, leaving the laps and angle of advance to be found.

1622. Width of Steam Port.—In a case like that of a slide-valve engine, where the same passages are used both for admitting and exhausting steam, the steam ports should be designed to allow a free exhaust with the smallest possible back pressure. Evidently the high-pressure steam from the boiler does not require as large passages as the low-pressure exhaust steam, so that satisfactory action can be obtained by giving a port opening during admission less than the full width of the port, but wide enough to prevent a reduction in the pressure of the steam, called **wire-drawing.** Experiments show that a mean steam velocity of 100 feet per second, or 6,000 feet per minute, will give a good exhaust, and that a port opening of from .6 to .9 the width of the port, according to the conditions, will give a free admission.

The width of the port for a given area depends, of course, upon its length, which should be made equal to the diameter of the cylinder, or somewhat less.

EXAMPLE.—Given, diameter of cylinder, 12 inches; piston speed, 600 feet per minute; length of ports, 10 inches. What should be the width of the ports and port opening?

SOLUTION.—With a mean steam flow of 6,000 feet per minute, we would have the area of the port in square inches $\times$ 6,000 = the area of the piston in square inches $\times$ the piston speed in feet per minute, or,

$$\text{area of port} = \frac{\text{area of piston} \times \text{piston speed}}{6{,}000}.$$

$$\text{Here, } A = \frac{12^2 \times .7854 \times 600}{6{,}000} = 11.3 \text{ square inches.}$$

This, divided by 10, gives a width of port of $1\frac{1}{8}$ inches, nearly. Ans.

For the width of the port opening, we have $1\frac{1}{8} \times .9 = 1$ inch, nearly. Ans.

1623. Width of Exhaust Port.—The exhaust port must be wide enough to prevent a reduction of its area to less than the area of the steam ports when the valve is at its maximum displacement. In Fig. 451, where the valve is in that position, the opening S_3 into the exhaust port is equal to the width of the steam port S_1 or S_2. The total width E is equal to S_3, plus the distance the inside edge of the valve has traveled from mid-position, after taking out the distance it has traveled over the bridge. Hence, the

Rule.—*To find the width of the exhaust port, add together the width of the steam port, half the travel of the valve, and the inside lap; from their sum, subtract the width of the bridge.*

When designing a valve, the travel and inside lap are usually two quantities to be determined, making it necessary to leave the calculation of the width of the exhaust port until the last.

1624. Width of Bridge.—The width of the bridges between the steam and exhaust ports is generally made about equal to the thickness of the cylinder wall, so that the shrinking will be equal when the casting cools in the

mold. In any case, they must be wide enough, so that the outside edges of the valve will not uncover the exhaust port. From Fig. 442, it is clear that the maximum displacement one way must not be greater than the width of the bridge + the width of the steam port + the outside lap, $= b + s + l$; it should be $\frac{1}{4}$ inch less than this in medium-sized engines to insure a steam-tight joint when the valve is at maximum displacement.

1625. Amount of Lead.—In general, it can be stated that the lead in stationary engines varies from zero to $\frac{1}{4}$ inch, and is generally not far from $\frac{1}{16}$ inch. No *rule* can be given, however. The amount of lead must be determined for each particular case, sometimes by experiment after the engine is erected.

Lead serves to give the piston full steam pressure at the beginning of the stroke. The tendency of a small lead is to cause the piston to move under a reduced pressure through part of the stroke, especially when the ports are small and the clearance space large. On the other hand, little or no lead gives good results with some engines where the compression is sufficient to produce a pressure at the beginning of the stroke nearly or quite equal to boiler pressure. A quick-acting valve requires less lead than one opening slowly.

1626. Point of Cut-Off.—By turning to Figs. 452 and 453, it will be perceived that an early cut-off on a plain slide-valve engine necessitates an early compression, which becomes excessive when the cut-off takes place before about $\frac{2}{3}$ stroke. Hence, a plain slide-valve is seldom arranged to cut off earlier than $\frac{2}{3}$ or $\frac{3}{4}$ stroke, except in the case of high-speed engines and locomotives, where the compression is not so objectionable and, indeed, is often an advantage.

1627. General Problems.—Given, stroke of an engine, 18 inches; length of connecting-rod, 45 inches; cut-off, $\frac{2}{3}$ stroke; release, $\frac{11}{12}$ stroke; lead, $\frac{1}{8}$ inch; width of steam ports, $1\frac{1}{8}$ inches; maximum port opening, 1 inch; to

find the inside and outside laps, travel of the valve, the angle of advance, and to draw a section of the ports and valve.

At first it will be convenient to consider the motion as harmonic, neglecting the effect of the connecting-rod. In laying out a diagram, it is ordinarily inconvenient to draw the crank circle full size, and sometimes the eccentric circle is drawn to a reduced scale also. In this case, we will draw the crank circle to a scale of 1 inch = 6 inches, and the valve circle half size. All the measurements and parts of the diagram pertaining to the valve will, of course, be half size also.

1628. From the center *O*, Fig. 454, upon the stroke line, describe the crank circle *A C*. To find the outside lap,

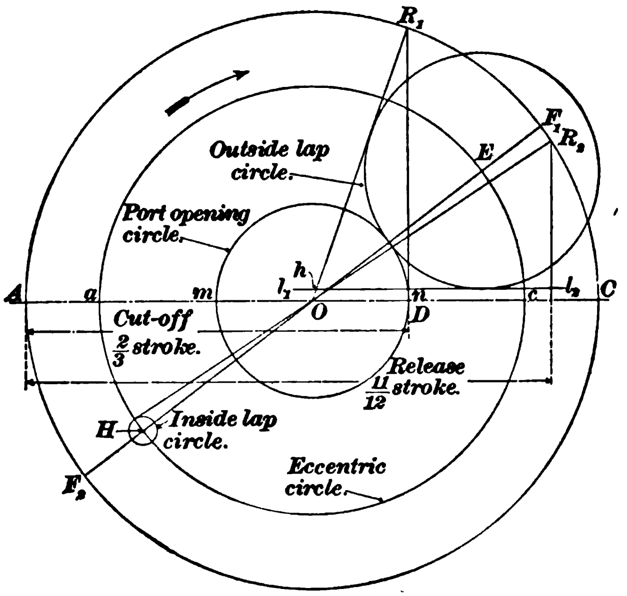


FIG. 454.

we have given the point of cut-off, the maximum port opening, and the lead. Hence, describe the circle *m n* about *O* with a radius equal to the maximum port opening; draw an indefinite line l_1 l_2 parallel to *A C* and above it a distance

equal to the lead; finally, draw the crank position $O\ R_1$ for cut-off at $\frac{3}{4}$ stroke, R_1 being perpendicularly above D, laid off on the stroke line, as shown in the figure. D happens to fall on the port opening circle in this instance, but does not necessarily do so. Now, it will be evident by reference to Figs. 446, 447, and 451, and the accompanying matter, that a circle drawn tangent to $O\ R_1$, $l_1\ l_2$, and circle $m\ n$ will be the outside lap circle, the radius of which will equal the outside lap of the valve. The center of this circle is found to be at E. It can be located readily by bisecting the angle $R_1\ h\ l_2$; the center must then fall at some point on the bisector.

1629. To determine the valve travel, we have simply to draw the eccentric circle $a\ c$, having the center O, through the point E. The diameter of this circle will be the travel.

1630. We have at once, also, the angle of advance by drawing the line $F_1\ F_2$ through points E and O, making the angle $F_1\ O\ C$ with $A\ C$.

1631. Finally, to obtain the inside lap, draw crank position $O\ R_2$ for $\frac{11}{12}$ of the stroke. $O\ R_2$, being beyond the angle of advance line $O\ F_1$, should properly be produced beyond O. As the laps are equal, this is not necessary, however. The circle with center at H drawn tangent to $O\ R_2$ produced will then be the inside lap. Measuring the diagram, we obtain the following dimensions, nearly: Travel, $4\frac{7}{16}$ inches; outside lap, $1\frac{7}{32}$ inches; inside lap, $\frac{1}{8}$ inch; angle of advance, 37°.

The section of the valve and ports is shown in Fig. 442, which is one-fourth size. To draw the valve, its length, or the distance between the edges of the valve faces, must be known, and is easily determined by first drawing a section of the ports. By the rule previously given, the width of the exhaust port should be $1\frac{1}{8} + 2\frac{3}{16} + \frac{1}{8} - 1 = 2\frac{7}{16}$ inches. It is drawn $2\frac{1}{2}$ inches wide, $1\frac{1}{4}$ inches on each side of the center line $m\ n$. The bridges are drawn 1 inch thick on the assumption that the cylinder walls are of that thickness, and the steam ports are $1\frac{1}{8}$ inches wide. Now, having completed the

section of the ports, the outside laps should be laid off outward from the outside edges of the steam ports and the inside laps from the inside edges. The valve faces will then be determined and the valve section can be completed.

1632. Equalizing the Cut-Off.—This valve is designed to cut off at $\frac{3}{4}$ stroke and to have a lead of $\frac{1}{8}$ inch, without considering the irregularity produced by the connecting-rod. Should an indicator be applied to the completed engine, however, it would show that the cut-off occurred *later* than $\frac{3}{4}$ stroke on the forward stroke and *earlier* on the return. In other words, steam would be admitted to the head end for a longer time than to the crank end. One way to overcome this is to give more lap to the end of the valve towards the head end of the cylinder, which will hasten its action during the forward stroke, and to reduce the lap on the other end, in order to retard the action on the return stroke.

1633. Fig. 455 shows a diagram for the valve laid out in this way. $O\,R_1$ and $O\,R_2$ are the crank positions at cut-off, and H is the center of the smaller outside lap circle. The points R_1 and R_2 are determined by laying off the distance $A\,n$, on the line $A\,C$, equal to $\frac{3}{4}\,A\,C$. Then, produce $A\,C$ to the left a distance equal to or greater than the length of the connecting-rod. With a radius equal to the length of the connecting-rod to the same scale as that to which the crank-pin circle was drawn—that is,

$$\frac{45}{6} = 7\tfrac{1}{2}'',$$

and a center on $A\,C$ produced, describe the arc $n\,R_1$, which cuts the crank-pin circle at R_1. R_1 is the position of the crank-pin when the steam is cut off from the head end of the cylinder. Also, lay off $C\,m$ equal to $\frac{3}{4}\,A\,C$, and with the same radius and a center on the line $A\,C$ produced describe the arc $m\,R_2$, intersecting the crank-pin circle at R_2. R_2 is the position of the crank-pin when the steam is cut off from the crank end of the cylinder. Draw $O\,R_1$ and $O\,R_2$. The valve travel remains the same; hence, drawing the eccentric

circle (sometimes called the valve circle), the center of the lap circle must lie on this circle. Consequently, bisecting the arc included between $O R_1$ and $n c$, the point E is obtained, which must be the center of the outside lap circle. Drawing $E H$ through O, it intersects the valve circle in H. Draw $O R_2$, and with H as a center describe a circle which shall be tangent to $O R_2$. This circle is the other outside lap circle.

From this it will be perceived that equalizing the cut-off by varying the laps is done at the expense of the lead, which in this case is nearly $\frac{1}{2}$ inch for the return stroke. It also causes an unequal port opening, which is of minor

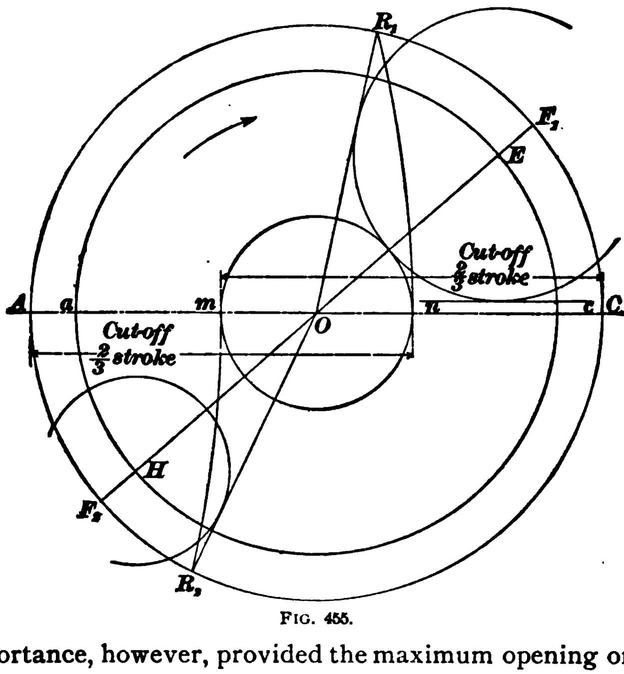


FIG. 455.

importance, however, provided the maximum opening on the end at which it is the least is sufficient.

1634. A better way of equalizing the cut-off is to provide for the use of a rocker which, if properly designed, will neutralize the irregularities due to the connecting-rod

at the points of cut-off, without disturbing the leads. The method of procedure is as follows:

First, determine from the valve diagram the travel, lap, angle of advance, etc., *for one end*, taking account of the connecting-rod. Next, lay out a diagram like Fig. 456, where $A\ C$ is the stroke of the cross-head, $D\ F\ B\ H$ is the crank-pin circle, and $h\ d\ f\ b$ is the valve circle, with a diameter in this case smaller than the travel of the valve, because of the multiplying effect of the rocker. Cut-off is to take place at $\frac{3}{4}$ stroke. Crank positions corresponding are $O\ F$ and $O\ H$, and we will let crank positions at admission be $O\ D$ and $O\ B$.

When a direct-acting rocker is used, the eccentric must be 90° + the angle of advance ahead of the crank. In the figure, eccentric positions $O\ f$ and $O\ h$ are laid off 90° + the angle of advance (as found from the diagram) ahead of crank positions $O\ F$ and $O\ H$. In like manner, eccentric

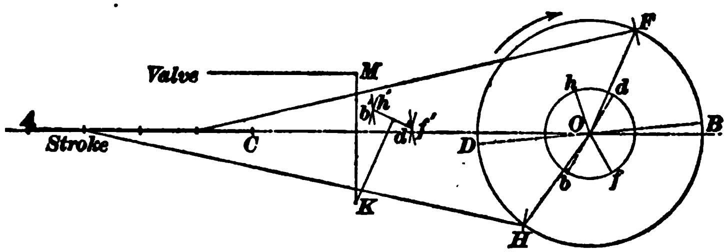


FIG. 456.

positions $O\ d$ and $O\ b$ are drawn corresponding to the points of admission.

Now, we know that a slide-valve is in the same position at both cut-off and admission. Hence, with a radius equal to the length of the eccentric-rod, and with admission and cut-off points d and f as centers, strike arcs d' and f'. The point of their intersection will be the point at which the eccentric-rod pin should be at admission and cut-off on the forwards stroke. In like manner, the intersection of arcs b' and h' gives the point for the return stroke. Connecting these points and erecting a perpendicular halfway between them, we have the central position of one arm of the rocker

The other arm $K M$ should be perpendicular to the valve stem at the central position, and the point of intersection K must be chosen so as to make the two lever arms proportional to the throw of the eccentric and the travel of the valve, respectively.

By methods similar to the foregoing, the release, or compression, may be equalized.

1635. Whenever, for any purpose, a rocker is used that either increases or diminishes the action of the eccentric on the valve, the valve's travel is to be used instead of the throw of the eccentric for all calculations and constructions connected with the valve diagram, as lap, lead, or cut-off.

1636. Modifications of the Slide Valve.—There are many modified forms of the slide valve, but the valve diagram can be applied to any of them that are operated by an eccentric. One of the most common modifications is the piston valve, which is illustrated in connection with the triple-expansion marine engine in Art. **1309.** When a piston valve takes steam at the ends and the exhaust steam passes out through the center, its action is in every respect like that of the plain slide valve. Sometimes, however, steam is admitted to the ports through the central part of the valve, and steam is exhausted at the ends. The steam lap, therefore, is on the inside and the exhaust lap at the ends of the valve, and the only difference is that the valve must move in the opposite direction to that in which it would move if it were a plain slide valve, and the eccentric would have to be moved around the shaft 180°.

1637. A **Trick valve** is shown in Fig. 457. It was designed to give a quick and full opening of the port with a small travel of the valve.

In Fig. 457, A shows the valve in mid-position, while B and C show the valve in two other positions. As will be seen, the valve is hollow, having a passage way H through it; otherwise, the valve corresponds very closely to the ordinary **D** valve before described. A movement to the right

of a distance equal to m will bring the edge p of the valve to the edge of the port S, as shown at B, so that any further movement to the right will admit steam to the cylinder. But this same movement has brought the edge f of the passage in line with the edge g, and any further movement to the right will admit steam to the passage, and, hence, to the left-hand port S, from beneath the valve (see C, Fig. 457). Suppose the valve to move, say $\frac{1}{16}''$ to the right from its position at B; then, the edge f will be $\frac{1}{16}''$ beyond the edge g, and edge p will be $\frac{1}{16}''$ beyond the outer edge of port S. This shows that a movement of the valve which would ordinarily have opened the port $\frac{1}{16}''$ had a **D** valve been used,

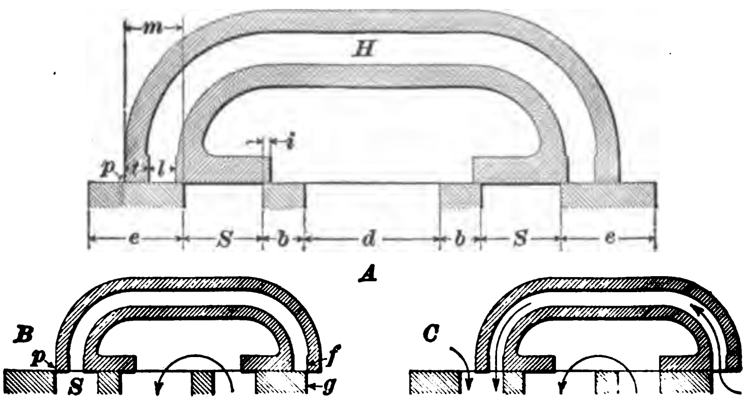


FIG. 457.

has opened the valve twice $\frac{1}{16}''$ or $\frac{1}{8}''$. C, Fig. 457, shows the valve in its extreme position to the right, giving full port opening to the steam and exhaust. The inside lap is shown by i, and the outside lap really equals m.

1638. The parts of the valve should have the following dimensions: $l = \frac{1}{2}(S - t)$; half travel $= S + i$; $e = 2m - t$. The width of the exhaust port d should be equal to $S + m + i + l - b$. The diagram is drawn as for a simple valve, remembering that the width of the opening to exhaust is S and of the opening to steam $2\,l$. In marine engines having a large diameter and short stroke, double-ported valves, as

shown in Fig. 458, are often used to obtain a sufficient port opening with a small travel. It will be seen to consist of two **D** valves, each with its ports and laps. Steam surrounds the outer valve and also the inner one, entering through *B* at the sides. Here, again, the diagram can be applied, the actual or total port opening being twice that of either valve considered separately.

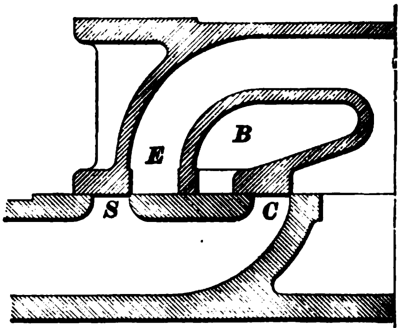


FIG. 458.

TO SET THE SLIDE VALVE.

1639. Principles Involved.—Setting a valve is more a matter of common sense than of rule. If the principles are understood there should be no difficulty.

After the valve and connections are constructed, there are usually two available means of adjustment—the valve spindle can be lengthened or shortened, and the eccentric can be moved about the shaft.

Lengthening or shortening the valve spindle serves to make the valve travel equally each way from mid-position. For example, if the valve travels an inch too far towards the head end, shortening the spindle by half that amount will pull the valve one-half inch towards the crank end and cause its displacement to be equal each way. Moving the eccentric simply hastens or retards the action of the valve, according as it is moved ahead or back; it alters the angle of advance.

1640. To Put the Engine on Dead Center.—Valve setting frequently involves setting the crank on the dead center, which may be accurately done as follows: Make a center-punch mark on the frame of the engine near the turned part of the rim of the fly-wheel. Place the crank a short distance from one dead point, and with the punch

mark as a center, describe an arc on the wheel rim with a tram, which is simply a steel rod with its ends bent at right angles and sharpened. Also, scratch a mark across the cross-head and guide with a scriber. Turn the engine past the center until the mark on the cross-head corresponds again with the line on the guide, and make another mark on the rim with the tram. With the center of the fly-wheel as a center, describe a circular arc on the fly-wheel rim which will intersect the two short arcs just described. Now, bisect the arc included between the points of intersection, and turn the wheel until this last point is at a distance from the punch mark on the frame equal to the length of the tram. The engine will then be on one center and it can be set on the other center in the same way.

Another less accurate but simpler way is to turn the engine over slowly through one revolution and to follow the cross-head with a scriber held in the hand. Just as the point of reversal is reached a line can be scratched on the guide, marking one center.

1641. To Set the Valve for Equal Lead.—It is first necessary to make the valve move centrally by adjusting the valve spindle and then to make it act at the right time by moving the eccentric.

Set the crank on a dead point, and give the eccentric the proper angular advance as near as can be judged. Measure the lead. Set the crank on the other dead point and again measure the lead. Then, move the valve on the spindle half the difference of the two leads, and finally give the valve the right lead by moving the eccentric. The lead should then come the same at the other end.

1642. A Second Method.—This method is convenient when it is difficult to turn an engine by hand. Loosen the eccentric and turn it around on the shaft to give maximum port opening at first one end and then the other. Make the openings equal by changing the length of the valve spindle by half their difference. Then, set the engine on a dead point and give the valve the proper lead by the eccentric.

SHIFTING ECCENTRICS.

1643. It is very common to regulate the speed of slide-valve engines by means of a throttling governor which varies the pressure of the steam before it enters the cylinder. Steam is often admitted to the cylinder without throttling, however, and the speed of the engine is then regulated by varying the time during which the steam is admitted; that is, by varying the point of cut-off. Slide-valve engines are often governed in this way, especially when made to run at high speed, the regulation being accomplished by shifting either the position or the throw of the eccentric.

1644. Changing the Angle of Advance.—One way of doing this is to have the eccentric loose on the shaft and connected with a governor in such a way that it will be rotated back and forth with the fluctuations of speed, thus changing the angle of advance. The effect of changing the angle of advance is shown in Fig. 459. Let $O\,F_1$ be one position of the eccentric. Cut-off occurs at crank position

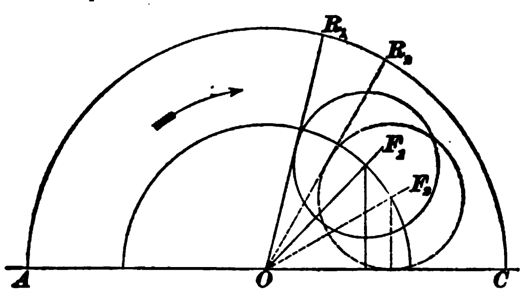


FIG. 459.

$O\,R_1$. Now, move the eccentric back to $O\,F_2$, which will decrease the angle of advance and bring the cut-off later in the stroke at $O\,R_2$. It will be seen, however, that the lead also varies by a large amount, which is not always desirable.

1645. Changing the Eccentricity.—Another way by which the cut-off can be changed is by varying the throw of the eccentric, leaving the angle of advance unchanged. Suppose, in Fig. 460, the eccentricity to be changed from

$O\,r_1$ to $O\,r_2$. The cut-off will change from crank position $O\,R_1$ to $O\,R_2$. The lead will also change, but in this case

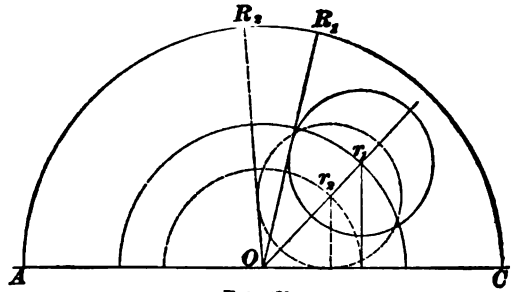


FIG. 460.

the later the cut-off the greater the lead, whereas in the other case where the angle of advance was changed the lead decreased as the cut-off became later.

1646. Changing Both Angle of Advance and Eccentricity.—A combination of the two methods, by diminishing the angle of advance and increasing the eccentricity, or increasing the angle of advance and decreasing the eccentricity at the same time, may be made to give a variable cut-off with constant lead. In Fig. 461 two positions of the eccentric are shown at $O\,F_1$ and $O\,F_2$. In the

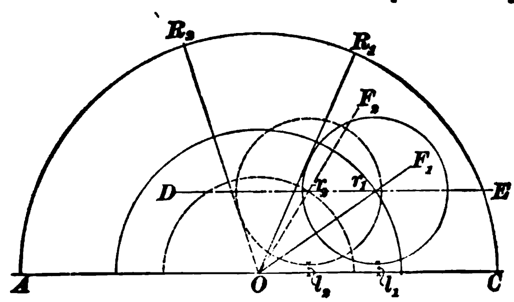


FIG. 461.

former position the eccentricity is $O\,r_1$, and in the latter $O\,r_2$, r_1 and r_2 being in the line $D\,E$, parallel to $A\,C$, which brings the lead l_1, equal to the lead l_2. The eccentric, therefore, has to shift across the shaft on the line $D\,E$, as indicated in Fig. 461.

1647. Shifting Eccentric with Variable Lead.—Fig. 462 shows in principle another method sometimes used that gives a variable lead to the valve. *A C* is a collar keyed to the shaft with lugs for bolts. The eccentric is slotted for the shaft and bolt *b*, the latter serving to clamp the eccentric which swivels on the stud *s*.

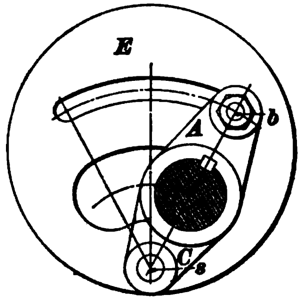


FIG. 462.

Many devices are used to vary the cut-off in some one of the foregoing ways by the action of the governor, some of which will be taken up with Shaft Governors.

DOUBLE VALVE GEARS.

1648. We have seen that an early cut-off with a plain slide valve is accompanied by an excessive compression and early release. To take advantage of the superior economy incident to the use of an earlier cut-off, and at the same time to avoid too early release and compression, double valves are used.

A double valve consists of a main valve, which is set to give the proper lead and compression, or exhaust, and a separate and independent cut-off valve. The latter is sometimes placed in a separate steam chest communicating with the steam chest of the main valve through ports under the cut-off valve. By cutting off the admission of steam to the main valve chest, steam is prevented from entering the cylinder whether the main valve is open for steam or not. A better and more common way, however, is to place the cut-off valve directly on the back of the main valve, which then acts as a valve seat for it. When this arrangement is used the cut-off valve is generally made in two parts, which may be separated or brought together by means of a right and left hand screw, thus varying the lap, and, hence, the cut-off. This arrangement is known as the Meyer valve.

1649. Meyer Valve.—In Fig. 463 a section of such a valve is shown. *A* is the main valve, which has two passageways *C* and *D*, the part between the passageways being an ordinary D valve. On the back of the main valve is the cut-off valve, consisting of two flat plates *B*, *B*, con-

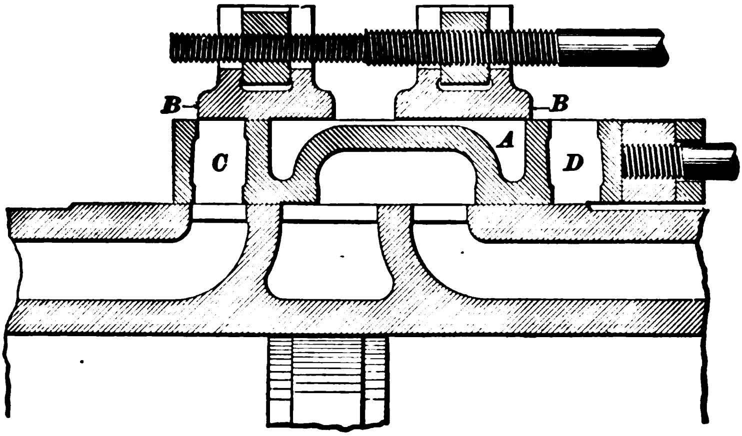


FIG. 463.

nected by a right and left hand screw. The main and cut-off valves are moved by separate eccentrics. A conventional way of representing a Meyer valve is shown in Fig. 464 where both parts are in mid-position—positions they can

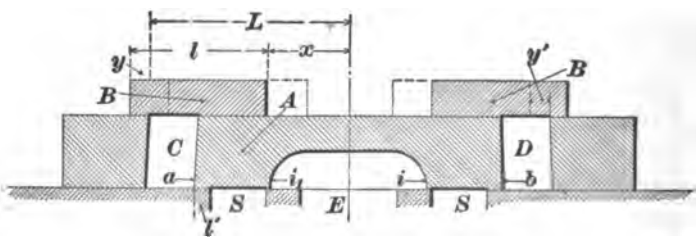


FIG. 464.

not occupy at the same time when connected with their eccentrics.

The cut-off valve acts solely to cut off steam, steam being admitted by the edges *a* and *b* of the main valve, Fig. 464. Cut-off is effected through the closing of the passages *C* and

D by the cut-off valve. It is evident, however, that the steam ports S, S in the cylinder will be closed by the edges a and b of the main valve at some point of its travel, so that if the cut-off valve is to serve its purpose it must act before the edges a and b cut off. Hence, to allow a wide range of cut-off, as well as to avoid excessive compression and a very early release, both of which are controlled by the main valve, the main valve should have a late cut-off.

THE CORLISS GEAR.

1650. A **Corliss gear** can not be laid out from the diagram like a plain slide valve. The dimensions of the parts must be proportioned by trial; but there are certain requirements that must be fulfilled, and these will be explained.

I.—The steam port should be opened rapidly, so as to avoid throttling the steam. This action is aided by the position of the eccentric which, in the Corliss engine, has only a slight angular advance, being nearly at 90° with the crank. The eccentric is, therefore, near mid-position at the points of admission and causes a prompt admission. To obtain the full benefit of this, the rods E and E' (Fig. 282, Art. **1287**) should be attached to the wrist-plate so that the movement of the attaching pin will be symmetrical on each side of the center of the wrist-plate A.

Following out the motion of the eccentric, it will be seen that, as it moves from mid-position, the valve will move less rapidly until, when fully open, it will be nearly at rest. It is not desirable, however, that the rapidity of motion should reduce too quickly; otherwise, as the piston nears mid-stroke, and, consequently, moves more quickly, some wire-drawing of the steam might occur. This is avoided by the arrangement of the rods and levers connecting the valves with the wrist-plate, the principle being similar to that of the slow-motion mechanism described in Art. **1444.**

Fig. 465 is a skeleton diagram of the wrist-plate and connections as found on one make of Corliss engines. The

center of the wrist-plate is O, and the center of the admission and cut-off valve for one end is at C. The parts of the diagram have the same letters as the corresponding parts of the diagram in Fig. 355, Art. **1448.** By plotting the motion, it will be seen that when the wrist-plate has moved through half its motion, or point b has reached point i, the end of the valve lever has been moved only through the arc $c\ i'$, while for the other half of the wrist-plate motion it moves through the greater arc $i'd$.

The action of the lever which connects with the exhaust valve C' is similar.

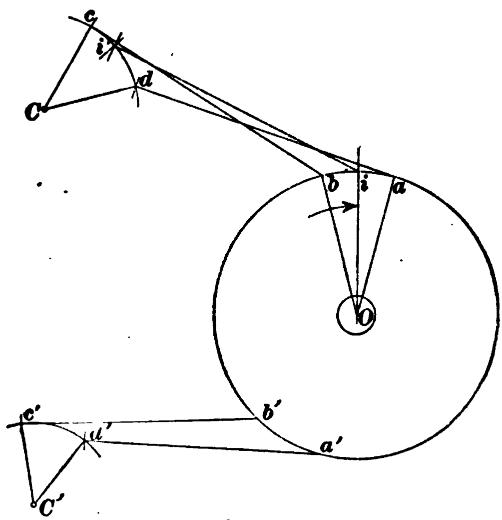


FIG. 465.

II.—The amount of travel of the wrist-plate should be great enough to cause the valve to give full port opening and over-travel from $\frac{1}{4}$ inch to $\frac{3}{8}$ inch besides. For, suppose the valve gave only full port opening for the extreme position of the eccentric; then, for short cut-offs, the valve would be tripped by the governor before the port was opened wide, and for later cut-offs the valve would move too slow, owing to the retarding action of the eccentric spoken of above. With over-travel this retarding action does not become serious until *after* the port is wide open.

III.—Having decided upon the valve travel, the lever arms and connections can be proportioned, and the next point to be considered is the angle which the lever arm *C c* in Fig. 465 should make with the rod *c b* connecting with the wrist-plate. It is clear that the greater this angle the more rapid the valve motion, but it should not be great enough to permit of there being any danger of the lever arm passing the center when in position *C d.*

1651. The Lap of the Valves.—The question of what lap to give the valves of a Corliss engine is somewhat complex.

A little consideration will show that cut-off by the action of the governor can occur only while the wrist-plate is moving one way. If the valve is not tripped before the motion of the wrist-plate reverses, the cut-off is given by the lap of the valve near the end of the stroke. If, therefore, the eccentric was set at right angles to the crank, its motion one way would continue until the piston reached about half stroke, so that cut-off could occur up to half stroke. Moreover, with the eccentric in this position, the valve would have the most rapid movement possible when opening.

But the eccentric can be at right angles to the crank only when there is no lap, and the greater the lap the more the eccentric must be set ahead to give the steam valves the proper lead. With the eccentric set ahead it will have reached the position of its greatest displacement either way *before* the piston reaches half stroke, the range of cut-off will be shortened, and the movement of the valve made less rapid.

It would, therefore, be advisable that the eccentric have no angular advance, and the valves no lap other than that necessary to make a steam-tight joint when closed, were it not for the exhaust valves. These valves are operated by the same eccentric as the admission valves, and could have no lap if the eccentric should have no angular advance.

To obtain a good steam distribution, however, both release and compression should occur *before* the end of the

stroke; with no lap they would take place *just at* the end of the stroke. The only way to bring about these events earlier is to add lap to the exhaust valves, and give the eccentric such angular advance as is necessary to bring about an early action. The reason why this is so should be clear from what has been stated before regarding the slide valve.

Since the eccentric is moved ahead, the admission valves must also have lap, and thus it will be seen that the whole question of lap on Corliss valves is that of making the exhaust valves act properly without interfering with the action of the steam valves. Separate eccentrics for the steam and exhaust valves will allow each to be set as it should be, but usually only one eccentric is used.

THE STEPHENSON LINK MOTION.

1652. In Fig. 466 is shown the **Stephenson link motion** as applied to locomotives. It differs from that shown in Fig. 316, Art. **1330,** in that a rocker *R* is used to actuate the valve, and that the link is suspended from above instead of being supported from below. The principle of the motion is the same, whatever the type of the engine to which it is applied.

In the figure *O* is the center of the driver axle. The two eccentrics *F* and *B* are keyed to the axle, and connect with the slotted link *L* through the eccentric rods *E* and *G*. The slide valve is attached by its stem *V* to the upper arm of the rocker *R*. The rocker in turn is connected with the link through the block *K* and the rocker pin *d*, the former being free to slide in the slot of the link.

The eccentric *F* is set to give the forward movement to the engine and the eccentric *B* to give a backward movement. The raising or lowering of the link is accomplished through the hanger *H* attached to the lower lever on the tumbling shaft *T*. The link itself is suspended from its saddle *S* by the hanger *H*, and the upper lever on the tumbling shaft connects with the reversing lever, not shown, through the reach rod *W*.

When the reversing lever is thrown forward, the link is lowered and the engine will run ahead; when thrown back, the link is raised and the engine will run backwards. In Fig. 466 the link is shown lowered so that the rocker pin *d* comes in line with the eccentric-rod *E*. If the link should be raised to the other extreme, *d* would be in line with eccentric-rod *G*. In either case the link is said to be in **full gear.** Should the rocker pin *d* be at the middle point of the link, the latter would be in **mid-gear.** If the link is

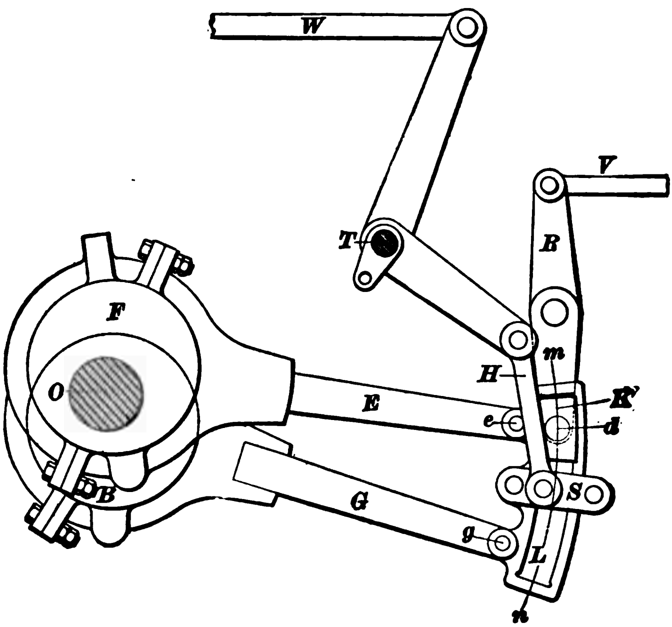


FIG. 466.

in full gear, so that the forward eccentric actuates the valve, it is said to be in *full gear forwards*, and when the backward eccentric actuates the valve, in *full gear backwards.*

1653. It will be remembered that when a reversing rocker intervenes between the eccentric and valve, the crank leads the eccentric by 90° minus the angle of advance of the eccentric. In Fig. 466, therefore, the crank falls within

the angle made by the two eccentrics, and, with the **link** lowered, *leads* eccentric *F*. If no rocker were used the effect would be to turn the crank around 180°, and it would *follow* eccentric *F*.

In what follows the link motion will be illustrated by skeleton diagrams. The link will be represented by an arc drawn through the curved slot, called the **link arc,** and each eccentric-rod will be represented by a line corresponding to the center line in Fig. 466, drawn from the center of the eccentric *F* to the point *d*. The eccentrics will be represented by straight lines connecting the center of the eccentrics with the center of the axle or shaft.

1654. Action of the Link Motion on the Valve.— While the link motion was designed primarily for reversing, it is found to be well adapted for a variable cut-off gear. Its action will be understood by reference to Figs. 467 and 468.

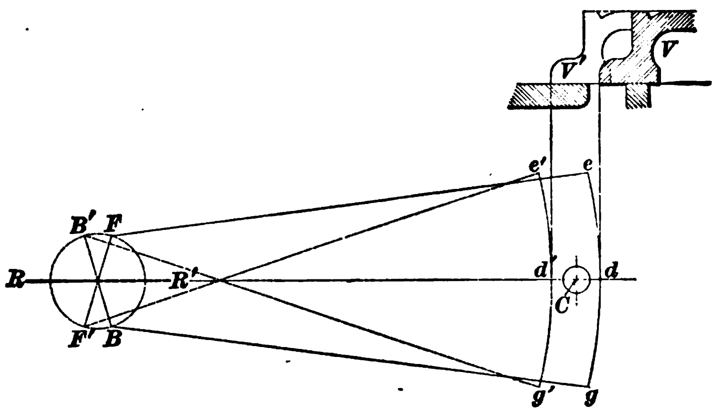


FIG. 467.

In Fig. 467 the solid lines represent the positions of the parts when the crank is on a center and the eccentrics are turned *towards* the link; the dotted lines show the positions when the eccentrics are on the side *opposite* to the link. For convenience, the valve is drawn directly above the link, and it is assumed that no rocker is used. When the crank is on the center at *R*, the link is at *eg*, and the point *d* is displaced a distance *Cd* to the right of the central position *C*. The

valve is also displaced a like amount, as shown at V. As the crank is on a center, the port is open an amount equal to the lead, and the displacement of the valve is equal to the lap plus the lead. In a similar manner, when the crank is at R', the eccentrics F and B will be at F' and B', respectively, and the link will be at $e'\,g'$, the valve then being displaced to the left a distance $C\,d'$ equal to the lap plus the lead, as shown at V'.

Now, in mid-gear this is the greatest possible displacement of the valve, its total travel being equal only to the distance $d\,d'$, or twice the lap plus twice the lead. This will

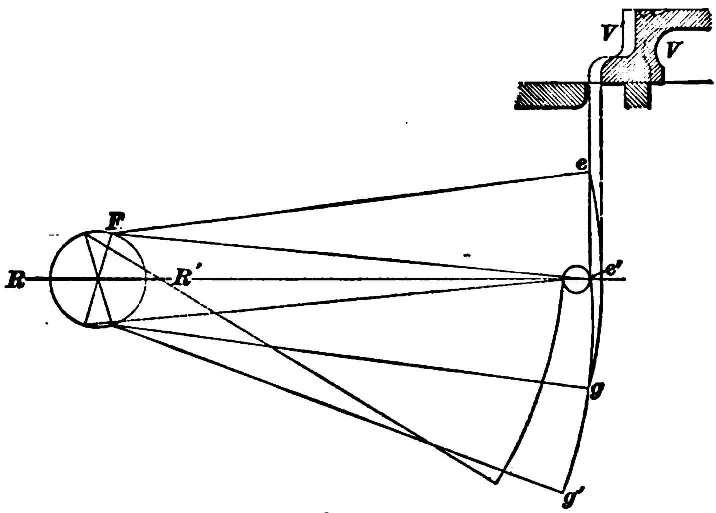


FIG. 468.

be evident by supposing the crank to be at R and to start to rotate in a right-handed direction. The tendency of the eccentric F will be to move point e of the link to the right, thus increasing the valve travel; but eccentric B will have a greater tendency to move the point g to the left, since it is approaching a position at right angles with the eccentric-rod, while eccentric F is moving away from such a position. From what we know of the action of the crank and connecting-rod, therefore, it is clear that B will have a greater influence on the movement of the link than F, and point d will

move to the left. *From this it follows that the greatest port opening in mid-gear is the lead, and that cut-off occurs very early in the stroke.*

1655. Fig. 468 shows the link in mid-gear at *e g*, and in full gear forwards at *e' g'*. In the latter position the valve is actuated almost entirely by the eccentric *F*, as in an ordinary slide-valve engine.

The travel of the valve, therefore, is now equal to the throw of the eccentric instead of twice the lap plus twice the lead, as before. Full port opening is now obtained, and cut-off occurs much later.

1656. Now, suppose the crank to be set on the dead center, as in Fig. 468, and the link to be shifted from full gear to mid-gear, or from *e' g'* to *e g*. The lead, it will be seen from the two positions *V* and *V'* of the valve, increases from a very small amount to the large lead obtained in mid-gear.

The above explanation of the valve's action may be summarized as follows:

In full gear the valve is under the control of one eccentric, which gives it a motion like that given to a plain slide valve by a single eccentric.

In passing from full gear to mid-gear, cut-off becomes earlier, and, hence, compression greater, the travel of the valve diminishes, which makes the port opening less; the lead increases.

In mid-gear the travel of the valve is equal to 2(lap + lead), and the maximum port opening is equal to the mid-gear lead.

1657. Opened and Crossed Rods.—As shown in the preceding figures, the rods are said to be **open.** If the eccentric-rods *E* and *G*, in Fig. 466, should be disconnected from the link and rod, and *E* should be bolted to the lower end of the link, and rod *G* to the upper end, we should have the arrangement shown in Fig. 469; in this case the rods are said to be **crossed.** The terms open and crossed are given according to the position of the rods *when the eccentrics are turned towards the link.* Thus, in Fig. 467, although

the rods are crossed when the eccentrics point *away* from the link, they are still called open links, because when turned *towards* the link they are not crossed. The action of crossed rods is different from open rods in that the lead *decreases* from full to mid-gear, in which position the motion usually gives no lead. With crossed rods the engine may be stopped by placing the link in mid-gear. This can not be done with open rods, where there is always a small port opening in mid-gear, unless the resistance to be overcome by the engine is so great that enough steam can not be admitted in mid-gear to run the engine.

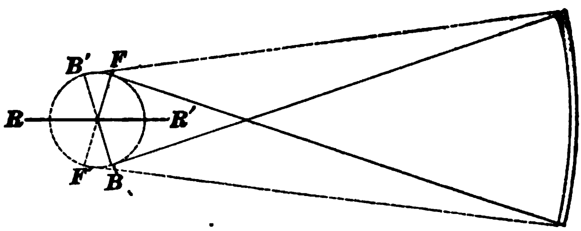


FIG. 469.

Open rods are mostly used, and as the principle of crossed rods is essentially the same, open rods alone will be considered.

DESIGNING THE LINK MOTION.

1658. The Valve and Ports.—The valve should be designed to meet the requirements when the link is in full gear. As in this position it is practically under the control of one eccentric, the valve diagram may be used.

Generally, the widths of the ports, the latest point of cut-off, the maximum port opening, and the full gear lead are known, or assumed, leaving the lap, travel, and angular advance to be found, the latter being the same for both eccentrics. When the link is to be used for reversing only, as in the case of hoisting engines, this part of the process is in all respects like designing a slide valve for an engine without the link motion. A hoisting engine is always run with the link in full gear, and the valve is designed to give the best results in this position.

If the link is to be used for varying the cut-off, however, as well as for reversing, as in the case of the locomotive, the valve should be given enough travel, so that it will not only open the port its full width in full gear, but will *over-travel* or will go beyond the inner edge of the port a certain amount, thus making what has heretofore been called the "port opening" greater than the width of the port. If this were not done the port opening would become so small for the intermediate points between full and mid-gear at which the engine would usually be run as to cause serious wire-drawing of the steam.

1659. As a basis to work upon, proportions of the valve and ports are here given, taken from locomotive practice. They will be convenient for reference in designing a link motion for any type of engine.

The following are average values: Latest point of cut-off, $\frac{7}{8}$ to .9 stroke; lead in full gear $\frac{1}{16}''$ to $\frac{1}{10}''$; lead in mid-gear, $\frac{3}{8}''$; inside lap, 0 to $\frac{1}{8}''$.

The accompanying table gives dimensions in inches from locomotives having cylinders over 15″ in diameter:

Width of Steam Port.	Width of Exhaust Port.	Outside Lap.	Valve Travel.
$1\frac{1}{4}$	$2\frac{1}{2}$	$\frac{3}{4}$	5
$1\frac{1}{4}$	3	$\frac{7}{8}$	$5\frac{1}{2}$
$1\frac{1}{4}$	$2\frac{1}{4}$	1	$5\frac{1}{4}$
$1\frac{1}{4}$	$2\frac{3}{4}$	$\frac{3}{4}$	$5\frac{1}{2}$
$1\frac{1}{4}$	$2\frac{1}{2}$	$\frac{3}{4}$	$5\frac{1}{2}$

The maximum port opening varied in these cases from $1\frac{5}{8}''$ to 2″.

1660. The Suspension of the Link.—As the eccentrics revolve, the link has a continued vibratory motion about its point of suspension and a swinging motion due to the vibration of its hanger. The result is more or less vertical motion, according to the point at which the hanger is

attached to the link, which, if excessive, will cause undue wear between the link and the block (*K* in Fig. 466). This motion between the block and the link is called the **slip.**

Ordinarily, the link is suspended at one of three points—the center of the link arc, the center of its chord, or at the lower end of the link. When at the first point, the slip is slight in mid-gear, and increases both ways; at the second, the slip increases from mid-gear, but is always *greater* than in the former case; while, at the third point, the action is good when the block is in the lower half of the link, but when in the upper half the slip is excessive. When the engine is to be run with the block at some one point in the link nearly all the time, the latter should be suspended from that point to reduce the slip as much as possible.

Both the hanger which supports the link and the lever to which its upper end is attached should be as long as possible. The object should be to so suspend the link that the saddle pin will always move approximately parallel to the center line of motion.

1661. Proportions of the Link.—The length of the link should not be less than $2\frac{1}{2}$ to 3 times the throw of the eccentric; if less, it is difficult to reverse the engine when the piston is near the end of the stroke, as the link then makes an obtuse angle with the valve stem. By the length of the link is meant the length *l* (Fig. 470) of the chord connecting centers of the link block when it is in the two extreme positions.

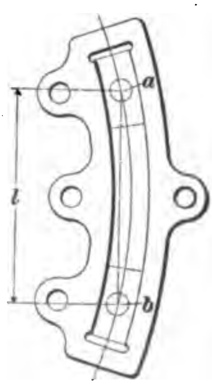


FIG. 470.

The radius of the link is generally made equal to the length of the eccentric rods; that is, equal to the distance from the center of the eccentric to the center of the link block when in full gear, or the distance *F d* in Fig. 466, or *F e'* in Fig. 468. This is the length by which the rods are represented in all link diagrams.

The purpose of curving the link is to make the valve move equally on both sides of a fixed point, no matter what is the position of the link. The radius above does not exactly accomplish this, but it is nearly correct. The effect of curving the link will be clearly seen from Fig. 471. The solid lines show the positions of the link in mid-gear when the crank is on each center. If the link were straight its two positions would be at *a b c* and *e h g*; the center of travel would be at the point *j*. The two positions of the link, if curved, would be at *a d c* and *e f g*, when the center of travel would fall half way between *d* and *f* or at *i*. Now, it will be observed that the point *i* is also the center of travel

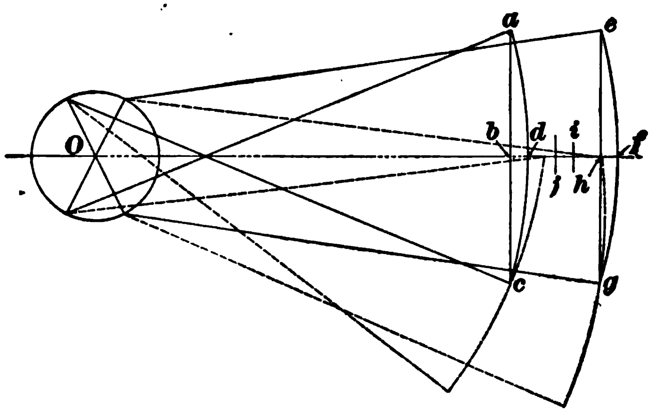


FIG. 471.

for the link when in full gear, as shown by the dotted lines, while the point *j* is to one side of the center. It is necessary, therefore, for the links to be curved.

1662. Eccentric-Rods.—The length of the eccentric-rod is to be taken equal to the distance from the center of the shaft to the middle position of the center of the link arc, or equal to *O i* in Fig. 471. It should not be *exactly* this distance, but any slight error can be corrected in setting the valves.

1663. Laying Out the Motion.—The method of laying out a link motion will be best explained by an

illustrative example. Let the following data be taken: Latest point of cut-off, .9 stroke; full gear lead, $\frac{3}{32}''$; width of steam port, $1\frac{1}{4}''$; width of bridge, $1''$; inside lap, $0''$. Assume the valve to over-travel $\frac{1}{2}''$ in full gear, giving a maximum port opening $1\frac{1}{4} + \frac{1}{2} = 1\frac{3}{4}''$.

Drawing the valve diagram, which is shown in Fig. 472, to a reduced scale, it is found that the travel $= 5''$, outside lap $= \frac{3}{4}''$, and the angle of advance of the eccentric $= 19°$.

From the rule for the width of the exhaust port the latter is found to be $2\frac{1}{2} + 1\frac{1}{4} + 0 - 1 = 2\frac{3}{4}''$. In a locomotive it would ordinarily be made less than this, say $2\frac{1}{2}''$, because in full gear, when the exhaust port is contracted the

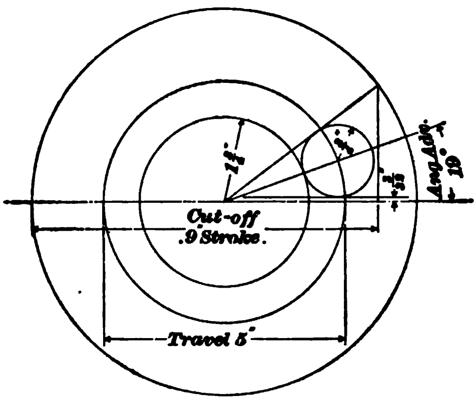


FIG. 472.

greatest amount, the engine is generally drawing a heavy load and runs at a slow speed, thus giving ample time for the steam to exhaust.

1664. In Fig. 473, let O be the center of the shaft and V the relative position of the valve and ports. Draw the center line $a\ x$ of the valve stem, and the center line $O\ C$ of the link motion. In locomotive practice this line usually makes a small angle with the center line of the valve spindle, to allow clearance under the boiler, but in this case we will assume the two to be parallel and that the motion is transmitted from the link to the valve stem through an equal armed rocker, each arm being $9''$ long. The position of the

rocker is usually fixed within certain limits by the design of the engine; in this case the center line will be taken at 4 ft. from the center O of the shaft, or $O\ C = 4$ ft. In the figure, the distance is proportionately shorter than this, so that the whole diagram may be represented on one page.

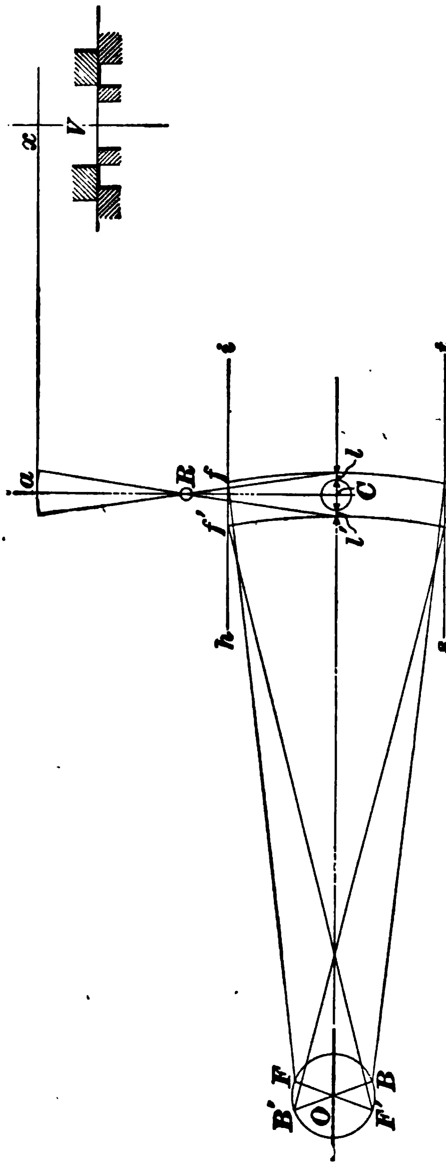


FIG. 473.

To proceed, describe the eccentric circle about O with a radius of $2\frac{1}{2}''$, and draw the center lines of the eccentrics for crank positions on the forward and backward centers. As a rocker is used, the crank leads the eccentric by an angle equal to 90° minus the angle of advance. When the crank is on the forward center, the forward eccentric is at F and the backward at B; when the crank is on the backward center, the forward eccentric is at F' and the backward eccentric at B'.

The length of the link is taken at 13", and it should now be drawn in full and mid-gear positions, so that its action at these points can be determined. Draw the lines *h i* and *s t* parallel to the center line of motion *O C*, and distant from it $6\frac{1}{2}$"; these will limit the ends of the link when in mid-gear. With *O C* as a radius, and *F* and *B* as centers, describe arcs cutting the horizontal lines just drawn at *f* and *b*; and with the same radius and *F'* and *B'* as centers, describe arcs cutting them at *f'* and *b'*. Now, with the same radius and with centers on the line *O C*, draw the link arcs *f b* and *f' b'*, which are the positions of the link in mid-gear when the crank is on each center. The mid-gear lead may now be examined by drawing the valve in the two positions corresponding to the above positions of the link, and, if not satisfactory, the proportions of the valve should be changed. An easier and better way, however, is to draw a circle about the point *C*, with a radius equal to the lap of the valve.

Then, since the valve is always displaced from mid-position an amount equal to the lap plus the lead when the crank is on a center, the spaces between the link arcs and the lap circle just drawn, marked *l* and *l'* in the figure, will be equal to the lead for each end of the stroke.

1665. To draw the link in full gear, describe arcs *f* and *f'*, Fig. 474, cutting the center line, with radii

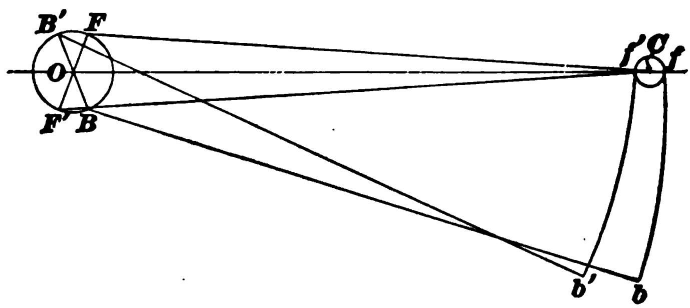


FIG. 474.

equal to *O C* and with centers *F* and *F'*, respectively. With the points of intersection at *f* and *f'* as centers, and radii equal to the length of the link, describe arcs *b* and *b'*, upon

which the lower end of the link must fall. Finally, with B and B' as centers, draw the link arcs $f\,b$ and $f'\,b'$. By drawing the lap circle about C, it may be determined how much the lead varies in full gear. Generally the variation can be compensated for by adjusting the length of the valve stem.

GOVERNORS.

1666. A **governor** is used to regulate the speed of a motor by varying the amount of energy supplied to it. In the water-wheel, for example, it raises or lowers the gate, thus supplying more or less water, and in the steam engine it varies either the quantity or pressure of the steam used. It is driven by the motor it regulates, and is usually so constructed that any variation in the speed of the motor will cause the governor to automatically regulate the speed. In most governors use is made of the centrifugal force of some rapidly revolving body, counteracted by some other force, as gravity or the tension of a spring.

PENDULUM GOVERNORS.

1667. One of the oldest and most common forms of governors is known as the pendulum governor, which is based upon the principle of a revolving pendulum. This form will be taken up first.

1668. A **simple revolving pendulum** is shown in Fig. 475. It may be considered to consist of a ball B suspended by a cord from the point O, the ball revolving about the vertical axis $O\ C$. *Theoretically*, a *simple* revolving pendulum is one having the mass of the ball B concentrated at its center and the cord or arm $O\ B$ of material without weight.

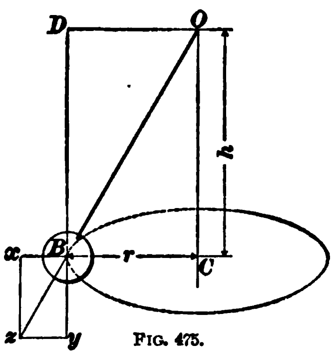


FIG. 475.

When the pendulum revolves about the axis at a uniform speed, the ball remains at a

constant distance r from the axis and at a constant distance $O\,C$ below O, the point of suspension. The latter distance is called the *height* of the pendulum, and is represented by h in the figure. Now, suppose the pendulum to be revolved at a greater speed; the centrifugal force, and, hence, r, will be increased and h will be correspondingly diminished.

When the pendulum is revolving there are three forces acting, namely: Gravity, which is equal to the weight of the ball and always acts downwards; centrifugal force, which acts horizontally outwards, and the tension in the cord. These three forces may conveniently be represented by the parallelogram of forces, as in Fig. 475, where $B\,y$ represents the weight of the ball, $B\,x$ its centrifugal force, and $B\,z$ the tension of the string. Of these, the weight $B\,y$ tends to turn the pendulum about the point O in a vertical plane towards the axis $O\,C$, while the centrifugal force $B\,x$ tends to turn it about O in a direction away from the axis.

Now, in order that the ball shall poise at a certain distance r from the axis, the moment of its weight about O must equal the moment of its centrifugal force about the same center, or, stated in the form of an equation,

$$B\,y \times O\,D\,(= B\,y \times r) = B\,x \times h.$$

But, letting the weight $B\,y$ of the ball $= B$, the centrifugal force $B\,x$ will be $.00034\,B\,r\,N^2$ (formula **19**, Art. **903**), where N is the number of revolutions.

Hence,

$$B \times r = .00034\,B\,r\,N^2 \times h, \text{ or}$$

$$h = \frac{1}{.00034\,N^2} \text{ feet} = \frac{35{,}294}{N^2} \text{ inches.} \qquad \textbf{(158.)}$$

The height of a revolving pendulum, therefore, is independent of the weight of the ball or the length of the string, and depends solely upon the number of revolutions.

1669. Pendulum Governor.—Figs. 476 and 477 show two forms of the pendulum governor, as used on certain classes of steam engines, and they differ in principle

from the simple revolving pendulum only in that the governor balls are connected with arms, links, etc., the weight of which modifies their action.

1670. In Fig. 476 the balls B, B are suspended from the collar C_1; this has the same effect as though they were suspended from the point O, at the intersection of the center lines of the arms. Two balls are used for the sake of symmetry and even action. The links l, l connect the arms with a lever collar C_2, which is free to turn in an annular groove in a sleeve U which can not turn, but can slide up and down on the spindle S. The sleeve extends into the standard M, and is connected with the rod R_1 by a stud working in a slot

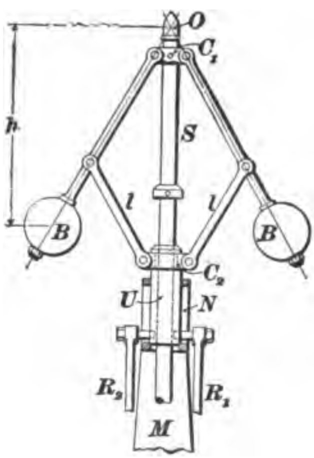


FIG. 476.

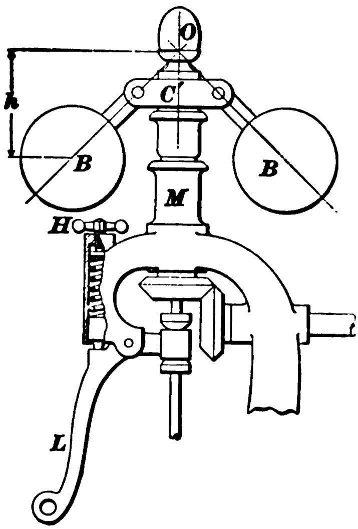


FIG. 477.

N. The spindle is driven by the engine through a belt and a pair of miter gears which are not shown. As the collar C_1 is pinned to the spindle, the balls revolve with the spindle, and, consequently, fly outwards, raising the rod R_1 by means of the links l, l, the collar C_2, and the sleeve M. On Corliss engines, on which this style of governor is largely used, the rod R_1 operates the mechanism that trips the steam valves

at the point of cut-off, thus regulating by varying the time during which steam is admitted to the cylinder.

1671. To prevent sudden fluctuations of the governor, a second rod R_2 operates a piston in a cylinder closed at both ends and filled with oil. The piston consists of two plates through which holes are drilled, and by turning the plates one way or the other they can be adjusted to allow more or less opening through the holes. This adjustment determines the freedom with which the oil can pass from one side of the piston to the other, and, hence, the freedom with which the piston can move. Other constructions are also used for the same purpose.

1672. Fig. 477 shows a throttling pendulum governor used to regulate an engine by varying the pressure of the steam. M is a part of the frame and is bored for a bearing in which turns a hollow spindle driven by miter gears. Inside of the spindle is the valve stem, the lower end of which can be seen. The balls are suspended from the collar C', which receives its motion from the spindle, and the arms to which the balls are attached are in the form of bell-cranks with their upper arms extending inwards and revolving in a groove about the end of the valve. As the speed increases the balls fly out and the upper arms of the bell-cranks lower the valve stem, thus partially closing the valve and shutting off the steam; with a decrease of speed the above operation is reversed. A cord can be attached to the lever L and carried to a place within convenient reach. Then, should the governor belt run off or break and the engine begin to "race," by pulling the cord the valve stem would be lowered and the engine brought to normal speed. The screw H is for regulating the distance the spindle can be lowered; that is, it fixes the lowest speed at which the engine can run. The governor in Fig. 477 is a Gardner governor.

1673. The height of a pendulum governor varies with the speed, and, as in the case of the simple revolving

pendulum, it is the perpendicular distance between the center of the governor balls and their equivalent point of suspension at the center line, or h in Figs. 476 and 477. It is evident that the height at which the balls stand at any particular speed determines the position of the regulating device for that speed, as the rods in Fig. 476 or the valve stem in Fig. 479.

Take, for example, the case of a throttling governor on a steam engine. When the engine is running "light," without any load, the governor valve will be open just wide enough to admit steam to the cylinder at the pressure necessary to keep the engine at the proper speed. When the engine is running "loaded," however, the valve must be opened wider. Now, this variation in the opening of the valve can be caused only by a variation in the height of the governor, which, in turn, is due to a change in speed. Hence, the governor cannot control the speed except within certain limits, which manifestly should not be far apart. A well-designed engine will not vary more than two per cent. either way from a certain mean speed, and the difference in the heights of the governor due to these two extremes of speed must be sufficient to move the throttle valve through its full range of action.

1674. To show clearly what may be expected of an ordinary pendulum governor, Table 36 has been prepared.

TABLE 36.

Revolutions.	Height in Inches.	Variation in Height, Inches.
200	.882	.035
150	1.569	.06
100	3.529	.14
50	14.118	.57

In the second column the approximate heights of a

pendulum governor at different speeds are given, computed by the formula

$$h = \frac{35,294}{N^2}.$$

In the third column the variation in height for a speed variation of 2% each way, or a total variation of 4% or $\frac{1}{25}$ of the mean number of revolutions, is given.

From the table we observe that at high speeds the heights and variations are very small. Thus, for a speed of 200 revolutions, the height is .882 inch, and it would be difficult to construct the governor. At 100 revolutions the height would be 3.529 inches. In this case, also, there would be constructive difficulties, especially if the governor were to be like Fig. 476. The allowable variation for this speed, moreover, is only .14 inch, a very small amount to control the working of a cut-off mechanism, or throttle valve, throughout its whole range of action.

1675. Weighted Pendulum Governor. — The weighted pendulum governor, or Porter governor, is designed to run at an increased height for a given speed, and to have a greater variation for a given variation in speed. This latter is called increasing the *sensitiveness* of the governor; a governor, for example, whose height varies one inch for 2% change in speed is more sensitive to that change than one whose height varies only one-half inch. In the Porter governor, the counterpoise weight is free to revolve and slide upon the spindle with the collar. It, therefore, adds to the weight of the balls by falling down through the links, or arms, but does not add to their centrifugal force, the result being that the height of the governor for any speed is increased.

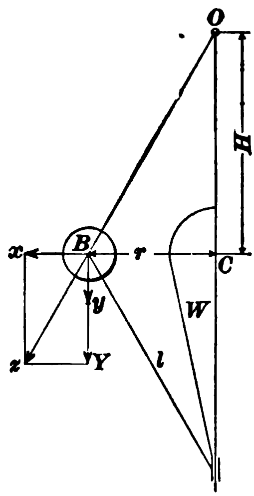


FIG. 478.

1676. In Fig. 478 the weight of the ball B is represented by the line $B\,y$, and its centrifugal force by $B\,x$. Suppose the effect of the counterpoise W is to add an additional weight $y\,Y$ to the weight of the balls. The arm will then have a direction $B\,z$, and the height of the governor will be H. Let $B = y\,B$ and $y\,Y = W$, whence, $B\,Y = B + W$. Then, taking moments about O, as in the case of the simple pendulum governor, we have for equilibrium

$$B\,x \times H = (B + W) \times r;$$

in which $W =$ entire weight of counterpoise and $B =$ weight of one ball.

But $B\,x = .00034\;B\;r\;N^2$ (formula for centrifugal force).

Hence, $(B + W)\;r = .00034\,B\,r\,N^2\,H$, or,

$$H = \frac{1}{.00034} \times \frac{\left(1 + \frac{W}{B}\right)}{N^2} \text{ feet} =$$

$$\frac{35{,}294\left(1 + \frac{W}{B}\right)}{N^2} \text{ inches.} \qquad \textbf{(159.)}$$

By reference to formula **158,** it will be seen that the height of a weighted governor equals the height of a simple governor multiplied by $\left(1 + \frac{W}{B}\right)$.

It follows from this, moreover, that for a given variation in speed, the variation in height will be correspondingly increased, making the governor more sensitive.

From formula **159,** it is evident that adding to the weight of the counterpoise W will increase the height of the governor. That is, the balls will drop lower, and the speed of the motor will increase until the centrifugal force of the balls is sufficient to restore equilibrium again. We thus have a method of increasing or diminishing the speed at which a motor will run by adding to or subtracting from the weight of the counterpoise.

1677. Spring governors are frequently used on throttling engines and other motors to regulate the opening of the valve. They are simply pendulum governors, with springs added to resist the action of the centrifugal force, thus increasing the height and sensitiveness in the same way that the weight operates with the Porter governor. Fig. 479 shows a Pickering governor with springs *s*, *s* attached to the spindle *l* and bearing against the governor arms. This governor has been previously described (see Art. **1290**).

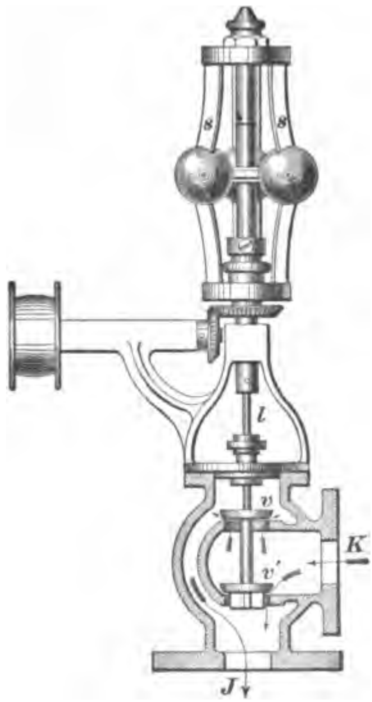


FIG. 479.

TO DESIGN A WEIGHTED GOVERNOR.

1678. Designing a governor consists mainly in so proportioning the parts that the forces acting will balance. It is convenient to assume the weight of the counterpoise as acting at the points B, B' in Fig. 480. W is the counterpoise, the weight of which is represented graphically by $a\ b$. Completing the parallelogram of forces, we have $a\ c$ as the pull upon the lower arm l. Laying this off at $B\ c'$ and again completing the parallelogram, we have $B\ b'$ as the resulting downward pull at B. If the upper and lower arms are *equal* (when continued to the center line) as in the figure, $B\ b'$ will equal $a\ b$; that is, the effect of the counterpoise acting at a will be the same as though its weight were transferred to each ball, making $2\ W$ when acting from B

and B'. If the upper and lower arms are not equal, the value of $B\,b'$ can easily be found by drawing the parallelograms. In what follows the arms will be considered equal.

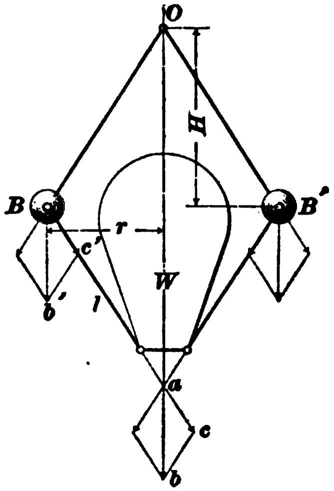


FIG. 480.

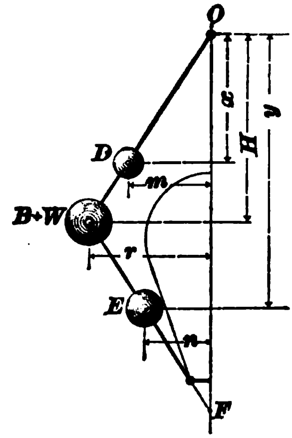


FIG. 481.

1679. In any governor the weight and centrifugal force of the arms exert an influence upon its action that can not be neglected. The effect of the arms is the same, however, as though their weights were concentrated at their centers of gravity, as at D and E in Fig. 481. Here there are four weights to be considered; the weight of the ball, counterpoise, and the two arms, which we will call B, W, D, and E, respectively. For a condition of equilibrium it will be necessary that the sum of the moments of these weights about O shall equal the sum of the moments of the centrifugal forces of B, D, and E, as in the simple case shown in Fig. 478. The sum of the moments of the weights is evidently

$$(B + W)\,r + D\,m + E\,n. \quad (a)$$

For the sum of the moments of the centrifugal forces we have, from the formula for centrifugal force, $.00034\,D\,m\,N^2\,x + .00034\,B\,r\,N^2\,H + .00034\,E\,n\,N^2\,y$. This, reduced, and with the dimensions expressed in inches, becomes

$$.000028\,N^2\,(D\,m\,x + B\,r\,H + E\,n\,y). \quad (b)$$

For equilibrium to exist (*a*) and (*b*) must evidently be equal so that

$$(B+W)r+Dm+En=.000028N^2(Dmx+BrH+Eny). \quad (160.)$$

1680. We have seen that to accomplish regulation the speed of the governor must increase or decrease. Suppose it to increase above mean speed. At first the height will not change, because there will be a certain amount of frictional resistance to overcome. When the speed has reached a certain point, however, the centrifugal force of the balls and arms will have increased enough to change the height, and the governor will regulate.

In ordinary practice it will be safe to assume a variation either way of 2% of the mean speed. Now, let R be the resistance in pounds, which is assumed to act at the collar, N the mean speed, and N_1 the speed just as the resistance is overcome. Then, the governor should be so designed that two per cent. increase of the mean speed will produce an increase in the centrifugal moments in (*b*) sufficient to balance the moment of the resistance R*, which is $R\,r$.

Hence, from (*b*),

$$.000028\,[(1.02\,N)^2-N^2]\,(Dmx+BrH+Eny)=Rr, \text{ or}$$

$$.00000114\,N^2\,(Dmx+BrH+Eny)=Rr. \quad (161.)$$

Finally, by transposing **160** and **161**, we obtain equations for determining the weight of the counterpoise and balls, as follows:

$$W=\frac{1}{r}[.000028N^2(Dmx+BrH+Eny)-(Br+Dm+En)]. \quad (162.)$$

$$B=\frac{R}{.00000114HN^2}-\frac{Dmx+Eny}{rH}. \quad (163.)$$

1681. The process is now to first draw the arms, their spread, and the height to be taken, of proportions suited to the size of the engine, or taken from some example at hand.

---

* The resistance is supposed to be transferred to the center of each ball in the same way that the weight of the counterpoise was treated.

Next, compute the weights and centers of gravity of one upper and one lower arm, as in Fig. 481, taking the weights in pounds, and dimensions in inches. Referring to Fig. 481, suppose they are found to be as follows:

$$x = 5. \qquad r = 8.$$
$$H = 10 \qquad n = 4.$$
$$y = 15. \qquad D = 6 \text{ lb.}$$
$$m = 4. \qquad E = 6 \text{ lb.}$$

Assuming $N = 200$ rev. and $R = 5$ lb., we have from **163**,

$$B = \frac{5}{.00000114 \times 10 \times 200^2} - \frac{(6 \times 4 \times 5) + (6 \times 4 \times 15)}{8 \times 10} =$$
4.96 lb.

That is, each ball must weigh 5 lb., nearly, to cause the governor to operate when the speed has varied two per cent.

From **162**, we have

$W = \frac{1}{8}.$ $[.000028 \times 200^2 (6 \times 4 \times 5 + 5 \times 8 \times 10 + 6 \times 4 \times 15) - (5 \times 8 + 6 \times 4 + 6 \times 4)] = 112.2$ lb.

W should include, beside the counterpoise, all the dead weight coming on the collar. Draw the counterpoise, and find whether it will allow the desired play of the arms. If not, the process must be repeated with other dimensions.

1682. When the governor is built it should be tested, owing to variations that will exist in the weights and sizes and the uncertainty of the resistance R. The counterpoise should be a little too large, so that metal can be turned off for adjustment, and the balls should be cast hollow and filled with lead to facilitate adding or removing weight.

SHAFT GOVERNORS.

1683. Modern high-speed engines, in which slide valves of one form or another are almost invariably employed, are regulated by governors that act upon the eccentrics and vary the points of cut-off according to methods already described. The governor is placed on the main shaft of the engine, from which it derives its name. It consists of

revolving weights whose centrifugal force is entirely balanced by springs.

Three different shaft governors will be described, each of which shifts its eccentric in a different way. No two makers of high-speed engines use shaft governors exactly alike; but if the principles of a few of them are understood, the student should have no difficulty with any.

1684. Buckeye Engine Governor.—The Buckeye engine has a special form of hollow slide valve, with cut-off valve inside, and is regulated by a governor that varies the point of cut-off by changing the angle of advance. The arrangement for effecting this is shown in Fig. 482. A governor wheel for supporting the parts of the governor is

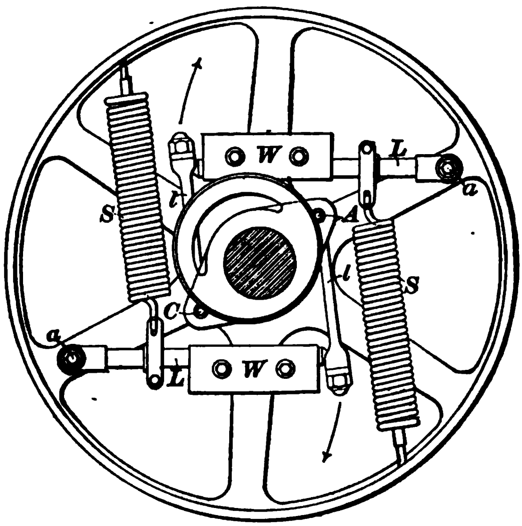


FIG. 482.

keyed to the shaft. *L*, *L* are two arms pivoted to two arms of the wheels at *a* and *a*. Two links *l*, *l* join these arms with lugs on the collar *A C*. The cut-off eccentric is fast to this collar and is loose on the shaft, while the eccentric for the main valve is keyed to the shaft. Now, as the speed increases the centrifugal force of the weights *W*, *W* increases, causing the ends of the arms *L*, *L* to fly out

towards the rim of the wheel and the eccentric to turn on the shaft. The centrifugal force of the weights is balanced by the tension of the springs *S*, *S*.

With a single-valve engine this form of governor could not be used, because, as we have seen, the variation of the angle of advance would produce too great a variation of the lead. But in the Buckeye engine, the admission of the steam is not governed by the cut-off valve, so that in this case, this form of governor is as good as any.

1685. Erie Engine Governor. — This governor, shown in Fig. 483, regulates the cut-off by moving the eccentric across the shaft. A frame *A C* is keyed to the shaft, and two bell-crank levers *L*, *L*, pivoted to the frame, carry the weights *W*, *W* at one end and at the other end are connected with the eccentric in the manner shown. As the weights fly out, the eccentric is moved down (in the figure), the angle of advance is increased, the throw of the eccentric diminished and the cut-off shortened. The spring *S*, by being compressed, resists the action of the weights.

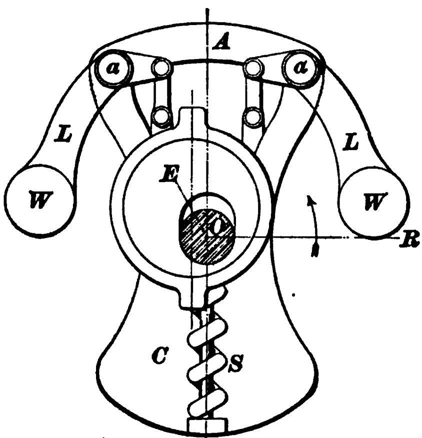


FIG. 483.

It will be well to notice here that in this governor the eccentric moves in a line at right angles to the center line *O R* of the crank. Why this is so will be clear upon reference to the diagram for this form of shifting eccentric in Fig. 461. There the eccentric is supposed to move in the line *D E* parallel to *A C*. But *A C* is the line from which the angle of advance is laid off. Hence, as when no rocker is used, the angle between the crank and eccentric must be

90° + the angle of advance; the relative position of the crank must be at right angles to $A\ C$ and below it, or at right angles to line $D\ E$ in which the eccentric moves. As the Erie engine has no rocker, the relative crank and eccentric positions must be $O\ R$ and $O\ E$, and the eccentric moves at right angles to the crank.

1686. Straight-Line Engine Governor.—Fig. 484

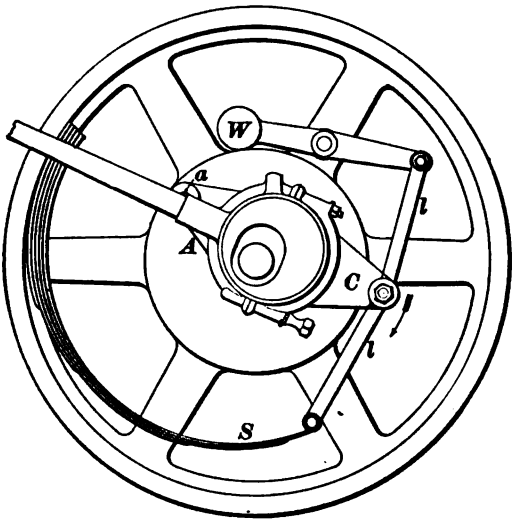


FIG. 484.

shows the principle of the governor used on the Straight-Line engine. The eccentric is on a plate $A\ C$, pivoted at a. As the weight W flies out, the eccentric is shifted about the center a, the links l, l moving in the direction of the arrow and compressing the flat spring S. In this case the governor is attached directly to one of the fly-wheels of the engine.

MACHINE DESIGN.

INTRODUCTORY.

THE DESIGN OF DETAILS.

1901. Rules and formulas for designing many of the most common details of machines are given in the following pages. In some cases these rules are based on considerations of strength, as developed in the subject of Strength of Materials; in others, the wear to which the parts are to be subjected has been the principal element in determining the given proportions. In all cases, however, the practice of successful designers has been followed in preference to mere theoretical principles.

1902. The first work a young designer is called upon to do is usually that of making drawings of details of machines, the general plans of which have been developed by his superiors. He will be given the leading dimensions of these details, and will be required to make a drawing from which the pattern-makers, blacksmiths, and machinists can finish them ready for their places in the completed machine.

1903. In most shops such parts as bolts, nuts, screws, pipe fittings, etc., and often other simple parts of machines, are bought from factories, where they are made in large quantities by special machinery. If the shop is a large one, there may be a separate department where these parts are made according to fixed standards. The designer should know what the practice of the shop in this regard is, and in all cases make his details to conform with these standards. He should also know the kind of material available, the methods employed by the shop in working this material, and

the capacity and principal dimensions of the tools and appliances for doing the work, in order that the detail as designed may be built in the most economical manner. When designing a machine part it is well to keep the processes that will be used in making it in mind; this will often prevent constructions that would be very difficult and expensive if built with the machinery in use in the shop for which the design is made.

1904. In making designs of details it is always advisable to draw them to as large a scale as can be used conveniently. The scales commonly used for small details are full size; and for larger ones 6 inches, 3 inches, or $1\frac{1}{2}$ inches = 1 foot may be used. A scale of 4 inches or 2 inches = 1 foot should never be used if it can possibly be avoided.

1905. Remember that the object of a detail drawing of a machine part is to show the workman in the clearest possible manner how the part is to be made and finished, so that it will take its proper place in the completed machine and do the work for which it is intended. The designer must, therefore, be very careful to make the drawing show the form and dimensions of each portion as clearly as possible; the drawing should also show plainly the kind and quality of material to be used, and the finish, if any, to be given the different surfaces. Use sections wherever the general views do not show the form with perfect clearness.

It is well for a designer to imagine himself in the position of a man in the shop who knows nothing of the machine, and study his drawing carefully to see if anything can possibly be lacking that will be required to make the ideas he wants carried out perfectly clear.

1906. Ordinary dimensions are expressed in feet and inches, and the fractions $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, etc., of an inch. Never use the fractions $\frac{1}{3}$, $\frac{1}{6}$, or $\frac{1}{6}$ in dimensions, as the scales which mechanics use are not divided in these fractions. The most common scales in use by mechanics for ordinary work are two-foot rules divided into inches, numbering from 1 to 24;

for this reason many draftsmen give all dimensions less than two feet in inches and fractions of an inch, and dimensions greater than two feet in feet, inches, and fractions of an inch.

Unless great accuracy is required, decimals are never used in giving dimensions. If decimal values are obtained from the calculations they are expressed in the nearest $\frac{1}{8}$, $\frac{1}{16}$, or $\frac{1}{32}$ of an inch, according to the degree of accuracy required. In particular cases, where extreme accuracy is needed, the dimensions may be expressed in decimals; as, for example, the pitch of gear teeth.

THE GENERAL DESIGN OF A MACHINE.

1907. The methods to be employed in the general design of a machine will vary so much with different conditions that no fixed rules or methods of procedure can be given. The first thing necessary is a thorough knowledge of the work the machine must do, together with its location and surroundings and the conditions under which it must do its work. Keeping these in mind, the designer must apply his knowledge of the principles of applied mechanics, strength of materials, and the design and construction of details, in such a way as to accomplish the desired end in the simplest and most direct manner consistent with the conditions imposed.

All machines consist of different combinations of a few simple principles; and, in order to be successful, the designer must become thoroughly acquainted with these principles and the relation they bear to each other. A study of machines that have been built for similar work is of great assistance in suggesting ideas for the new machine.

In many cases it will be necessary to make more or less complete drawings of a number of different plans, before a satisfactory result will be obtained. A combination that appears feasible at first will be found to be impracticable when drawn out in detail, and the parts proportioned so as to give the necessary strength. In other cases the motion of some part may be found to be limited in such

a way as to interfere with the proper working of the machine. The difficulty or expense of manufacture may also make some otherwise good design impracticable.

1908. In all work, whether designing details or more complicated combinations, keep all calculations, notes, and sketches in such a form that they can be preserved for future reference. Date these notes and give them such titles as will be required to make their purpose perfectly clear. In this way ideas that may be impracticable for the particular case for which they were originally developed can be kept for a possible future use; and the results of many hours spent in calculation will be preserved so as to make a repetition unnecessary. Some engineering establishments provide their draftsmen with books made of manila paper bound in board covers; and all calculations, notes, and sketches are made in these books instead of on loose sheets of paper that will soon be lost.

1909. The following practical rules are often neglected by inexperienced designers:

Make all parts that are subject to wear or breakage accessible for the purpose of inspection, repairs, or renewal.

Provide means for adjusting all parts that are subject to wear.

Make careful provision for lubrication.

Use links and rotating pieces for guiding motion in preference to slides.

Use cranks, levers, belts, and gear-wheels for transmitting motion in preference to cams, screws, or worm-wheels.

Wherever possible, make the motion of all parts positive; that is, avoid the use of weights or springs for producing motion.

Use through bolts or T *head bolts instead of tap bolts or studs, wherever it can be done.*

1910. Designers are often required to furnish an estimate of the weight and cost of a machine from the draw-

ings. This is done in the following manner: The volume of the different details is estimated by means of the principles of mensuration; the weight can then be obtained by multiplying the volume of each piece by the weight of a cubic unit of the material of which it is composed, as given in a table of specific gravities. When the weights are known, the cost of the material is easily found from the known market values. The time that will be required for fitting and finishing the different pieces is then estimated and charged for according to the rates paid for that work. In this way the cost of the machine may be estimated with a degree of accuracy that will depend on the knowledge the estimator has of the time required to do different kinds of work in the shops, and his skill in making approximate calculations of the volumes of irregular-shaped bodies.

MATERIALS USED IN MACHINE CONSTRUCTION.

1911. Cast Iron.—This metal, which has already been briefly referred to in Art. **1332**, etc., is used very largely in the construction of machine parts, particularly those that must be massive; for example, the frames and beds of engines, lathes, planers, etc. It is not well suited for parts subjected to shocks or for parts requiring strength and elasticity.

The great advantage of cast iron is the ease with which it may be given any desired form. Shapes that could not possibly be forged from wrought iron may be cast with comparative ease. The operation of casting is as follows: A pattern is first made of the exact shape of the required part; this pattern is usually made of pine, though metal is sometimes used when the castings are small and a great number are to be made. The pattern is placed in a bed of sand or loam, in which it leaves, after being removed, an impression or cavity called the *mold*. The melted metal is poured into the mold, and, after cooling, the casting is with-

drawn and finished to the required dimensions. Cast iron contracts in cooling about *one-eighth of an inch per foot in each direction;* on account of this contraction, commonly called the **shrinkage,** the pattern must be made that much larger than the required casting. In practice this is always done by using a **shrink rule** in constructing the pattern. The shrink rule is about $\frac{1}{8}''$ longer per foot than the standard rule.

1912. A serious difficulty experienced in the use of cast iron is its liability to be thrown into a state of internal stress, on account of inequality of cooling after being poured into the mold. It is a matter of experience that the amount of contraction depends upon the size and thickness of the casting. In general, thick and heavy parts contract more than thin ones; consequently, a casting composed of both thick and thin parts will sometimes differ from the form desired. Again, one part of a casting may cool and solidify while another part is still in a molten condition. The contraction of the latter must, therefore, strain the part already solidified. The casting is thus thrown into a state of internal stress which must to some degree reduce its effective strength.

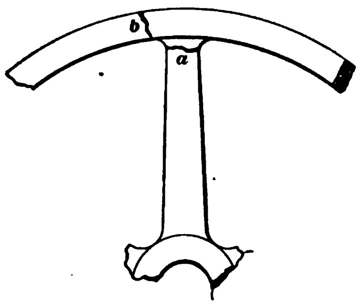


FIG. 592.

Take, for example, the case of a pulley. When the rim is thin, but rigid, it is liable to contract and solidify first, and the subsequent contraction of the arm may induce a fracture, as shown at *a*, Fig. 592. If, however, the arms set first, the subsequent contraction of the rim may cause a fracture, as shown at *b*.

1913. When pulleys are cast with thin rims which are not rigid, the casting often takes the form shown in Fig. 593. The rim is drawn in at the points where it joins the arms, because the arms solidify and contract

after the rim has set, and the latter, not being sufficiently rigid to withstand the pull of the arms, is distorted.

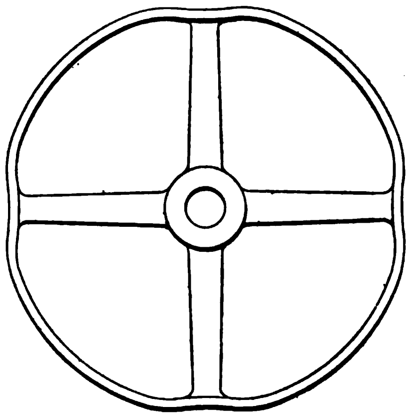


FIG. 593.

These internal stresses make cast iron an unreliable material for the construction of parts requiring strength; and it should be the aim of the designer to prevent these stresses as far as may be by making all parts that are to be cast as uniform in thickness as possible, and avoid having a large boss, or hub, appear in a comparatively thin part, or having a very thick part meet a very thin one.

1914. It is found that, in cooling, the iron crystals arrange themselves perpendicularly to the surface of the

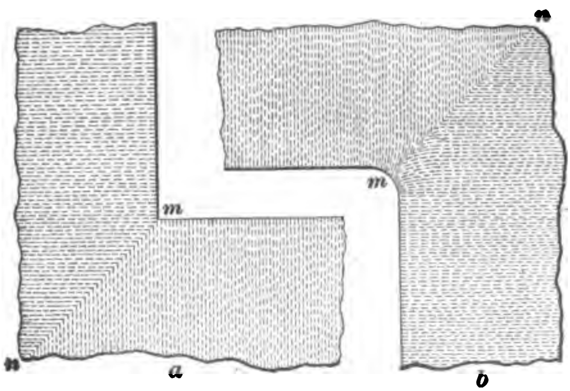


FIG. 594.

casting. For this reason an inside angle (*a*, Fig. 594) is a source of weakness, the casting having a tendency to break through the line *m n*. Such corners should be rounded, as shown at *b*, Fig. 594, in which case the crystalline arrangement renders the casting much stronger.

1915. Chilled castings are made by lining the whole or a part of the mold with cast iron which is protected by a thin coating of loam. The cast-iron lining is a good conductor of heat, and the molten iron is thus cooled off quickly, or *chilled*. The sudden cooling of the casting prevents the carbon from separating from the iron with which it is in chemical combination, and as a result the portion of the casting which is chilled is of white, hard iron. Usually, the chilling extends to the depth of from $\frac{1}{8}$ to $\frac{3}{8}$ inch; the interior of the casting is of soft, gray cast iron, which is best for resisting shocks, while the chilled surface is very hard and durable.

For malleable cast iron see Art. **1337.**

1916. Wrought Iron and Steel.—The leading properties of these metals have already been given. They are used for parts of machines requiring strength and elasticity, such as shafts, bolts, piston rods, and connecting-rods of engines, etc.

Wrought iron or steel machine parts are forged or rolled to approximately the required shape, and then finished upon the lathe, planer, or other tool. Parts may also be cast of steel the same as cast iron. When so made they are called **steel castings.**

1917. Copper.—This metal is used principally for making tubes, steam pipes, expansion joints, and similar details, for condensers, boilers, engines, etc. It can be hammered or rolled into sheets or drawn into wire; it may be cast or forged, but can not be welded. The tenacity of cast copper is about 21,000 lb. per square inch; of forged copper, about 30,000 lb. per square inch. The tenacity of copper may be increased by hammering, wire-drawing, or rolling, but it is at the same time rendered hard and brittle. The toughness may be restored by annealing.

1918. Bronze, or Gun-Metal.—This is an alloy composed of copper and tin, in the proportion of 90 parts of copper to 10 parts of tin. The metal has a tenacity of about 35,000 lb. per square inch. It is largely used for the

bearings of rotating machine parts. The bronze being softer than the iron wears more rapidly, and thus lengthens the life of the rotating part. The hardness of the bronze may be increased by increasing the proportion of the tin; for bearings required to sustain a great pressure the bronze may be composed of 86 parts of copper to 14 parts of tin. A very soft bronze is composed of 92 parts of copper and 8 parts of tin. This quality of bronze is used for making gear-wheels which are subjected to severe shocks.

1919. Phosphor-Bronze.—This is made by alloying ordinary bronze with from 2 to 4 per cent. of phosphorus. It is now being largely used instead of ordinary bronze, and is also employed in place of iron and steel in the construction of propeller blades, pump rods, etc. The softer phosphor-bronze has a tensile strength of about 45,000 lb. per square inch; the hardest varieties may have a tenacity as high as 65,000 lb. per square inch, while hard, unannealed wire has, in some cases, a tenacity of 140,000 lb. per square inch.

1920. Manganese-Bronze.—This is also called white bronze, and is an alloy of ordinary bronze and ferromanganese. It is equal in strength and toughness to mild steel, and may be forged into nuts, bolts, rods, etc. It resists the corroding action of sea water, and is, therefore, much used for propellers. Both manganese-bronze and phosphor-bronze are largely used in marine work.

1921. Brass.—Brass is composed of copper and zinc in the proportion of two parts of the former to one of the latter. Its tenacity is about 25,000 lb. per square inch. Brass is used for condenser tubes and for various fittings, such as valves, cocks, etc.

1922. Wood.—This material is used to a limited extent in machine construction; for example, oak and lignum-vitæ are sometimes used for bearings; beech and hornbeam for cogs of mortise wheels; pine, cherry, and mahogany for patterns.

FASTENINGS.

SCREWS, BOLTS, AND NUTS.

1923. Screws are used in machine construction for three different purposes:

1. As a fastening for clamping or joining parts together. 2. For the transmission of motion. 3. For producing pressure. Screws used as fastenings are called **bolts.**

FORMS OF SCREW THREADS.

1924. The V Thread.—Screw threads are usually triangular or square in section, the triangular form being best for bolts, and the square form best for screws transmitting motion. The **Seller's triangular** or **V thread,** commonly called the **American thread,** or **United States standard,** which is used in the United States, is shown in Fig. 595. Fig. 596 is an enlarged section of the thread.

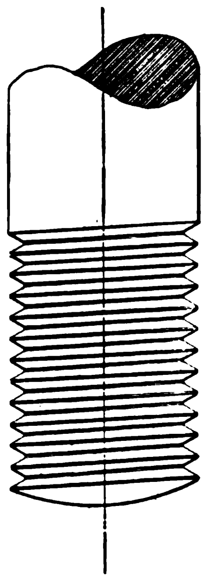


FIG. 595.

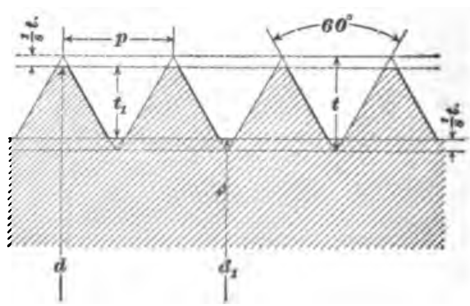


FIG. 596.

The *angle* between the sides of the thread is 60°. The distance p from one thread to the next is called the **pitch** of the screw. As shown in the figure, a section of a single thread is an equilateral triangle, the altitude of which is t; to form the United States standard thread $\frac{1}{8}$ the altitude

of the triangle is cut off from the apex, and the angle at the root is filled in to a like depth. Hence, the *real depth* of the thread, t_1, is $\frac{3}{4}$ the altitude of the triangle; that is, $t_1 = \frac{3}{4} t$.

$$\text{But} \quad t = p \cos 30^\circ = .866\,p;$$

$$\text{hence,} \quad t_1 = \tfrac{3}{4}\, t = .65\,p. \qquad \textbf{(207.)}$$

1925. V threads are sometimes cut without the flat top and bottom, the section being a full equilateral triangle; in this case they are commonly called **sharp V threads.**

1926. The pitch of the thread depends upon the diameter of the bolt; it may be obtained approximately by the following formula, in which d represents the diameter of the bolt:

$$p = .24\sqrt{d + .625} - .175''. \qquad \textbf{(208.)}$$

The diameter d_1 at the root of the thread may be found by the following formula:

$$d_1 = d - 2\,t_1 = d - 1.3\,p. \qquad \textbf{(209.)}$$

The diameter d_1 must always be used in calculating the strength of a bolt.

Letting n represent the number of threads per inch in a screw, we have

$$n = \frac{1}{p}. \qquad \textbf{(210.)}$$

$$\text{Consequently,}\ d_1 = d - \frac{1.3''}{n}. \qquad \textbf{(211.)}$$

EXAMPLE.—The external diameter of a bolt is $1\frac{3}{8}$ inches. Find the pitch, the number of threads per inch, the depth of thread, and the diameter of bolt at root of thread.

SOLUTION.—

$p = .24\sqrt{1.375 + .625} - .175'' = .164''.$

$n = \frac{1}{p} = \frac{1}{.164} = 6$, nearly. Use 6 threads per inch, then, $p = \frac{1}{6} = .167''$. Ans.

$t_1 = .65\,p = .65 \times .167 = \frac{7}{64}''$, nearly. Ans.

$d_1 = d - 2\,t_1 = 1\frac{3}{8} - \frac{7}{32} = 1\frac{5}{32}''$. Ans.

Table 43 gives the number of threads per inch, diameter of bolt at root of thread, and effective area of bolt at root of thread, United States standard sizes:

TABLE 43.

Diameter of Screw in Inches. d.	Number of Threads per Inch. n.	Diameter at Bottom of Threads in Inches. d_1.	Area at Bottom of Threads in Square Inches. a.
1/4	20	.185	.0269
5/16	18	.240	.0452
3/8	16	.294	.0679
7/16	14	.345	.0935
1/2	13	.400	.1257
9/16	12	.454	.1619
5/8	11	.507	.2019
3/4	10	.620	.3019
7/8	9	.731	.4197
1	8	.838	.5515
1 1/8	7	.939	.6925
1 1/4	7	1.064	.8892
1 3/8	6	1.158	1.0532
1 1/2	6	1.283	1.2928
1 5/8	5 1/2	1.389	1.5153
1 3/4	5	1.490	1.7437
1 7/8	5	1.615	2.0485
2	4 1/2	1.711	2.2993
2 1/4	4 1/2	1.961	3.0203
2 1/2	4	2.175	3.7154
2 3/4	4	2.425	4.6186
3	3 1/2	2.629	5.4284
3 1/4	3 1/2	2.879	6.5099
3 1/2	3 1/4	3.100	7.5477
3 3/4	3	3.317	8.6414
4	3	3.567	9.9930
4 1/4	2 7/8	3.798	11.3292
4 1/2	2 3/4	4.027	12.7366
4 3/4	2 5/8	4.255	14.2197
5	2 1/2	4.480	15.7633
5 1/4	2 1/2	4.730	17.5717
5 1/2	2 3/8	4.953	19.2676
5 3/4	2 3/8	5.203	21.2617
6	2 1/4	5.423	23.0978

1927. The Square Thread.—In Fig. 597 is shown a screw with a **square thread,** and in Fig. 598 an enlarged section of the thread. As the name implies, the section of the thread is a square, each side of which is one-half the pitch.

The pitch of the square thread is usually taken *double* that of the triangular thread

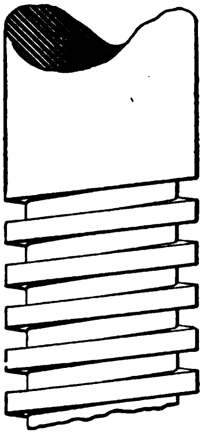


FIG. 597.

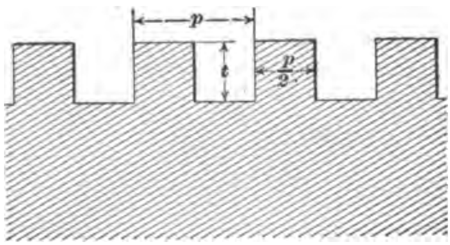


FIG. 598.

for the same diameter of bolt; for example, the pitch of a square thread on a 2-inch bolt or rod is $\frac{1}{2}$ inch. Approximately, the pitch is $\frac{1}{5}$ the diameter of the screw; that is,

$$p = \frac{d}{5} = \frac{d_1}{4}. \qquad \textbf{(212.)}$$

$$\text{Hence, } d_1 = \frac{4}{5} d. \qquad \textbf{(213.)}$$

$$\text{Also, } t = \frac{p}{2} = \frac{d}{10} = \frac{d_1}{8}. \qquad \textbf{(214.)}$$

The edges of the threads should be very slightly rounded off so as to prevent them from being accidentally flattened, which would cause a nut to bind upon the thread of the screw. When this rounding off is carried far enough, the thread takes the form shown in Fig. 599. This thread is especially adapted to withstand rough usage.

1928. The modification of the square thread, shown in Fig. 600, is frequently used for the lead screws of lathes. The section of the thread, instead of being square, tapers slightly from root to point. This taper is given not only because a thread of this form is much easier to cut than the

square thread, but because it enables the nut, which is made

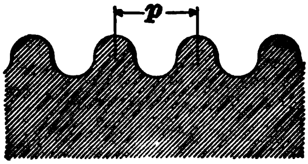


FIG. 599.

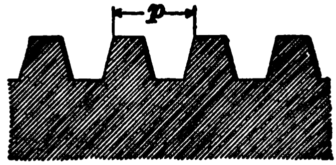


FIG. 600.

in two parts, to readily engage or disengage with the screw.

1929. The **trapezoidal screw thread** is shown in Fig. 601. One face of the thread is perpendicular to the axis of the screw, and the other is inclined at an angle of 45°. From the construction of the figure it is evident that $t = p$, the pitch. To form the actual thread an amount equal to $\frac{1}{8}\,t$ is cut off from top and bottom of the triangle; hence, the real depth t_1 is $\frac{3}{4}\,t$.

When this thread is used, it is generally for communicating motion, or where great resistance without any bursting tendency is required. The usual dimensions are given by the following formulas, in which the letters have the same meaning as before:

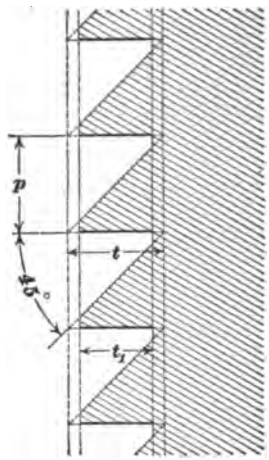


FIG 601.

$$p = \frac{2\,d}{15} = \frac{d_1}{6}. \qquad (215.)$$

$$t_1 = \frac{d}{10} = \frac{d_1}{8}. \qquad (216.)$$

For example, supposing the diameter of the screw to be $1\frac{1}{2}$ inches, the pitch of a trapezoidal thread is $p = \frac{2\,d}{15} = \frac{2 \times 1\frac{1}{2}}{15} = \frac{1}{5}''$; the number of threads per inch is $\frac{1}{p} = \frac{1}{\frac{1}{5}} = 5$, and the depth of the thread is $t_1 = \frac{d}{10} = \frac{1\frac{1}{2}}{10} = .15''$.

1930. The relative advantages of the various forms of screw threads may be shown by a consideration of the

forces acting on the thread. Usually the load on a bolt or screw acts in the direction of its axis; that is, a bolt used as a fastening is in tension, while a screw used to produce pressure is in compression. In either case the load is carried by the reaction between the surface of the thread of the screw and the surface of the thread in the nut. Suppose, in Fig. 602, the load to be upon the side $m\ l$ of the thread, and let the reaction of the thread at the point A be represented by R, which must, of course, be perpendicular to $m\ l$. This reaction R may be resolved into two forces, one, P, parallel to the axis of the bolt, and the other, Q, perpendicular to the axis. Then, P represents the portion of the load carried by the surface A of the screw, while the force Q tends to burst the nut. Now, for a given load the force P will remain the same whatever the angle of the thread may be. On the other hand, the forces Q and R will increase as the angle $m\ l\ n$ decreases. The friction between two surfaces is proportional to the perpendicular pressure between them. Consequently, *the greater the angle of a screw thread, the greater is the friction between the bolt and nut, and also the greater is the force tending to burst the nut.*

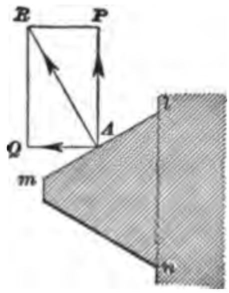


FIG. 602.

In the case of the square thread, the angle between the sides is zero, and hence there is no force tending to burst the nut. The reaction R becomes equal to the load P; therefore, the friction of a square thread is less than that of a triangular thread. On the other hand, the triangular thread is nearly twice as strong as a square thread. Thus, in Fig. 602, the shearing surface of a single triangular thread is $\pi d_1 \times$ distance $n\ l$, or $\pi d_1 p$, nearly, while it will be seen by referring to Fig. 598 that the shearing surface of a single square thread is $\pi d_1 \times \frac{1}{2} p = \frac{\pi d_1 p}{2}$.

It follows, therefore, that the triangular thread is better for fastenings, and the square thread for transmitting motion.

The trapezoidal thread combines the good features of both the triangular and the square threads. It has the same shearing section as the former, and the same friction as the latter. In this care must be taken, however, to use the screw so that the pressure comes on the flat side of the thread, for if it is put upon the inclined side the friction and bursting force on the nut are both greater than for a 60° triangular thread. The trapezoidal thread is used in the breech mechanism of large guns.

In England the **Whitworth system** of triangular threads is in use. The angle of the Whitworth thread is 55° and the point is rounded instead of being cut flat.

1931. Multiple-Threaded Screws.—It is plain that a nut will advance a distance equal to the pitch of the thread for each revolution of the screw. When a screw is used to transmit motion, it is often desirable to have the nut advance a considerable distance for one revolution, and this may necessitate a pitch altogether too large for the diameter of the screw. This difficulty is obviated by cutting two or more parallel threads, each having the same pitch.

These screws are termed **multiple-threaded** screws; when the screw has two threads, it is called a **double-threaded** screw; when it has three threads, a **triple-threaded** screw, and when it has four threads, a **quadruple-threaded** screw.

In Fig. 603 is shown a single square-threaded screw, and in Fig. 604 a double square-threaded screw, both screws having the same diameter and pitch. It is apparent that the root diameter of the double-threaded screw is much larger than that of the single-threaded one; it is, consequently, stronger, and is, therefore, to be pre-

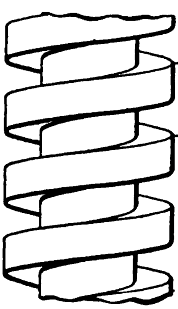


FIG. 603.

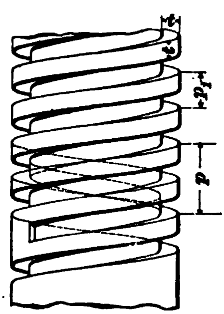


FIG. 604.

ferred. The distance (p_1, Fig. 604) between *two consecutive threads* of a multiple-threaded screw is equal to the pitch p, divided by the number of threads (2, 3, or 4, according to whether the screw is double, triple, or quadruple-threaded), and is called the **divided pitch** of the thread.

The dimensions of the thread are based upon this divided pitch; that is, $t = \frac{p_1}{2}$. Besides being stronger for the same pitch than the single thread, the multiple thread has the advantage of having a greater wearing surface than the single thread.

1932. Gas-Pipe Threads.—The rules for the pitches and depth of screw threads do not apply to gas-pipe threads, since the calculated depth of the thread would in that case be greater than the thickness of the pipe.

The following table gives the standard dimensions of steam, gas, and water pipes:

TABLE 44.

Nominal Diameter in Inches.	Thickness in Inches.	Actual Internal Diameter in Inches.	Actual External Diameter in Inches.	Threads per Inch. n.	Pitch of Threads.
$\frac{1}{8}$	.068	.270	.405	27	.037
$\frac{1}{4}$	.088	.364	.540	18	.056
$\frac{3}{8}$	.091	.494	.675	18	.056
$\frac{1}{2}$	.109	.623	.840	14	.071
$\frac{3}{4}$	.113	.824	1.050	14	.071
1	.134	1.048	1.315	$11\frac{1}{2}$	.087
$1\frac{1}{4}$	.140	1.380	1.660	$11\frac{1}{2}$	.087
$1\frac{1}{2}$	.145	1.611	1.900	$11\frac{1}{2}$	.087
2	.154	2.067	2.375	$11\frac{1}{2}$	.087
$2\frac{1}{2}$	.204	2.468	2.875	8	.125
3	.217	3.061	3.500	8	.125
$3\frac{1}{2}$	.226	3.548	4.000	8	.125
4	.237	4.026	4.500	8	.125
$4\frac{1}{2}$	.247	4.508	5.000	8	.125
5	.259	5.045	5.563	8	.125
6	.280	6.065	6.625	8	.125
7	.301	7.023	7.625	8	.125
8	.322	7.982	8.625	8	.125
9	.344	9.001	9.688	8	.125
10	.366	10.019	10.750	8	.125

1933. Threads may be right-handed or left-handed. To determine whether a screw is right or left-handed, hold it so that its axis will be horizontal; if the slope of the thread (from top to bottom) is from left to right, it is right-handed; otherwise the thread is left-handed. For nuts the above rule should be reversed. The threads of screws for general use are right-handed, and are so shown in the previous figures. Screws having left-handed threads are made only for special purposes.

STRENGTH OF SCREW BOLTS.

1934. Usually the stress on a bolt acts in the direction of its axis; that is, the bolt is in tension.

Let W = load on bolt in pounds;
S_t = *safe* working stress in pounds per square inch;
a = area of cross-section of bolt at root of thread;
d = nominal (outside) diameter of bolt in inches;
d_1 = diameter at root of thread in inches.

Then, if the bolt is in tension,

$$W = a\,S_t; \text{ or, } a = \frac{W}{S_t}. \qquad \textbf{(217.)}$$

The value of the nominal diameter d (corresponding to the value of a) obtained from formula **211** may be found from Table 43.

1935. For bolts subjected to a constant tension, S_t may be 8,000 lb. per sq. in. More often the tension varies between zero and its maximum value; in this case S_t may be taken as 6,000 lb. per sq. in. For cylinder-head bolts, and, in general, for bolts used to make a steam-tight joint, S_t may vary from 3,000 lb. for small cylinders to 6,000 for very large ones. Ordinarily S_t may be taken as 4,000 or 4,500 lb. per sq. in. All the above values are for wrought-iron bolts.

EXAMPLE.—Find the diameter of a wrought-iron bolt which is to sustain a steady load of $4\frac{1}{2}$ tons.

SOLUTION.—From formula **217**,

$$a = \frac{W}{S_t} = \frac{4\frac{1}{2} \times 2,000}{8,000} = 1.125 \text{ sq. in.}$$

From Table 43, the value of d lies between $1\frac{3}{8}''$ and $1\frac{1}{2}''$. The latter value should be taken. Ans.

1936. For screws transmitting motion, the following formula may be used:

$$W = 3,000\,d_1^{\,2} = 1,920\,d^2 \text{ (since } d_1 = \tfrac{4}{5}d\text{)},$$

$$\left.\begin{aligned} \text{or } d_1 &= .0183\sqrt{W} \\ d &= .0228\sqrt{W} \end{aligned}\right\} \qquad \textbf{(218.)}$$

For screws of this character, the least number of threads in the nut that are necessary to prevent excessive wear is given by the following formula, in which n_1 = the number of threads in the nut:

$$n_1 = \frac{W}{300\, d_1^2} = .0052\frac{W}{d^2}. \qquad \textbf{(219.)}$$

The above formula applies to square and trapezoidal threads, and is based upon the assumption that the pressure on the thread per square inch of projected area should not be greater than 700 lb. per sq. in.

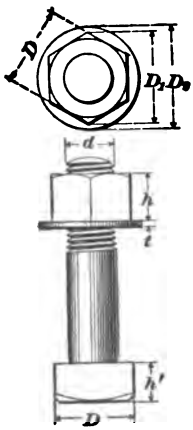


FIG. 605.

EXAMPLE.—A square-threaded screw 1½ inches in diameter transmits motion to a load of 4,000 pounds. What is the least allowable number of threads in the nut?

SOLUTION.—

$$n_1 = .0052\frac{W}{d^2} = \frac{.0052 \times 4,000}{(1\tfrac{1}{2})^2} = 9\tfrac{1}{4}. \quad \text{Ans.}$$

PROPORTIONS OF BOLTS AND NUTS.

1937. The dimensions of the nut and bolt head are made to depend upon the diameter of the bolt. The standard form of bolt and nut is shown in Fig. 605. The bolt has a square head and hexagonal nut with washer. The washer is used to give a smooth seat for the nut to be screwed up against.

The following proportions are usually adopted;

Diameter of nut or head across flats,

$$\left.\begin{aligned} D &= 1\tfrac{1}{2}\,d + \tfrac{1}{8}'' \text{ for rough work.} \\ D &= 1\tfrac{1}{2}\,d + \tfrac{1}{16}'' \text{ for finished work.} \end{aligned}\right\} \qquad \textbf{(220.)}$$

Height of nut,

$$\left.\begin{aligned} h &= d \text{ for rough work.} \\ h &= d - \tfrac{1}{16}'' \text{ for finished work.} \end{aligned}\right\} \qquad \textbf{(221.)}$$

Thickness of washer, $t = .15\,d$. **(222.)**

Diameter of washer, $D_2 = 1\tfrac{1}{8}\,D_1$. **(223.)**

The above proportions for diameters D hold for both hexagonal and square nuts. The diameter across corners D_1 may be found from the geometry of the figure. Thus, for hexagonal nuts,

$$D_1 = \frac{D}{\cos 30°} = \frac{D}{.866} = \left\{ \begin{array}{l} 1.73d + .14'' \text{ for rough nuts.} \\ 1.73d + .07'' \text{ for finished nuts.} \end{array} \right\} \quad (224.)$$

For square nuts,

$$D_1 = \frac{D}{\cos 45°} = D\sqrt{2} = \left\{ \begin{array}{l} 2.12d + .18'' \text{ for rough nuts.} \\ 2.12d + .09'' \text{ for finished nuts.} \end{array} \right\} \quad (225.)$$

Height of head,

$$\left. \begin{array}{l} h' = \frac{3}{4}d + \frac{1}{16}'' \text{ for rough bolts.} \\ h' = d - \frac{1}{16}'' \text{ for finished bolts.} \end{array} \right\} \quad (226.)$$

EXAMPLE.—Required, the various dimensions of a finished bolt and hexagonal nut, the bolt being $1\frac{1}{2}$ inches in diameter.

SOLUTION.—Diameter of nut across flats $= D = 1\frac{1}{2}d + \frac{1}{16} = 1\frac{1}{2} \times 1\frac{1}{2} + \frac{1}{16} = 2\frac{5}{16}$ in.

Diameter of nut across corners $= D_1 = \frac{D}{.866} = \frac{2.3125}{.866} = 2\frac{11}{16}$ in., nearly. Ans.

Side of square bolt head $= D = 2\frac{5}{16}$ in. Ans.

Height of nut $= h = d - \frac{1}{16} = 1\frac{1}{2} - \frac{1}{16} = 1\frac{7}{16}$ in. Ans.

Height of bolt head $= h' = d - \frac{1}{16} = 1\frac{1}{2} - \frac{1}{16} = 1\frac{7}{16}$ in. Ans.

Diameter of washer $= D_2 = 1\frac{1}{8}D_1 = 1\frac{1}{8} \times 2\frac{11}{16} = 3$ in., nearly. Ans.

Thickness of washer $= t = .15d = .15 \times 1\frac{1}{2} = \frac{1}{4}$ in., nearly. Ans.

WRENCHES.

1938. The usual forms of solid wrenches are shown in

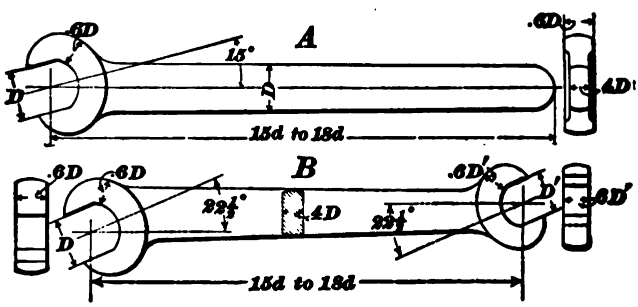


FIG. 606.

Fig. 606, in which that shown at A is used for hexagonal nuts, and that at B for square nuts. The length may be

from 15 to 18 times the diameter of the bolt for which it is to be used. In the figures d represents the diameter of the bolt. The other proportions are given in terms of the diameter across the flats of the nut as shown in the figure.

FORMS OF BOLT HEADS.

1939. The ordinary square bolt head has been shown in Fig. 605. Other forms are shown in Figs. 607 to 615. In Fig. 607, the hexagonal bolt head is similar to a hexagonal nut, and has the same dimensions except that the height h' may be less. Usually $h' = \frac{2}{3}\,d$ to d.

Fig. 608 shows a hexagonal head with a collar or flange, which is added to give an increased bearing surface. A

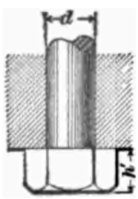


FIG. 607.

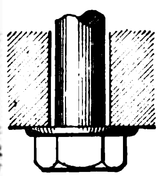


FIG. 608.

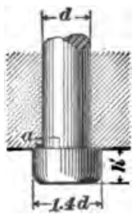


FIG. 609.

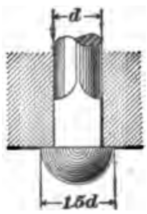


FIG. 610.

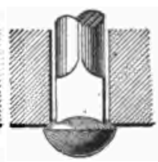


FIG. 611.

cylindrical head is shown by Fig. 609, and a hemispherical head by Fig. 610. The height h' of the former may be from $.5d$ to $.8d$; that of the latter is $\frac{3}{4}d$. The diameter of these heads is as shown by the figures. Fig. 611 shows a bolt head with a hemispherical bearing surface resting on a seat of the same shape. This bolt may lean to one side while the head will still remain in contact with its seat all the way round.

An eye-bolt is shown in Fig. 612. The cross-section of the eye through the hole should equal or exceed the area of the bolt.

That is, referring to this figure, $2\,a\,b = \frac{1}{4}\,\pi\,d^2$ or

$$a\,b = .39d^2.$$

In good practice $a\,b$ is at least equal to $\frac{1}{2}d^2$. To calculate the diameter d_1 of the pin passing through the eye, we observe that the pin is in double shear, the shearing surface being twice the area of it; or $2\left(\frac{\pi\,d_1^2}{4}\right) = \frac{1}{2}\,\pi\,d_1^2$.

The strength of this pin in shear should equal the strength of the eye-bolt in tension; therefore, letting S_s represent

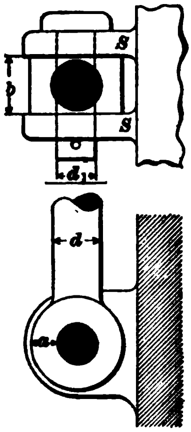


FIG. 612.

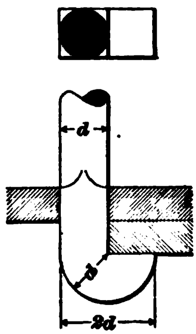


FIG. 613.

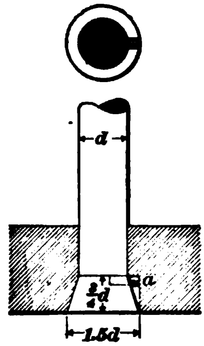


FIG. 614.

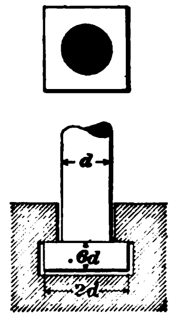


FIG. 615.

the *safe* shearing stress per square inch, and S_t the *safe* tensile strength, we have

$$\tfrac{1}{2}\pi d_1^2 S_s = \tfrac{1}{4}\pi d^2 S_t,$$

$$\text{or } d_1 = d\sqrt{\frac{S_t}{2S_s}}.$$

But the ratio $\frac{S_t}{S_s}$ is usually about 1.25;

hence, $\quad d_1 = d\sqrt{.625} = .8\,d$, nearly. **(227.)**

If, however, the pin is overhung; that is, if there is but one of the nibs S, S, instead of two, as shown in Fig. 612, it will be in a single shear, and

$$d' = d\sqrt{\frac{S_t}{S_s}} = 1.1\,d, \text{ nearly.} \qquad \textbf{(228.)}$$

Fig. 613 shows the head of a **hook bolt.** This form of bolt is used when it is undesirable to weaken one of the connected pieces by a bolt hole. The proportions are shown in the figure. The **countersunk head** is shown by Fig. 614, and the **T head** by Fig. 615.

1940. The ordinary method of *attaching a bolt to stonework* is shown in Fig. 616. The head is long and rectangular, and is made *jagged* with a cold chisel; the hole is made

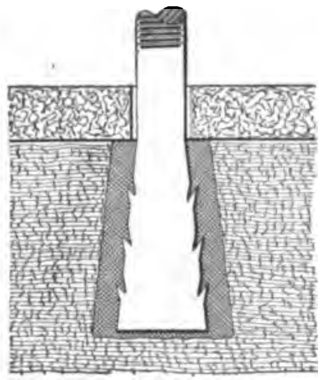


FIG. 616.

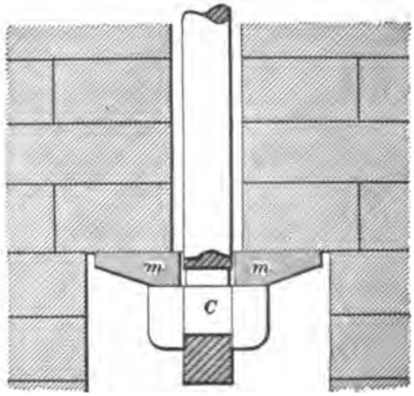


FIG. 617.

larger at the bottom than at the mouth. The bolt head is placed in the hole and the remaining space is then filled with melted lead or sulphur.

1941. The ordinary method of fixing the **foundation bolts** which fasten an engine bed to its foundation is shown in Fig. 617. These foundation bolts have no solid heads, but are long rods threaded on one end for a nut, and have a slot in the other end through which passes a **cotter** *C*, which rests against a cast-iron or wrought-iron washer *m*. This washer and cotter form the head of the bolt.

The bolt head is within a recess formed in the foundation, so arranged as to be accessible. The area of the washer bearing against the foundation, multiplied by the safe compressive strength of the material of the foundation, should be equal to the tension carried by the bolt. For example, the tensile strength of wrought iron is about 20 times the compressive strength of brick. Hence, the bearing area of a washer resting against a brick foundation should be 20 times the cross-section of the bolt.

1942. Various devices are used to prevent a bolt from *turning* while the nut is being screwed up. A common

method is to make the neck of the bolt next to the head **square,** as shown in Figs. 610, 611, and 613. The *bolt hole* is also made square. Another way is to insert a *pin a* into the neck, close to the head, as shown in Figs. 609 and 614. The projecting part of the pin fits into a recess cut out to receive it.

FORMS OF NUTS.

1943. The *common hexagonal nut* has been shown in Fig. 605. Ordinarily, both hexagonal and square bolt heads and nuts are *chamfered off* at an angle of 30° or 40°. Other forms of nuts are shown in Figs. 618 to 622. The **flanged**

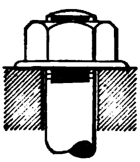


FIG. 618.

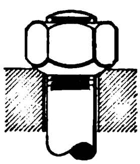


FIG. 619.

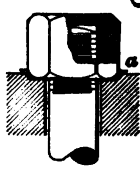


FIG. 620.

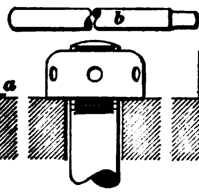


FIG. 621.

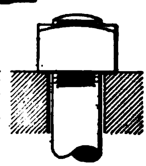


FIG. 622.

nut, Fig. 618, is useful when the bolt hole is larger than the bolt, as it covers the hole and gives a greater bearing surface. Fig. 619 shows a nut with a **spherical bearing surface** and the seat shaped to correspond. This nut will bear upon the seat all around, whether the bolt be perpendicular or inclined to the seat. A **cap nut** is shown in Fig. 620. This form of nut is used to prevent the leakage of a fluid past the screw threads. To prevent leakage past the seat, the nut is screwed down on a soft, thin copper washer *a*. Fig. 621 shows a **round nut,** and Fig. 622 an ordinary **square nut.** The round nut is provided with holes in its circumference, as shown, and is screwed up by inserting a bar *b* in one of the holes.

LOCKING NUTS.

1944. All nuts are slightly loose on their bolts, a small clearance being necessary to permit them to turn freely. When a nut is subject to vibration it is liable to slack back and

allow the bolt to become loose. To prevent this slacking back, various locking arrangements have been devised. A common device is the **locknut,** or **jam nut,** shown in Fig. 623. Two nuts are used, one of which is about half as thick as the ordinary nut. The load is thrown on the outer nut, which should, therefore, be the thicker one. In practice it is common to place the thin one on the outside, because the wrench is generally too thick to act on it when placed below the other. The jam nut is not always satisfactory as a method of locking.

1945. The nut may be effectively locked to the bolt by the use of a set-screw, as shown in Fig. 624. To prevent the point of the set-screw from injuring the thread, a piece of iron or steel *m* may be let into the nut. The piece is screwed along with the nut, and acts as a shield interposed between the set-screw and thread.

1946. A good method of locking a nut is shown in Fig. 625. The lower portion of the nut is turned down, and

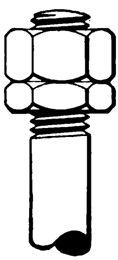


FIG. 623

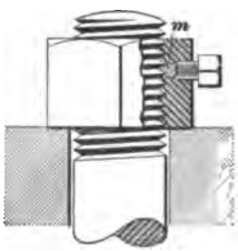


FIG. 624.

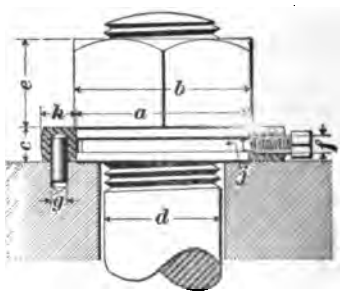


FIG. 625.

a groove is cut in the center of the circular portion. A collar is fastened by means of a pin to one of the pieces to be connected, and the circular part of the nut is fitted into this collar. The nut is then bound to the collar by a set-screw passing through the latter, the point of the set-screw engaging into the groove turned in the nut. The following proportions have proved very satisfactory, in which *d*, the diameter of the bolt, is taken as a unit. All dimensions are in inches:

$$\left.\begin{array}{ll} a = 1\tfrac{1}{2}d - \tfrac{1}{16}''; & f = \tfrac{1}{8}d + \tfrac{1}{8}''; \\ b = 1\tfrac{1}{2}d + \tfrac{1}{8}''; & g = \tfrac{1}{8}d + \tfrac{1}{16}''; \\ c = \tfrac{1}{4}d + \tfrac{1}{4}''; & h = \tfrac{1}{4}d + \tfrac{1}{4}''; \\ \multicolumn{2}{c}{e = \tfrac{3}{4}d.} \end{array}\right\} \quad (229.)$$

1947. Fig. 626 shows a device for locking a nut by means of a **stop plate.** The plate is fastened to one of the pieces through which the bolt passes.

It is so shaped that the bolt may be locked at intervals of $\frac{1}{12}$ of a revolution. Suitable proportions for this stop plate are shown in the figure, in which d, the diameter of the bolt, is taken for the unit, except the distance between the center of the bolt and screw for which D, the diameter between the parallel sides of the nut, is taken for the unit. All dimensions are in inches.

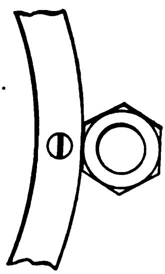


FIG. 627.

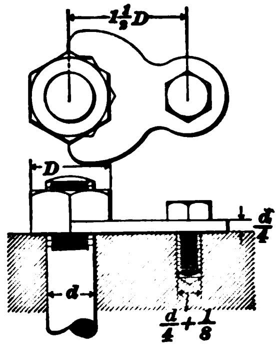


FIG. 626.

1948. In Fig. 627 is shown another form of *stop plate*, which may be conveniently used when the bolts are set in a circle, as, for example, on engine cylinder heads.

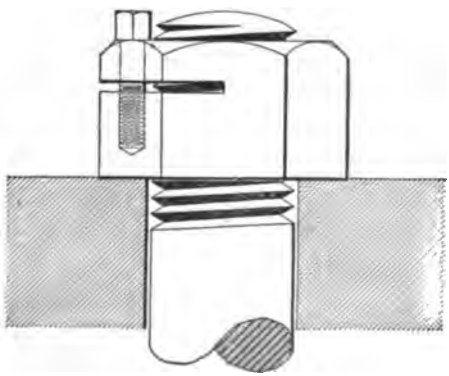


FIG. 628.

1949. In Fig. 628 is shown a different manner of locking the nut. In this the nut is sawed half way through, and the parts connected by a small screw. When the nut is screwed home the small screw is tightened, thereby greatly increasing the friction between the bolt and the nut.

1950. A convenient locking device is **Grover's spring washer,** shown in Fig. 629. The washer when not held

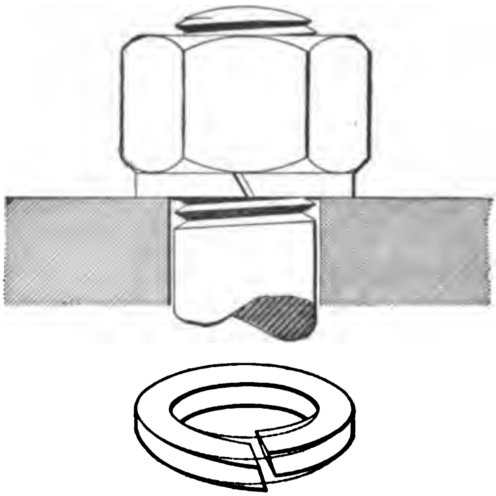


FIG. 629.

down by the nut has the form shown in the lower part of the figure; when the nut is screwed down tightly, the washer is flattened out and its elasticity keeps the nut tight on the bolt.

FORMS OF BOLTS AND SCREWS.

1951. Bolts.—A **stud bolt,** or **stud,** is shown in Fig. 630. Each end of the stud has a screw thread cut on it. One end screws into one of the pieces to be connected, the other carries a nut.

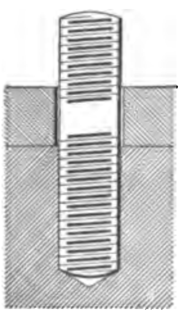


FIG. 630.

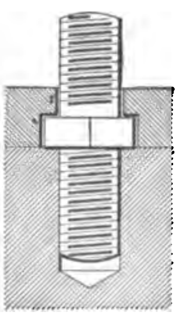


FIG. 631.

A stud having a **collar** is shown in Fig. 631. The collar may be square or round; it serves as a shoulder against which to screw up the stud, and, when square, is a convenient place to apply a wrench.

A **tap bolt,** shown in Fig. 632, is a bolt not requiring a nut. It is screwed directly

into one of the pieces to be connected, the head pressing upon the other piece.

Fig. 633 shows a tap bolt having a countersunk head. This style of bolt is called a **patch bolt,** and, as its name implies, it is used in making patches in boilers, etc. The diameter of the neck of the projection to which the wrench

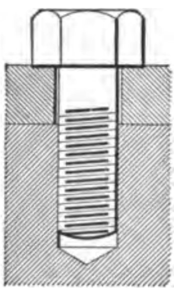


FIG. 632.

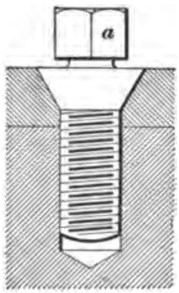


FIG. 633.

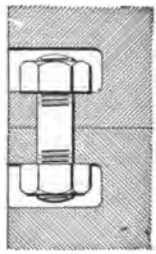


FIG. 634.

is applied is smaller than the root diameter of the bolt, so that the projection will break off instead of breaking the bolt when too much force is applied to it.

Fig. 634 shows a bolt having a nut at each end instead of a head and nut.

1952. Screws.—In Figs. 635, 636, and 637 are shown different forms of machine screws slotted for a screwdriver. Fig. 635 is called **a countersink head screw;** Fig. 636,

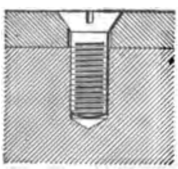


FIG. 635.

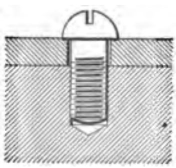


FIG. 636.

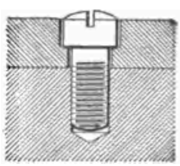


FIG. 637.

a **button head screw,** and Fig. 637, a **fillister head screw.** When the countersink head screw is used, the hole in the piece which is to be held tight is countersunk so that the head of the screw is flush as shown.

1953. Set-screws are screws or bolts which are used to press against a piece, and by friction to prevent it from

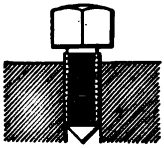


FIG. 638.

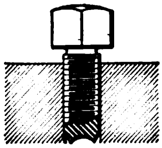


FIG. 639.

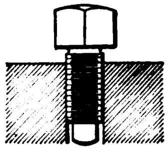


FIG. 640.

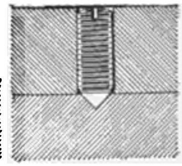


FIG. 641.

moving or rotating relatively to another piece. For example, a set-screw may be screwed through the hub of a pulley, and by pressing against the shaft will prevent the pulley from turning on the shaft. Various forms of set-screws are shown in Figs. 638, 639, 640, and 641.

Fig. 638 is called a **cone-point set-screw**; Fig. 639, a **cupped set-screw**; Fig. 640, a **round pivot-point set-screw,** and Fig. 641, a **headless cone-point set-screw.**

1954. Bolts in Shear.—Usually bolts are in direct tension, but constructions occur in which a bolt may be placed in shear.

The strength of a bolt in shear is about $\frac{4}{5}$ that of a bolt in tension; that is, the shearing strength of wrought iron is about $\frac{4}{5}$ the tensile strength. Hence, the diameter of a bolt in single shear should be $\sqrt{\frac{5}{4}} = 1.1$ that of a bolt in tension under the same load; and the diameter of a bolt in double shear should be $\sqrt{\frac{5}{4 \times 2}} = .8$ that of a bolt in tension under the same load.

1955. Knuckle Joint.—The knuckle joint, Fig. 642, is an example of a bolt in shear. Since the bolt is in double shear, it need be theoretically only .8 the diameter of the rod. The bolt wears, however, and since it should at no time be less than .8 the diameter of the rod, the bolt and rod are made equal in diameter.

The other proportions are in terms of the diameter of the bolt. All dimensions are in inches.

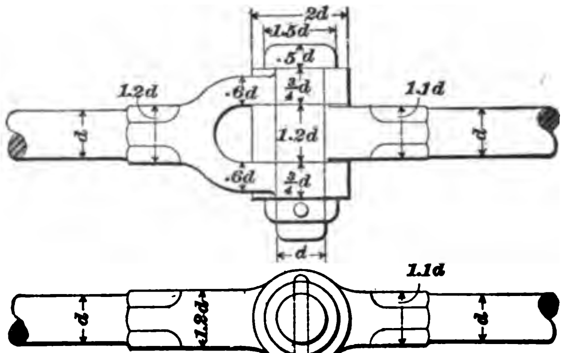


FIG. 642.

Other examples of bolts in shear may be seen in pin-connected iron bridges.

EXAMPLES FOR PRACTICE.

1. Calculate the diameter of a wrought-iron bolt which is to sustain a varying load of 2,300 pounds. Ans. $\frac{7}{8}$ in.

2. What steady load may be safely sustained by 5 bolts $1\frac{1}{8}''$ in diameter? Ans. 13.86 tons.

3. A screw with square threads transmits motion to a load of 1,500 pounds. Calculate the diameter of the screw, the number of threads per inch, and the necessary number of threads in the nut.

Ans. $\begin{cases} \text{Diameter} = \frac{7}{8} \text{ in.} \\ \text{Threads per inch} = 6. \\ \text{Threads in nut} = 10. \end{cases}$

KEYS.

1956. Keys are iron or steel wedges used to secure wheels, cranks, or pulleys to shafts. It is the duty of the key to prevent the relative rotation of the pieces connected; if, for example, the pieces in question are a pulley and shaft, the function of the key is to prevent the pulley from turning on the shaft. Generally, the key will also prevent a wheel or pulley from moving lengthwise along the shaft.

FORMS OF KEYS.

1957. The **concave key** is shown in Fig. 643. The key is hollowed out to fit the shaft, and holds by friction alone; hence, it is suitable only for light work.

Fig. 644 shows a shaft with a **flat key.** A flat surface is cut on the shaft to receive the key, which is, consequently, more effective than the concave key.

1958. The **sunk key,** Fig. 645, is much more effective than either of the above mentioned, since it is impossible for the pulley to slip on the shaft without shearing off the key.

A slot called the **key-way** is cut lengthwise in the shaft,

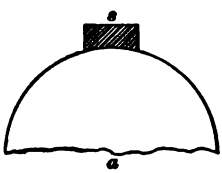


FIG. 643.

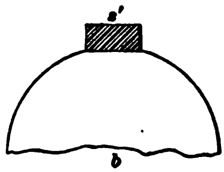


FIG. 644.

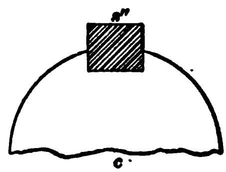


FIG. 645.

and another one is cut in the hub of the pulley. The key is accurately fitted and driven in.

Two kinds of sunk keys are in common use: 1. The

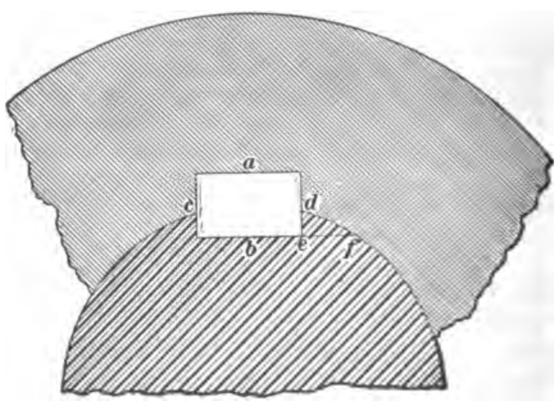


FIG. 646.

rectangular keys of the form shown in Fig. 646, which are used for fastening cranks, gear-wheels, etc. 2. Square keys,

Fig. 645, which are used on machine tools, and, in general, on work requiring accurate fitting.

The keys of the former class are driven in tightly and usually fit at the top and bottom as well as at the sides; they are unsuited for parts which require nice fitting, because they are liable to spring the parts out of true. The square key, on the other hand, does not fit tightly at the top and bottom, but rather at the sides.

1959. Large wheels or pulleys may be fastened to the shaft by using two, three, or four keys. Fig. 647 shows a method of keying a piece to a square shaft. When several keys are thus used the wheel or pulley may be centered on

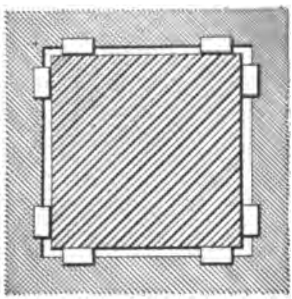


FIG. 647.

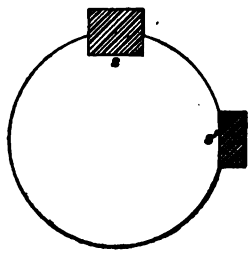


FIG. 648.

the shaft by means of the keys. If a pulley is accidentally bored a little too large for the shaft, it may be prevented from rocking by using both a sunk key and a flat key, as shown in Fig. 648.

The flat key is placed at a distance around the shaft of about 90° from the sunk key, and the pulley is thus made to bear on the shaft at three points.

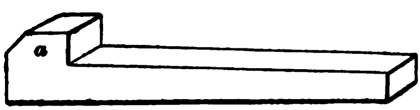


FIG. 649.

When a key cannot be conveniently driven out from the small end, it is necessary to make it with a gib head *a*, as shown in Fig. 649. The head forms a shoulder to drive against.

1960. Sliding, or feather, keys are used where it is desired to prevent a piece from rotating on a shaft, and, at the same time, allow it to slide lengthwise. The key is

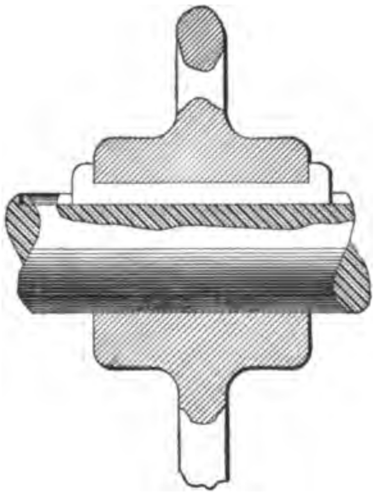


FIG. 650.

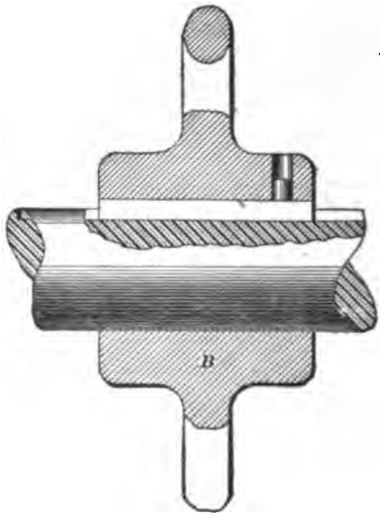


FIG. 651

usually fastened to the piece, and is free to slide in the keyway of the shaft, though the operation is sometimes reversed, and the key is fastened to the shaft. Various methods of fixing the key to the wheel or pulley are shown in Figs. 650, 651, and 652. In Fig. 652, the key is dovetailed in section, and driven tightly into the hub *m*.

When the hub of a wheel in which there is a feather abuts against a collar or a bearing it is evident that the feather must not project, and in such a case the feather key, Fig. 652, or the flush feather key, Fig. 651, may be used; otherwise, the key may have gib heads, as shown in Fig. 650.

1961. Round, or pin, keys may be used when the piece is shrunk on to the shaft, as for example, a small crank, as shown in Fig. 653. A hole is drilled partly in the shaft and partly in the crank, and a round pin is driven in the hole, as shown in the figure.

1962. To facilitate the driving in and the removal of keys they are usually tapered. The taper varies from $\frac{1}{64}$ to $\frac{1}{100}$ or $\frac{1}{150}$, the smaller tapers being used on the most

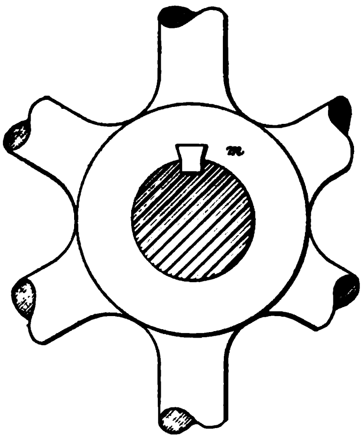


FIG. 652.

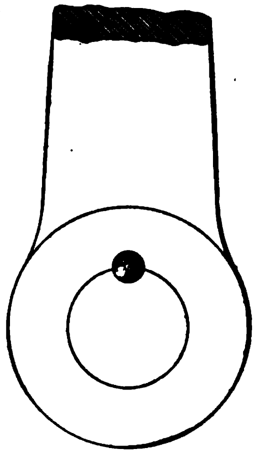


FIG. 653.

accurate work. By a taper of $\frac{1}{64}$ is meant that the decrease in thickness is $\frac{1}{64}$ the length of the key. Square keys and feather keys do not require a taper.

NOTE.—To prevent any misunderstanding, the word *taper*, when used in this subject, will mean the gradual diminution in size of a slender object. Thus, should it be stated that a certain conical piece has a taper of 2 inches per foot, it would be meant that were the conical piece one foot long the diameter at one end would be 2 inches larger than at the other.

STRENGTH AND PROPORTIONS OF KEYS.

1963. A sunk key is subjected to two kinds of stresses. The twisting of the piece on the shaft tends to shear the key, and also to crush it by compression.

Let b = width of key in inches;
t = thickness of key in inches;
l = length of key in inches;
S_s = *safe* shearing stress allowable in pounds per square inch;
S_c = *safe* crushing stress allowable in pounds per square inch;

P = force in pounds acting at rim of wheel or pulley;
R = radius of wheel or pulley in inches;
d = diameter of shaft in inches.

The shearing area of the key is $b\,l$; hence, the safe resistance of the key to shearing is $b\,l\,S_s$. Taking moments about the center of the shaft, we have

$$b\,l\,S_s \times \tfrac{1}{2}d = P \times R,$$

or
$$b\,l = \frac{2PR}{S_s d}. \quad (a)$$

Suppose the key to be half bedded in the shaft, the crushing area is $\frac{1}{2}\,t\,l$, the resistance to crushing $\frac{1}{2}\,t\,l\,S_c$. If the key is designed to be equally strong against shearing and crushing, the shearing resistance must equal the crushing resistance, or

$$b\,l\,S_s = \tfrac{1}{2}\,t\,l\,S_c,$$

or
$$b = \tfrac{1}{2}\,t\frac{S_c}{S_s}. \quad (b)$$

If now we assume the crushing strength of the material to be double the shearing strength, $\frac{S_c}{S_s} = 2$; and we obtain $b = t$. S_c is really not double S_s, but on account of the friction between the key and shaft, there is little danger of crushing, and a small factor of safety may be used. In any case, b is not to be less than t, and for practical reasons it is generally made greater.

For shearing, we may use a factor of safety of about 10, giving a safe shearing stress S_s of 5,000 lb. per sq. in. for wrought iron, and 7,000 lb. for steel. Then equation (a) above becomes

$$\left.\begin{aligned} b\,l &= \frac{PR}{2,500\,d} \text{ for wrought iron.} \\ b\,l &= \frac{PR}{3,500\,d} \text{ for steel.} \end{aligned}\right\} \qquad \textbf{(230.)}$$

Instead of the twisting moment PR of formula **230**, it may be more convenient to use the horsepower transmitted by the shaft, and its number of revolutions.

Let N = number of revolutions per minute;
H = horsepower.

Then, a point on the circumference of the wheel or pulley moves $2\pi RN$ inches per minute, or $\frac{2\pi RN}{12}$ feet per minute.

Hence, if a force P constantly acts at the circumference of the wheel, the work done per minute is $\frac{2\pi RN}{12} \times P$ foot-pounds.

Therefore, $\frac{2\pi RNP}{12} \div 33{,}000 = H$, the horsepower.

$$\frac{12 \times 33{,}000 \times H}{2\pi N} = PR = 63{,}025\,\frac{H}{N}. \qquad \textbf{(231.)}$$

Formula **231** will be frequently used hereafter, and should be carefully studied.

Substituting the value of PR from formula **231**, in formula **230**, we have

$$\left.\begin{aligned} bl &= \frac{25H}{dN} \text{ for wrought iron.} \\ bl &= \frac{18H}{dN} \text{ for steel.} \end{aligned}\right\} \qquad \textbf{(232.)}$$

Formulas **230** and **232** may be used in calculating the sizes of keys for large work. For small shafts the sizes given by **230** and **232** are much smaller than are used in actual practice.

1964. Designers usually adopt some standard ratio between the depth and width of the key, the ratio varying from $\frac{1}{2}$ to $\frac{5}{6}$. We shall adopt the ratio $\frac{2}{3}$; that is, $t = \frac{2}{3}b$.

EXAMPLE.—The maximum pressure on an engine crank-pin is 12,500 pounds, and the length of crank is 10 inches. Suppose the diameter of the shaft to be 5 inches, and the length of the key the same. What should be the dimensions of a wrought-iron key to hold the crank to the shaft? The crank is not to be shrunk on the shaft.

SOLUTION.—Using formula **230**,

$$b = \frac{PR}{2{,}500\,dl} = \frac{12{,}500 \times 10}{2{,}500 \times 5 \times 5} = 2''. \quad \text{Ans.}$$

$$t = \tfrac{2}{3}b = \tfrac{2}{3} \times 2 = 1\tfrac{1}{3}'', \text{ say } 1\tfrac{11}{32}''. \quad \text{Ans.}$$

1965. In common designing, the sizes of keys are determined by empirical formulas, which give an excess of strength. For an ordinary sunk key, the following proportions may be adopted:

$$\left.\begin{array}{l} b = \frac{1}{4} d. \\ t = \frac{2}{3} b = \frac{1}{6} d. \end{array}\right\} \qquad (233.)$$

Using formula **233** in the example of the crank-shaft above,

$$b = \tfrac{1}{4} d = \tfrac{1}{4} \times 5 = 1\tfrac{1}{4}''.$$
$$t = \tfrac{1}{6} d = \tfrac{5}{6} = \tfrac{13}{16}'', \text{ nearly.}$$

The key is sunk for $\frac{1}{2}$ its depth in the shaft.

The following empirical formulas give good results:

$$\left.\begin{array}{l} b = \dfrac{d}{7} + \dfrac{1''}{4}. \\ t = \dfrac{d}{12} + \dfrac{3''}{16}. \end{array}\right\} \text{For driven pulleys.} \qquad (234.)$$

$$\left.\begin{array}{l} b = .2\, d + .16''. \\ t = .1\, d + .16''. \end{array}\right\} \text{For driving pulleys.} \qquad (235.)$$

When d is less than $1\frac{3}{16}$ in.,

$$\left.\begin{array}{l} b = \dfrac{d}{3}. \\ t = \dfrac{d}{5}. \end{array}\right\} \qquad (236.)$$

Formulas **233** and **235** give nearly the same results; thus, for a $2\frac{1}{2}$-inch shaft, **233** gives $b = \frac{2\frac{1}{2}}{4} = \frac{5''}{8}$, and $t = \frac{3}{8}''$, nearly. **235** gives $b = .66$ in. $= \frac{11}{16}''$, and $t = .41'' = \frac{7}{16}''$. Unless otherwise stated, the student may use formula **233** in solving his problems.

For sliding feather keys, the following formulas give the proportions used in ordinary practice :

$$\left.\begin{array}{l} b = \dfrac{d}{8} + \dfrac{1''}{16}. \\ t = \dfrac{3\, d}{16} + \dfrac{1''}{16}. \end{array}\right\} \qquad (237.)$$

1966. In some instances pulleys may be keyed to a large shaft and yet transmit a small amount of power. In such cases the dimensions of a key, if based upon the actual diameter of the shaft, would be much too large, and the proportions should be based upon the diameter of a shaft which would be necessary to transmit the power of the pulley in question, and no more. Letting H represent the horsepower transmitted by the pulley, and N represent the revolutions per minute of the shaft, we may take

$$d = 5\sqrt[3]{\frac{H}{N}}, \qquad \textbf{(238.)}$$

and use this value of d in formula **236.**

EXAMPLE.—A pulley transmitting 2 horsepower is keyed to a shaft 4 inches in diameter making 120 revolutions per minute. Determine the dimensions of the key.

SOLUTION.—The diameter of a shaft to transmit 2 horsepower is

$$d = 5\sqrt[3]{\frac{H}{N}} = 5\sqrt[3]{\frac{2}{120}} = 1.28''. \quad \text{Ans.}$$

From formula **236,**

$$b = \frac{d}{8} = \frac{1.28}{8} = \tfrac{7}{16}''. \quad \text{Ans.} \qquad t = \frac{d}{5} = \frac{1.28}{5} = \tfrac{1}{4}''. \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE.

1. Calculate the dimensions of an ordinary sunk key for a shaft $3\frac{1}{4}$ inches in diameter. Ans. $\frac{9}{16}'' \times \frac{7}{8}''$.

2. Calculate the dimensions of a feather key for a shaft $2\frac{3}{4}$ inches in diameter. Ans. $\frac{13}{32}'' \times \frac{9}{16}''$.

3. A wrought-iron key is used to fasten a fly-wheel on a 6-inch shaft. If the maximum pressure on the crank-pin is 15,000 lb., and the crank radius is 16 inches, what should be the dimensions of the key, its length being 8 inches? Ans. $2'' \times 1\frac{11}{32}''$.

4. A pulley transmits $3\frac{1}{4}$ horsepower, and is keyed to a shaft 5 inches in diameter, making 150 revolutions per minute. Calculate the dimensions of the key. Ans. $\frac{1}{4}'' \times \frac{5}{16}''$.

COTTERS.

1967. A **cotter** is an iron or steel bar which is driven through one or both of two pieces to be connected, and holds them together by its resistance to shearing at two *transverse*

cross-sections. An example of a cotter was shown in Fig. 617. The cotter *C* passes through the foundation bolt, and is subjected to a shearing stress along the dotted lines.

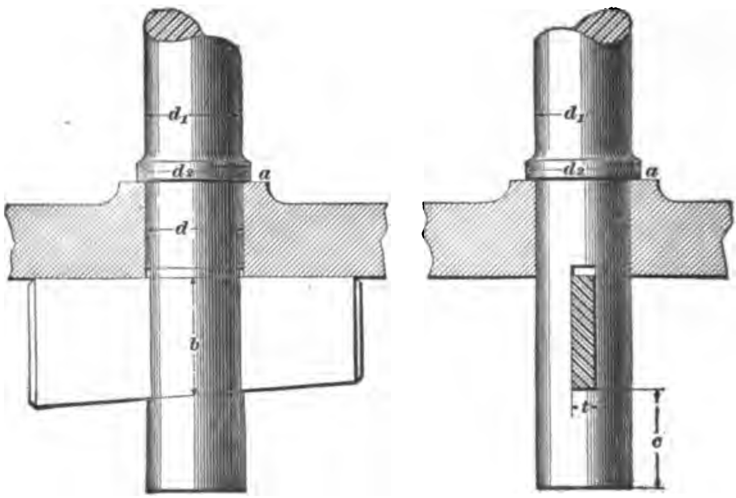


FIG. 654.

A simple form of a cotter is shown in Fig. 654. The cotter passes through the rod only, and acts when the rod is in tension. The enlargement, or collar *a*, on the rod prevents any downward movement, and, therefore, resists thrust.

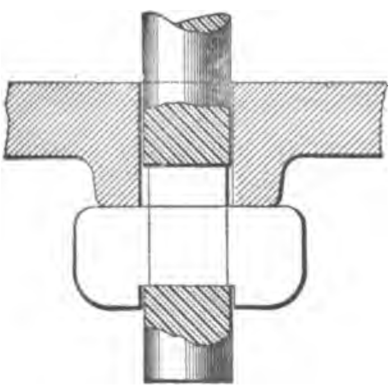


FIG. 655.

Fig. 655 shows a cotter with gib ends. Since in this case the rod is not provided with a collar, this arrangement will resist tension only. In the arrangement shown in Fig. 656, the cotter is divided into two parts, the one with hooked ends being called the **gib**, and the other the cotter.

In this construction the rod should be placed in tension only. If it is necessary to provide for thrust the end of the rod should be tapered as shown in Fig. 657.

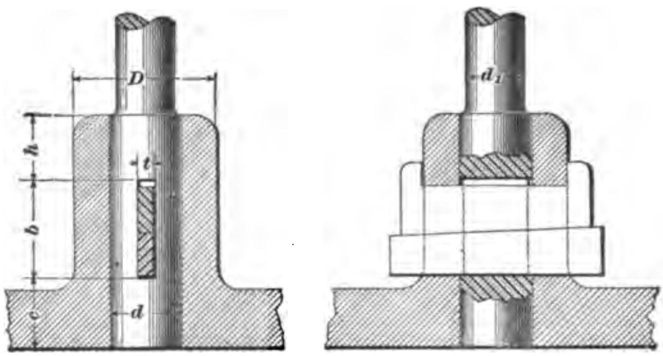


FIG. 656.

Fig. 658 shows an arrangement in which a rod is cottered into a socket. As shown in Figs. 656 and 658, the cotter is long and tapered; it serves, therefore, as a means of adjusting the length of the connected pieces. By driving the cotter farther in, the total length of the two pieces is lessened, and *vice versa*.

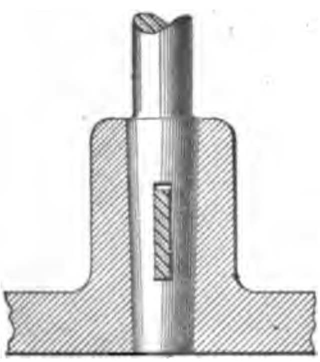


FIG. 657.

A cotter may be used to connect two straps *m* and *n* to a rod *l* shown in Fig. 659. When driven down the friction between the cotter and lower strap must cause the latter to open out as shown by the dotted lines. Hence, it is desirable in such a case to use a cotter combined with a gib, as shown in Fig. 660, or with two gibs as shown in Fig. 661.

The gibs serve to keep the straps from spreading. In Fig. 662 the side *a b* of the gib and *c d* of the cotter

are parallel to each other, and perpendicular to the axis of the rod, and the taper comes between the gibs and cotter.

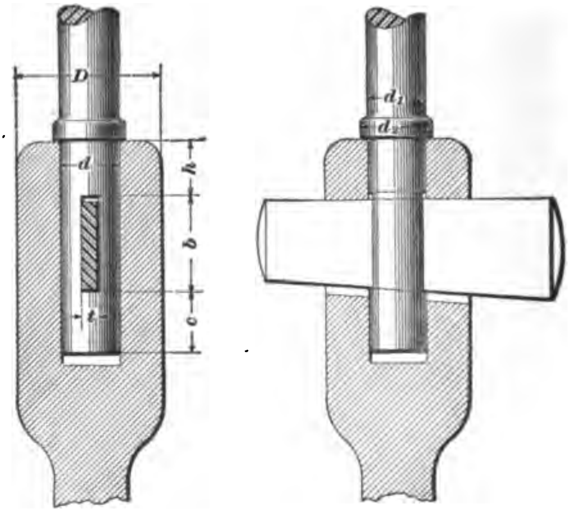


FIG. 658.

1968. Strength and Proportions of Cotters.—In designing a cotter connection the following points must be taken into account:

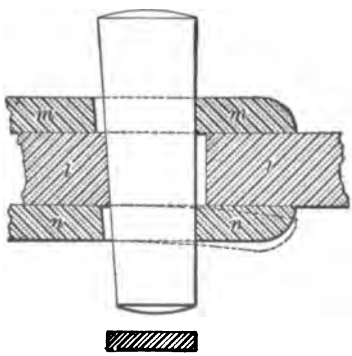


FIG. 659.

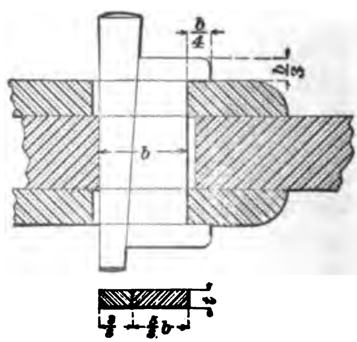


FIG. 660.

Referring to the illustration of the cotter, Fig. 654—

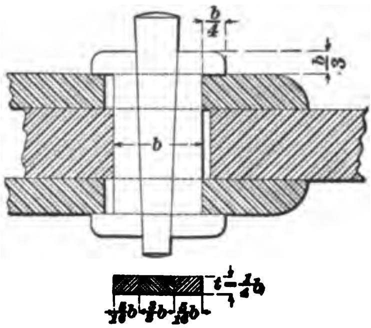


FIG. 661.

1. The cross-section $b\,t$ of the cotter must be sufficient to withstand the shearing force.

2. The thickness t must be great enough to provide against failure by crushing.

3. The two diameters d_1 and d should be so designed that the rod is of uniform strength throughout.

Let P = force in pounds exerted on the rod;

S_s = *safe* shearing strength of cotter in pounds per square inch;

S_c = *safe* compressive strength of cotter or rod in pounds per square inch;

S_t = *safe* tensile strength of rod in pounds per square inch.

The various diameters and other dimensions are indicated on the figures.

Consider the arrangement shown in either Fig. 656 or Fig. 657 and conceive a section taken through the cotter hole. The net area of the rod is $\left(\frac{\pi}{4}d^2 - d\,t\right)$, very nearly; the shearing area of the cotter is $2\,b\,t$; the area of the cotter subject to crushing is $d\,t$, very nearly; the area of the socket subject to tension is $\frac{\pi}{4}(D^2 - d^2) - (D - d)t$; finally, the area of the smaller part of the rod is $\frac{1}{4}\pi d_1^2$.

Hence,
$$P = \left(\frac{\pi}{4}d^2 - d\,t\right)S_t. \qquad (a)$$
$$P = 2\,b\,t\,S_s. \qquad (b)$$
$$P = d\,t\,S_c. \qquad (c)$$
$$P = \left[\frac{\pi}{4}(D^2 - d^2) - (D - d)\,t\right]S_t. \qquad (d)$$
$$P = \frac{\pi}{4}d_1^2\,S_t. \qquad (e)$$

Suppose both rod and cotter are made of the same material, either wrought iron or steel, and take $\frac{S_s}{S_t} = \frac{4}{5}$, as was done before in Art. **1954.**

Experience shows that S_c may be double S_t; that is, $\frac{S_c}{S_t} = 2$.

$$\text{Hence, } \frac{S_c}{S_s} = \frac{2}{\frac{4}{5}} = 2\tfrac{1}{2}.$$

Now, combining equations (b) and (c),

$$2\,b\,t\,S_s = d\,t\,S_c; \text{ or, } b = \frac{d\,S_c}{2S_s} = \frac{d}{2} \times 2\tfrac{1}{2} = 1.25\,d.$$

Combining equations (a) and (c), we have

$$\left(\frac{\pi}{4}d^2 - d\,t\right) = d\,t\frac{S_c}{S_t} = 2\,d\,t;$$

whence, $3\,d\,t = \frac{\pi}{4}d^2$, and $t = \frac{\pi}{12}d = .26\,d = \text{say } \frac{1}{4}d.$

Combining equations (a) and (d), and taking $t = \frac{\pi}{12}d$, it will be found that

$$D = \frac{4}{3}d = 1.3\,d.$$

Combining equations (a) and (e), and taking $t = \frac{\pi d}{12}$,

$$\frac{\pi}{4}d_1^2 = \frac{\pi}{4}d^2 - d\,t = \frac{\pi}{4}d^2 - \frac{\pi}{12}d^2; \text{ whence,}$$
$$d_1^2 = \tfrac{2}{3}\,d^2, \text{ and } d_1 = .816\,d.$$

It was shown above that to have the same tensile strength as the rod, the diameter D of the socket or boss should be 1.3 d.

To prevent failure from crushing, however, the bearing surface of the socket should equal that of the rod, or

$$(D - d)\,t = d\,t. \quad \text{Hence, } D = 2\,d.$$

The diameter d_2 of the collar, Fig. 658, should be such that the bearing surface of the collar is at least equal to that of the cotter in the rod. Hence,

$$\frac{\pi}{4}\left(d_2^2 - d^2\right) = d\,t = \frac{\pi}{12}d^2;$$

whence, $\frac{\pi}{4}d_2^2 = \frac{\pi}{3}d^2$, and $d_2 = d\sqrt{\frac{4}{3}} = 1.15\,d.$

Collecting the above results, we have

$$\left.\begin{aligned} b &= 1\tfrac{1}{4}\,d; \\ t &= \tfrac{1}{4}\,d; \\ d_1 &= .816\,d; \\ D &= 2\,d; \\ d_2 &= 1.15\,d; \\ h &= c = \tfrac{3}{4}\,d \text{ to } 1\tfrac{1}{4}\,d. \end{aligned}\right\} \qquad \textbf{(239.)}$$

1969. If a steel cotter be used in a wrought-iron rod, b may be made equal to d, the other dimensions remaining the same as above.

For a cotter of the form shown in Fig. 660 or 661 it is good practice to make $t = \frac{1}{4}\,b$, and $b\,t = \frac{5}{4} \times$ sectional area of strap.

The width b is the same whether a single cotter, a gib and cotter, or two gibs and cotter are used. The other proportions are shown on the figures.

Example.—Suppose in Fig. 661 the strap is $\frac{7}{8}'' \times 3\frac{1}{2}''$; find the thickness of the cotter, and combined width of gibs and cotter.

Solution.—$b\,t = \frac{5}{4} \times \frac{7}{8} \times 3\frac{1}{2} = \frac{245}{64}$. Making $t = \frac{1}{4}\,b$, as stated above,

$$b\,t = \tfrac{1}{4}\,b^2 = \frac{245}{64};$$

whence, $\qquad b = \sqrt{\frac{245}{16}} = 3.91'' = 3\frac{15}{16}''.$ Ans.

$t = \frac{1}{4}\,b = 1$ inch, nearly. Ans.

The width of cotter is, then, $\frac{3}{8}\,b$, or $1\frac{1}{2}''$, and the width of each gib is $\frac{5}{16}\,b$, or $1\frac{7}{32}''$.

1970. Taper of Cotters.—The greatest allowable taper that a cotter may have without danger of slacking back is about $\frac{1}{7}$.

Usually, the taper is $\frac{1}{24}$ to $\frac{1}{48}$ when the cotter is not secured. If fastened by a set-screw or bolt and nut the

cotter may have a taper of $\frac{1}{6}$ or $\frac{1}{8}$. The taper is found by dividing the increase in width by the length of cotter. Thus, if a cotter is 2 inches wide at one end, $2\frac{1}{2}$ inches wide at the other, and 12 inches in length, the taper is $(2\frac{1}{2} - 2) \div 12 = \frac{1}{24}$.

1971. Locking arrangements for cotters are shown in Figs. 662 and 663. In Fig. 662 the cotter is held

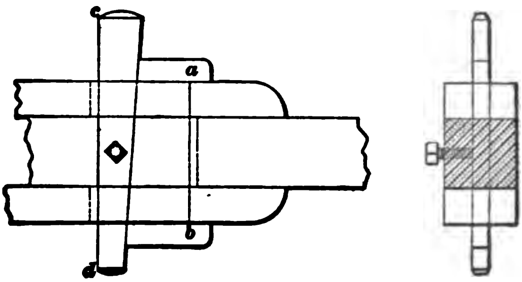


FIG. 662.

by a set-screw, the point of which fits into a groove cut into the cotter. The diameter of the set-screw may be $\frac{1}{3}b + \frac{1}{8}''$. In the arrangement shown in Fig. 663 the end of the cotter is a screw, and the cotter is secured by a nut on an extra seat. This method is used where the cotter has an excessive taper.

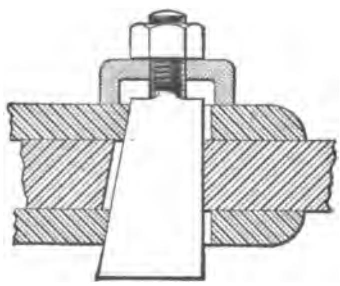


FIG. 663.

FIG. 664.

1972. A split pin is a form of cotter which is used not to firmly connect two pieces, but to prevent them from separating entirely. Small split pins are of the form shown in Fig. 664. When large pins are used for this purpose they are solid and tapered.

EXAMPLES FOR PRACTICE.

1. Find the dimensions of a rod and socket of the form shown in Fig. 658, assuming $S_t = 6{,}000$ pounds. The load or pull on the rod is 4,600 pounds.

Ans. $\begin{cases} d = 1\frac{3}{16} \text{ in.}; & b = 1\frac{1}{4} \text{ in.}; \\ d_1 = 1 \text{ in.}; & t = \frac{19}{64} \text{ in.}; \\ d_2 = 1\frac{5}{8} \text{ in.}; & D = 2\frac{5}{8} \text{ in.}; \\ h = 1 \text{ in.} \end{cases}$

2. A cotter of the form shown in Fig. 655, resists a pull of 8,200 pounds. Find the necessary breadth and thickness on the assumption that S_s is 4,000 pounds, and that the thickness is one-fourth the breadth.

Ans. $\begin{cases} b = 1.27 = 1\frac{1}{4} \text{ in.} \\ t = \frac{5}{16} \text{ in.} \end{cases}$

3. A cotter and two gibs connect two straps to a rod, as shown in Fig. 661. Supposing the pull on the rod to be 9,000 pounds, and taking $S_s = 5{,}400$ for steel, find the dimensions of cotter and gibs.

Ans. $\begin{cases} t = \frac{15}{32} \text{ in.} \\ \text{Width of cotter} = \frac{11}{16} \text{ in.} \\ \text{Width of gibs} = \frac{9}{16} \text{ in.} \end{cases}$

4. In example 3, (*a*) what should be the *net* section of the strap to be equal in strength to the cotter? Assuming the thickness of strap to be $\frac{1}{4}$ the width, (*b*) what would be its actual dimensions?

Ans. $\begin{cases} (a) \ .85 \text{ sq. in.} \\ (b) \ 2\frac{1}{8} \text{ in.} \times \frac{17}{32} \text{ in.} \end{cases}$

5. Calculate the dimensions of a steel cotter which fastens a wrought-iron rod $2\frac{5}{8}$ in. in diameter. Ans. $2\frac{5}{8}$ in. $\times$ $\frac{21}{32}$ in.

6. A cotter is $1\frac{3}{4}$ inches wide in the middle and tapers on each side. If the cotter is 18 inches long, what is its width at each end? Assume that the taper is $\frac{1}{4}$ inch to the foot. Ans. $2\frac{1}{8}$ in. and $1\frac{3}{8}$ in.

ROTATING PIECES.

JOURNALS.

1973. Journals are the cylindrical portions of rotating pieces which turn within bearings and form the supports. Journals which are situated at or near the end of a shaft, axle, or other rotating piece, are termed **end journals.** Any journal situated between two end journals is called a **neck journal.**

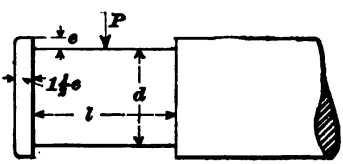


FIG. 665.

The ordinary form of an end journal is shown in Fig. 665. It consists simply of a

cylinder with collars at each end to prevent end play in the bearing.

The length of the step, seat, or bearing on which the journal rests, is, however, often made slightly shorter than the journal, permitting a slight motion lengthwise, and securing uniform wear.

SIZE AND PROPORTIONS OF END JOURNALS.

1974. The chief element in the design of a journal moving slowly or intermittently is strength. When journals run constantly at considerable velocity, strength is not so important a consideration as durability and freedom from liability to heat.

The dimensions of an end journal to give sufficient strength may be calculated by considering the journal as a cantilever uniformly loaded.

Let l = length of journal in inches;
d = diameter of journal in inches;
P = total load on journal in pounds;
S_f = *safe* stress of material in flexure.

Then, from the Table of Bending Moments, the bending moment is $\frac{w l^2}{2} = \frac{Pl}{2}$, and the resisting moment is $\frac{S_f I}{c}$, which for a circular section is (see Table of Moments of Inertia),

$$S_f \frac{\frac{\pi d^4}{64}}{\frac{1}{2} d} = S_f \frac{\pi d^3}{32}.$$

$$\text{Hence,} \quad \frac{Pl}{2} = S_f \frac{\pi d^3}{32}, \qquad (a)$$

$$\text{or } d = \sqrt{\frac{16 P}{\pi S_f} \times \frac{l}{d}} = 2.26 \sqrt{\frac{P}{S_f} \times \frac{l}{d}}. \qquad \textbf{(240.)}$$

Formula **240** gives the diameter of the journal when the ratio $\frac{l}{d}$ is assumed.

EXAMPLE.—Find the diameter and length of a wrought-iron journal on which there is a load of 1,200 pounds. Assume $S_f = 8{,}500$ pounds and $\frac{l}{d} = 1.4$.

SOLUTION.—Using formula **240**,

$$d = 2.26\sqrt{\frac{P}{S_f} \times \frac{l}{d}} = 2.26\sqrt{\frac{1{,}200}{8{,}500} \times 1.4} = 1'', \text{ nearly. Ans.}$$

$$l = 1.4\,d = 1.4''. \text{ Ans.}$$

1975. The **bearing surface,** or **projected area,** of a journal is the length multiplied by the diameter; that is, it is the area of the projection of the journal on a plane. The total load on the journal divided by the projected area gives the pressure per square inch of projected area—a quantity which will be denoted by p.

Hence, $p = \frac{P}{l\,d}$, or $P = p\,l\,d$. $\quad$ (*b*)

In order that the journal may not heat, the pressure p must not exceed a certain limit determined by experience. When this pressure is too great, the oil used to lubricate the journal is squeezed out, and the journal heats rapidly.

From equation (*a*), $P = \frac{\pi S_f d^3}{16\,l}$.

From equation (*b*), $P = p\,l\,d$.

Hence, $\frac{\pi S_f d^3}{16\,l} = p\,l\,d$, or $\pi S_f d^2 = 16\,p\,l^2$; and

$$\frac{l}{d} = \sqrt{\frac{\pi S_f}{16\,p}}. \qquad (c)$$

Substituting this value of $\frac{l}{d}$ in **240,** we obtain, after a slight reduction,

$$d = \sqrt{\frac{4P}{\sqrt{\pi S_f p}}} = 1.5\sqrt{\frac{P}{\sqrt{S_f p}}}. \qquad \textbf{(241.)}$$

From equation (*b*), $l = \frac{P}{d\,p}$. $\quad$ **(242.)**

Formula **241** may be used to compute the diameter of a journal when the pressure p per square inch of projected area is fixed. The length may then be obtained from formula **242.**

EXAMPLE.—Compute the length and diameter of a steel journal sustaining a load of 12,000 pounds. The safe stress S_f is 14,000 pounds, and the pressure per square inch of projected area is not to exceed 750 pounds.

SOLUTION.—Using formula **241**,

$$d = 1.5\sqrt{\frac{P}{\sqrt{S_f p}}} = 1.5\sqrt{\frac{12{,}000}{\sqrt{14{,}000 \times 750}}} = 2.89'', \text{ say } 2\tfrac{7}{8}''. \quad \text{Ans.}$$

Hence,
$$l = \frac{P}{dp} = \frac{12{,}000}{2\frac{7}{8} \times 750} = 5\tfrac{9}{16}''. \quad \text{Ans.}$$

1976. The pressure p per square inch of projected area may be taken at from 400 to 800 pounds, when the journal runs constantly at a speed under 150 revolutions per minute. For journals which run slowly or intermittently, p may be much greater, while for journals running faster than 150 revolutions per minute, the pressure p should vary inversely as the number of revolutions. That is, letting N = number of revolutions per minute, $p = \frac{a}{N}$, where a is a constant.

Another consideration affecting the allowable pressure p is the direction of the load. In some journals the load acts only in one direction, generally downwards; in others, as, for example, crank-pins and cross-head pins, the direction of the load changes at every revolution. In the latter case, the pressure p may be twice as great as in the former, because the change in the direction of the load permits a more perfect lubrication of the bearing. When, however, the direction of the load is variable, the safe stress S_f must be taken smaller than when the direction is constant.

1977. The following values of p for different kinds of journals are taken from Unwin:

TABLE 45.

PRESSURE ON BEARINGS AND SLIDES.

Kind of Journal Bearing.		Pressure per Sq. In. of Projected Area, p.
Bearings on which the load is intermittent and the speed slow, such as crank-pins of shearing machines		3,000 lb.
Cross-head neck journals		1,200 lb.
Crank-pins of large, slow-speed engines	800 to	900 lb.
Crank-pins of marine engines, usually	400 to	500 lb.
Main crank-shaft bearings—marine engines (slow)		600 lb.
Main crank-shaft bearings—marine engines (fast)		400 lb.
Locomotive driving axle journal	180 to	350 lb.
Railway journals		200 lb.
Fly-wheel shaft journals	150 to	250 lb.
Small engine crank-pins	150 to	200 lb.
Slipper slide blocks, marine engines		100 lb.
Stationary engine slide blocks	25 to	125 lb.
Stationary engine slide blocks, usually	30 to	60 lb.
Propeller thrust bearings	50 to	70 lb.
Shaft in cast-iron steps or seats		15 lb.

For journals not given above, the value of p must be determined by the judgment of the designer. The value adopted should seldom or never exceed 750 pounds. For journals running faster than about 150 revolutions per minute, p should vary inversely as the number of revolutions. For example, if a given journal is allowed a bearing pressure of 300 pounds at 150 revolutions, it should only be allowed a bearing pressure of $\frac{300 \times 150}{250} = 180$ pounds if it is required to run at 250 revolutions.

1978. The permissible working stress S_f may be taken as follows:

TABLE 46.

Direction of Load Constant.		Direction of Load Variable.	
Steel	$S_f = 14{,}000$	Steel	$S_f = 12{,}000$
Wrought iron	$S_f = 8{,}500$	Wrought iron	$S_f = 7{,}000$
Cast iron	$S_f = 4{,}000$	Cast iron	$S_f = 3{,}000$

NECK JOURNALS.

1979. A neck journal similar to an engine cross-head pin may be considered as a beam supported at both ends and uniformly loaded. Consequently, the bending moment is $\frac{w\,l^2}{8} = \frac{P\,l}{8}$. (See Table of Bending Moments and Deflections.)

$$\text{Hence, } \frac{P\,l}{8} = \frac{S_f \pi d^3}{32}, \text{ (see } (a)\text{, Art. } \mathbf{1974}),$$

$$\text{and } d = \sqrt{\frac{4P}{\pi S_f} \times \frac{l}{d}} = 1.13\sqrt{\frac{P}{S_f} \times \frac{l}{d}}. \qquad \mathbf{(243.)}$$

Formula **243** may be used to calculate a neck journal when $\frac{l}{d}$ is given or assumed.

But, $P = p\,d\,l$, (see (b), Art. **1975**).

$$\text{Hence, } \frac{P\,l}{8} = \frac{p\,d\,l^2}{8} = \frac{S_f \pi d^3}{32}, \text{ or } \frac{l}{d} = \sqrt{\frac{\pi S_f}{4p}}. \qquad (d)$$

Substituting this value of $\frac{l}{d}$ in formula **243**, we obtain

$$d = 1.06\sqrt{\frac{P}{\sqrt{p\,S_f}}}. \qquad \mathbf{(244.)}$$

From $P = p\,d\,l$, $\qquad l = \frac{P}{p\,d}. \qquad \mathbf{(245.)}$

The same values of S_f and p may be taken as for end journals.

It will be seen by comparing formulas **241** and **244** that for the same load the neck journal need be but about $\frac{3}{4}$ the diameter of the end journal; and a comparison of

equations (c) and (d) shows that the ratio $\frac{l}{d}$ is double in a neck journal what it would be in an end journal.

NOTE.—In using formulas **241** and **244** it will be found most convenient to use logarithms. Thus, applying logarithms to formula **241** it becomes $\log d = \log 1.5 + \frac{1}{2}[\log P - \frac{1}{2}(\log p + \log S_f)]$.

EXAMPLE.—Find the length and diameter of a wrought-iron neck journal, the load being 9,600 pounds, variable in direction. Allow a bearing pressure of 600 lb. per square inch.

SOLUTION.—Using formula **244**,

$$d = 1.06\sqrt{\frac{P}{\sqrt{p\,S_f}}} = 1.06\sqrt{\frac{9{,}600}{\sqrt{600 \times 7{,}000}}}, \text{ or}$$

$$\log d = \log 1.06 + \tfrac{1}{2}[\log P - \tfrac{1}{2}(\log p + \log S_f)] =$$

$$.02531 + \tfrac{1}{2}[3.98227 - \tfrac{1}{2}(2.77815 + 3.84510)] = .36068;$$

$$\text{whence, } d = 2.295'' = 2\tfrac{19}{64}'', \text{ or nearly } 2\tfrac{5}{16}''. \quad \text{Ans.}$$

$$l = \frac{P}{p\,d} = \frac{9{,}600}{600 \times 2\frac{5}{16}} = 7'', \text{ nearly.} \quad \text{Ans.}$$

1980. It may sometimes be desirable to make the length of a journal greater than the calculated value, simply as a matter of taste. To give the journal the same strength, its diameter must be increased in the proportion given by the following formula:

$$\frac{l_1}{l} = \left(\frac{d_1}{d}\right)^3, \qquad \textbf{(246.)}$$

where l and d are the original and l_1 and d_1 the new lengths and diameters of the journal, respectively.

For example, suppose a wrought-iron end journal to be subjected to a load of 350 pounds. Assuming $\frac{l}{d} = 1.4$, the dimensions of the journal will be

$$d = 2.26\sqrt{\frac{P}{S_f} \times \frac{l}{d}} = 2.26\sqrt{\frac{350}{8{,}500} \times 1.4} = .5426''.$$

$$l = 1.4\,d = 1.4 \times .5426 = .7596''.$$

Say the diameter is $\frac{9}{16}''$ and the length $\frac{3}{4}''$.

Though a journal of these dimensions would be sufficiently stiff and durable for the load, it is rather small to

look well. Suppose the length is made double the calculated value; that is, $l_1 = 2l$. Then, from formula **246**,

$$d_1 = d\sqrt[3]{\frac{l_1}{l}} = .5426\sqrt[3]{2} = .6837'' = \tfrac{11}{16}'', \text{ nearly.}$$

$$l_1 = 2l = 2 \times \frac{3}{4} = 1\frac{1''}{2}.$$

FRICTION OF JOURNALS.

1981. The work expended in overcoming the friction of a journal is

$$W = fPN\frac{\pi d}{12} \text{ foot-pounds per minute;}$$

where f = coefficient of friction,
and N = revolutions per minute.

It is therefore apparent that with the same load and speed, the work expended against friction is directly proportional to the diameter of the journal. Consequently, when a journal runs under a steady, uniform load, it is preferable to make the diameter as small as possible, consistent with strength, and obtain the desired projected area by adding to the length of the journal.

For example, take two journals, one 2 inches in diameter and 6 inches long, the other 4 inches in diameter and 3 inches long. They each have the same projected area, 12 inches. The latter journal, however, requires double the work to overcome friction that the former requires, and, besides, contains twice as much material. Hence, the $2'' \times 6''$ journal is preferable for a steady load. On the other hand, the $4'' \times 3''$ journal is $\frac{4^3 \times 6}{2^3 \times 3} = 16$ times as strong as the other, and would be preferred in situations where the load is variable, and the journal is liable to shocks, as in the case of crank-pins of high-speed engines.

1982. The height of the journal collars, Fig. 665, may be

$$e = \frac{d}{10} + \frac{1''}{8}. \qquad \textbf{(247.)}$$

The width of the outer collar is $1\frac{1}{2}e$. It is good practice to turn the journal with a fillet in the corner, as the shaft or axle is more liable to crack and fracture if the shoulder is turned with a square corner.

The fillet may be a circular arc drawn with a radius equal to $\frac{e}{2}$.

PIVOTS AND COLLAR JOURNALS.

1983. A **pivot journal** is shown in Fig. 666. It differs from an ordinary journal in that the direction of pressure is parallel to the axis of the shaft instead of perpendicular to it. The bearing area is, therefore, the area of the end of the pivot, $\frac{1}{4}\pi d^2$.

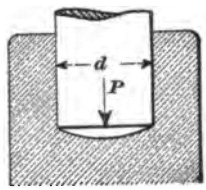


FIG. 666.

The diameter of a flat pivot may, therefore, be found at once by assuming a value for the pressure per square inch of projected area and solving for the area. The following formulas give good results for ordinary cases:

Let P = load on pivot;
d = diameter of pivot;
N = revolutions per minute of pivot.

TABLE 47.

	Wrought-Iron or Steel Pivot on Gun-Metal Bearing.	Cast-Iron Pivot on Gun-Metal Bearing.	Iron or Steel on Lignum-vitæ Bearing, Moistened with Water.	
Case I—Pivot turning very slowly or intermittently ...	$d = .035\sqrt{P}$	$d = .05\sqrt{P}$	....	**(248.)**
Case II — Revolutions per minute less than 150....	$d = .05\sqrt{P}$	$d = .07\sqrt{P}$	$d = .035\sqrt{P}$	**(249.)**
Case III — Revolutions per minute more than 150...	$d = .004\sqrt{PN}$	....	$d = .035\sqrt{P}$	**(250.)**

EXAMPLE.—What should be the diameter of a steel pivot on gun-metal bearings, the load being 200 lb., and the number of revolutions 320 per minute?

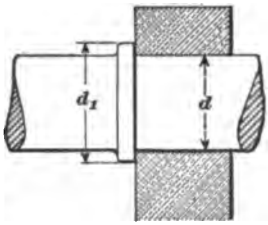


FIG. 667.

SOLUTION.—

$d = .004\sqrt{PN} = .004\sqrt{200 \times 320} = 1''$, nearly. Ans.

1984. In a **collar journal** the direction of the load is parallel to the axis of the shaft, but the bearing area is formed by a collar raised on the shaft. A journal with one collar is shown in Fig. 667, and one with several collars in Fig. 668.

Letting d_1 and d represent the diameters of collars and shaft, respectively; b, the number of collars; p, the pressure per square inch of projected area, and P, the total load or thrust, we have

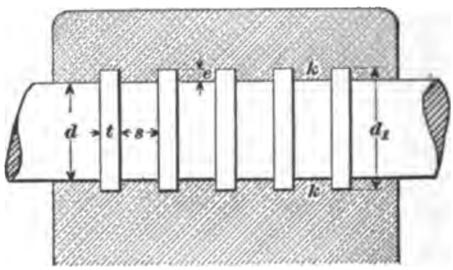


FIG. 668.

$\frac{1}{4}\pi(d_1^2 - d^2)\ b\ p = P.$

The value of p is usually taken at 60 lb. per square inch for the thrust bearings of propeller shafts, and this value should not be much exceeded in any case.

$$\text{Then, } d_1^2 - d^2 = \frac{P}{15\,\pi\,b},$$

$$\text{or } d_1 = \sqrt{d^2 + \frac{P}{15\,\pi\,b}}. \qquad \textbf{(251.)}$$

The number of collars b depends upon the judgment of the designer. The larger the number the smaller is the diameter, and the less will be the wear and work of friction. On the other hand, when many collars are used, there is danger of bringing all the thrust on one or two.

In Fig. 668 the dimension $e = \frac{1}{2}(d_1 - d)$. Usually the thickness t of the collars is $= .8\ e$, and the width s of the space is equal to the thickness t, unless the annular encircling

rings k are hollow for water circulation, or unless they are lined with white metal.

In the latter case, $s = 2$ to $2\frac{1}{2}\,t$.

EXAMPLES FOR PRACTICE.

1. Find the proportions of a cast-iron end journal, turning slowly under a steady load of 15,000 lb., assuming the length equal to the diameter. Ans. $4\frac{5}{8}'' \times 4\frac{5}{8}''$.

2. Find the proportions of a wrought-iron end journal, turning under a load of 8,600 lb., assuming the safe pressure per square inch of projected area to be 650 lb. The direction of the load is variable. Ans. $3'' \times 4\frac{7}{16}''$.

3. Find the dimensions of a steel neck journal, working under a load of 12,000 lb., direction variable, the allowable bearing pressure to be 1,200 lb. per sq. in. Ans. $1\frac{7}{8}'' \times 5\frac{5}{16}''$.

4. Find the minimum dimensions of a wrought-iron end journal, the required conditions being that the bearing pressure shall not exceed 750 pounds, and the length shall be one and one-fourth times the diameter. The load on the journal is 9,600 pounds, and variable in direction. Ans. $3\frac{3}{16}'' \times 4''$.

5. Find the diameter of a wrought-iron pivot running at a speed of 80 revolutions per minute and bearing a load of 800 pounds, gun-metal bearing. Ans. $1\frac{7}{16}$ in.

6. Assuming 5 thrust collars on a shaft 10 inches in diameter subjected to an end thrust of 20,000 pounds, find the diameter and thickness of collars. Ans. $\begin{cases} d_1 = 13.6''. \\ t = 1\frac{7}{16}''. \end{cases}$

SHAFTS.

1985. Shafts may be divided into three classes, according to the kind of stress to which they are subjected:

(1) Shafts subjected chiefly to torsion or twisting, as, for example, line shafting in mills and shops, and, in general, shafts used to transmit power.

(2) Shafts subjected chiefly to bending action, such as the axles of gears, etc.

(3) Shafts subjected to both twisting and bending, such as engine crank-shafts.

LINE SHAFTING.

1986. Line shafting is a term applied to the long and continuous lines of shafting used in mills, factories, and shops for the distribution of power. The shaft is principally

strained by torsion, but there is always in addition a bending action due to the weight of the shaft itself, and the pulleys carried by it, and also due to the tension of belting or the thrust of gearing.

In calculating the diameter of a shaft for a given twisting moment, two things must be considered: 1. Strength. 2. Stiffness. Very large shafts or short shafts need be calculated for strength only; but in long lines of shafting of small diameter attention must be paid to stiffness and rigidity.

The twisting moment is $P\ R$, where P is the twisting force and R the distance between the line of action of the force and the center of the shaft. In case of a belted pulley, shown in Fig. 669, the force P is the difference in the ten-

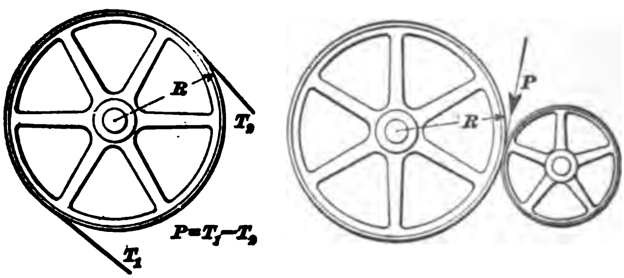


FIG. 669.

sions of the tight and slack sides of the belt; the distance R is the radius of the pulley. Or, again, R may be the radius of a gear attached to the shaft in question.

It is usually more convenient to express the twisting moment in terms of the horsepower transmitted by the shaft, and number of revolutions.

By formula **231**, $PR = 63{,}025\,\frac{H}{N}$. The resistance of a shaft to twisting is given by a formula similar to formula **116**, $M = S_4\,\frac{I}{c}$, Art. **1398**.

Let S_s = the safe shearing strength of shaft;
J = polar moment of inertia of cross-section of shaft about neutral axis;
c = distance from neutral axis to outermost fiber.

Then, the moment of resistance to twisting is $\frac{S_s J}{c}$.

Where a plane section revolves, or is conceived to revolve, about a line lying in the plane of the section, as, for example, a circular section revolving about its diameter, the moment of inertia obtained with reference to this axis is termed the **rectangular moment of inertia,** and is nearly always denoted in text-books by I. When, however, the section revolves about an axis perpendicular to the plane of section, as, for example, a buzz saw or fly-wheel, the moment of inertia found with respect to this axis is termed the **polar moment of inertia,** and is nearly always denoted in text-books by J. Those moments of inertia used in the subject of Strength of Materials were rectangular moments. For a solid circular section, $J = 2I = 2 \times \frac{\pi d^4}{64} = \frac{\pi d^4}{32}$; and for a hollow circular section,

$$J = \frac{\pi(d^4 - d_1^4)}{32}.$$

The reason for using the safe shearing stress S_s in the expression $\frac{S_s J}{c}$ is that its value agrees very closely with the value for torsion; it is quite different from the safe stress for flexure. The student may obtain the value of S_s by simply dividing the ultimate shearing stress by the proper factor of safety; i. e., $S_s = \frac{S_s}{f}$.

Placing the twisting and resisting moments equal to each other, we have

$$PR = 63{,}025 \frac{H}{N} = \frac{S_s J}{c} = S_s \frac{\frac{\pi d^4}{32}}{\frac{d}{2}} = S_s \frac{\pi d^3}{16}.$$

Whence, $d = \sqrt[3]{\frac{16PR}{\pi S_s}} = 1.72\sqrt[3]{\frac{PR}{S_s}}$. **(252.)**

Or, $d = \sqrt[3]{\frac{16 \times 63{,}025 H}{\pi N S_s}} = 68.5\sqrt[3]{\frac{H}{N S_s}}$. **(253.)**

Let $\sqrt[3]{\frac{16}{\pi S_s}} = k$, and $\sqrt[3]{\frac{16 \times 63{,}025}{\pi S_s}} = k_1$.

Then, $d = k\sqrt[3]{PR} = k_1\sqrt[3]{\frac{H}{N}}$, **(254.)**

as in formula **124**, Art. **1416**.

The values of k and k_1 depend upon the value of the safe stress S_s assumed for the material of the shaft.

If we take $S_s = 3{,}400$ for cast iron;
$S_s = 6{,}800$ for wrought iron;
$S_s = 9{,}000$ for steel,

we obtain the values of k and k_1 given in Table 32, Art. **1416**.

1987. Formulas **252** and **253** are general, and may be used for any material; the proper value of S_s being left to the judgment of the designer. Ordinarily, the values of S_s given above are those used in good practice. Under exceptional circumstances it may be necessary to use lower values, particularly if the shaft is subjected to shocks, or if the stress changes suddenly or violently.

It has been shown that for strength the diameter of a shaft is proportional to the cube root of the twisting moment. A similar theoretical treatment would show that for *stiffness* the diameter of the shaft is proportional to the *fourth root* of the twisting moment.

Hence, for shafts of small diameter or of considerable length the diameter may be calculated by the formula $d = c\sqrt[4]{Pr} = c_1\sqrt[4]{\frac{H}{N}}$ and Table 31, Art. **1415**.

1988. Speed of Line Shafting.—The speed of a shaft is fixed largely by the speed of the driving belt or the diameters of the pulleys upon it. In general, machine shop

shafts run about 120 to 150 revolutions per minute; shafts driving woodworking machinery, about 200 to 250 revolutions per minute; in cotton mills, the practice is to make the shaft diameter smaller, and run at a higher speed. Line shafts should generally not be less than $1\frac{3}{4}$ inches in diameter.

1989. Distance Between Bearings.—This distance should not be great enough to permit a deflection of more than $\frac{1}{100}''$ per foot of length. Hence, when the shaft is heavily loaded with pulleys, the bearings must be closer than when it carries only a few pulleys.

Below are given the maximum distances between the bearings of different sizes of continuous shafts which are used for the transmission of power:

TABLE 48.

Diameter of Shaft in Inches.	Distance Between Bearings in Feet.	
	Wrought-Iron Shaft.	Steel Shaft.
2	11	11.5
3	13	13.75
4	15	15.75
5	17	18.25
6	19	20
7	21	22.25
8	23	24
9	25	26

Pulleys which give out a large amount of power should be placed as near a hanger as possible.

SHAFTS SUBJECTED TO BENDING.

1990. Under this head are included the axles of large water-wheels, gear-wheels, etc., which are loaded transversely. The axle is not generally strained by twisting, and

may be treated as a beam transversely loaded. The mode of procedure may, perhaps, be best shown by an example.

EXAMPLE.—An axle 12 feet long, between centers of bearings, carries a wheel weighing 9 tons; the wheel is 4 feet from one end of the shaft. The axle is of wrought iron. Required, the dimensions.

SOLUTION.—Let the line mn, Fig. 670, represent temporarily the axle in question; P, the load of 9 tons, and R_1 and R_2, the reactions.

Lay off to scale the load P on the vertical through R_1, and, choosing a pole O, draw the rays $O1$, $O3$, and ab, and cb parallel to them. Then, abc is the bending moment diagram. $O2$ is drawn parallel to ac, and gives the reaction $R_1 = 1\text{-}2 = 3$ tons, and $R_2 = 2\text{-}3 = 6$ tons.

The lengths and diameters of the journals may now be calculated. For the journal nearest the load, we have, from formula **241**,

$$d = 1.5\sqrt{\frac{R_2}{\sqrt{S_f p}}}, \text{ and}$$

assuming that $p = 700$ and $S_f = 8{,}500$,

$$d = 1.5\sqrt{\frac{12{,}000}{\sqrt{8{,}500 \times 700}}} = 3\tfrac{3}{8}'', \text{ nearly.}$$

$$l = \frac{R_2}{p\,d} = \frac{12{,}000}{700 \times 3\frac{3}{8}} = 5\tfrac{1}{8}'', \text{ nearly.}$$

For the other journal,

$$d' = 1.5\sqrt{\frac{R_1}{\sqrt{S_f p}}} = 1.5\sqrt{\frac{6{,}000}{\sqrt{8{,}500 \times 700}}} = 2\tfrac{3}{8}''.$$

$$l_1 = \frac{6{,}000}{700 \times 2\frac{3}{8}} = 3\tfrac{5}{8}''.$$

$$d_4 = d' + 2e, \text{ where } e = \tfrac{1}{10}d' + \tfrac{1}{8}'' = .3625'' = \tfrac{3}{8}'', \text{ nearly.}$$

$$\text{Hence,}\quad d_4 = 2\tfrac{3}{8}'' + \tfrac{3}{4}'' = 3\tfrac{1}{8}''.$$

$$\text{Likewise,}\quad d_5 = d + 2(\tfrac{1}{10}d + \tfrac{1}{8}'') = 4\tfrac{5}{16}''.$$

The resisting moment of a beam is $S\frac{I}{c}$ (see Art. **1398**), and since the safe stress S remains constant for the same beam, the resisting moment is proportional to $\frac{I}{c}$. For a beam of circular section

$$\frac{I}{c} = \frac{\frac{\pi d^4}{64}}{\frac{d}{2}} = \frac{\pi d^3}{32}.$$

Hence, since $\frac{\pi}{32}$ is a constant quantity, the resisting moment at any section of a circular shaft is proportional to the cube of the diameter at that section. Conversely, the diameter of the shaft at any section must be proportional to the cube root of the resisting moment at that

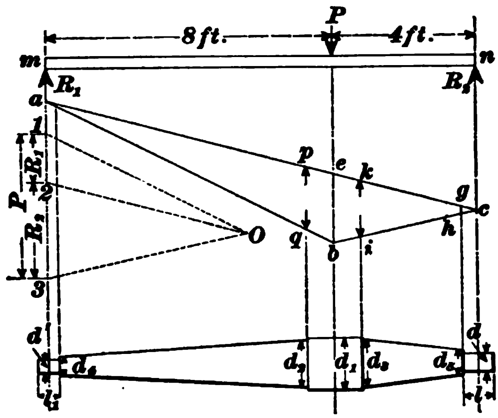


FIG. 670.

section, and to have the shaft of equal strength throughout, the diameter at each section should be proportional to the cube root of the *bending moment* at that section.

$$\text{Hence,}\quad \frac{d_1}{d} = \sqrt[3]{\frac{\text{moment } b\,e}{\text{moment } g\,h}}.$$

By measurement, the moment $b\,e$ is 288 inch-tons, and the moment $g\,h$ is $15\frac{3}{8}$ inch-tons.

$$\text{Whence,}\quad \frac{d_1}{d} = \sqrt[3]{\frac{288}{15\frac{3}{8}}} = 2.65;$$

$$\text{or, } d_1 = 2.65\,d = 2.65 \times 3\tfrac{3}{8} = 9 \text{ inches, nearly.}$$

The shaft should be bossed at this point to allow for cutting the key-way. The depth of the sunk key $= \frac{1}{6}\,d_1 = 1\frac{1}{2}$ in. (See formula **233.**)

The depth of the key-way is, therefore, $\frac{3}{4}$ inch, and the diameter of the bossed portion $10\frac{1}{2}$ inches at least. The diameters d_2 and d_3 may be easily found by measuring the moments $p\,q$ and $k\,i$, whence,

$$d_2 = d_1\sqrt[3]{\frac{q\,p}{b\,e}}, \text{ and } d_3 = d_1\sqrt[3]{\frac{i\,k}{b\,e}}.$$

In a similar manner may be found the dimensions of an axle bearing two or more transverse loads.

The axle may or may not be tapered as shown in Fig. 670. If it is not tapered, its diameter must be calculated from the maximum bending moment ($b\ e$, in this case). A straight shaft is easier to construct, but the end journals must be, of course, larger, and there is considerable loss by friction on that account.

SHAFTS SUBJECTED TO COMBINED BENDING AND TWISTING.

1991. A good example of a shaft coming under this head is an engine crank-shaft carrying a heavy fly-wheel.

Let B = bending moment;
T = twisting moment;
T_1 = the twisting moment, which would have the same effect as B and T acting together.

Then, $T_1 = B + \sqrt{B^2 + T^2}$. **(255.)**

The twisting moment T_1 is called the **ideal twisting moment,** and should be used in formula **252** to determine the diameter of the shaft.

EXAMPLE.—Calculate the diameter of a steel crank-shaft for an engine from the following data: The length of stroke is 10 feet. The maximum tangential force acting on the crank-pin is 25 tons. The shaft is 10 feet long between centers of bearings, and carries midway between bearings a fly-wheel weighing 55 tons. Assume the safe stress to be 9,000 lb. $= 4\frac{1}{2}$ tons.

SOLUTION.—The twisting moment T = maximum tangential force $\times$ length of crank = 25 tons $\times$ 5 feet = 125 foot-tons = 1,500 inch-tons.

The bending moment $= \frac{1}{4} P l = \frac{55 \times 10}{4} = 137.5$ foot-tons = 1,650 inch-tons.

Then, $T_1 = B + \sqrt{B^2 + T^2} = 1{,}650 + \sqrt{1{,}650^2 + 1{,}500^2} = 3{,}880$ inch-tons.

Now, using formula **252**,

$$d = 1.72 \sqrt[3]{\frac{P R}{S_s}} = 1.72 \sqrt[3]{\frac{T_1}{S_s}} = 1.72 \sqrt[3]{\frac{3{,}880}{4.5}} = 16\tfrac{3}{8}'', \text{ nearly. Ans.}$$

It will be noticed that, in the above solution, the units selected are inches and tons. Generally, when the bending and twisting moments are large, it is more convenient to use tons than pounds. Care must be taken, however, to

make the units consistent; for example, it would have been wrong to have taken the moments in *inch-tons*, and the safe stress in *pounds*.

1992. It is sometimes convenient to put formula **255** in another form.

Let $\frac{B}{T} = Z$; then, formula **255** becomes

$$T_1 = ZT + \sqrt{Z^2 T^2 + T^2} = T(Z + \sqrt{Z^2 + 1}).$$

$$\text{Let } Z + \sqrt{Z^2 + 1} = k; \text{ then, } T_1 = kT.$$

But, by formula **252,** the diameter is proportional to the cube root of the twisting moment. Therefore, if d is the diameter of shaft required by the twisting moment T, and d_1 the diameter required by the twisting moment T_1, $\frac{d_1}{d} = \sqrt[3]{\frac{T_1}{T}} = \sqrt[3]{k}$, or $d_1 = d\sqrt[3]{k}$.

The following table gives the values of $\sqrt[3]{k}$ for varying values of Z:

TABLE 49.

Z	$\sqrt[3]{k}$.	Z	$\sqrt[3]{k}$.	Z	$\sqrt[3]{k}$.
.2	1.068	1.2	1.403	2.2	1.665
.4	1.139	1.4	1.461	2.4	1.710
.6	1.209	1.6	1.516	2.6	1.753
.8	1.277	1.8	1.568	2.8	1.794
1.0	1.341	2.0	1.618	3.0	1.833

This table may be used advantageously for computing the diameter of shafts subjected to twisting and bending. The diameter is first computed for twisting alone, and is then multiplied by the value of $\sqrt[3]{k}$ corresponding to the ratio Z between the bending and twisting moments.

EXAMPLE.—A wrought-iron shaft transmits 150 horsepower at 125 revolutions per minute, and is at the same time subjected to a bending moment of 60,000 inch-pounds. Calculate the diameter of the shaft.

SOLUTION.—For twisting only,

$$d = 3.62\sqrt[3]{\frac{H}{N}} = 3.62\sqrt[3]{\frac{150}{125}}$$

See Art. **1416.**

By formula **231**, $T = P\,R = 63{,}025\dfrac{H}{N} = 75{,}630$ in.-lb.

$B = 60{,}000.$

Then, $\dfrac{B}{T} = Z = \dfrac{60{,}000}{75{,}630} = .8$, nearly.

From the above table $\sqrt[3]{k} = 1.277$.

Hence, $d = 1.277 \times 3.62\sqrt[3]{\dfrac{150}{125}} = 4\frac{7}{8}''$. Ans.

HOLLOW SHAFTS.

1993. It is plain that the outer fibers of a shaft are much more useful in resisting a twisting or bending strain than the fibers near the center; hence, if a shaft is made hollow, its weight will be diminished in much greater proportion than its strength. In other words, a hollow shaft is stronger than a solid one containing the same amount of material per unit length.

Let d_1 = outside diameter of hollow shaft;
d_2 = inside diameter of hollow shaft;
d = diameter of solid shaft having the same strength as hollow shaft.

The resisting moment (see Art. **1398**) is $\dfrac{S\,I}{c}$. Taking the axis of the shaft as the neutral axis, I becomes J; the values of J for both hollow and solid shafts are double those given in the Table of Moments of Inertia, where the neutral axis is the diameter of a cross-section.

For a solid shaft, $$\frac{S_s\,J}{c} = \frac{\dfrac{S_s\,\pi\,d^4}{32}}{\dfrac{d}{2}} = \frac{S_s\,\pi\,d^3}{16}.$$

For a hollow shaft, $$\frac{S_s\,J}{c} = \frac{\dfrac{S_s\,\pi\,(d_1^4 - d_2^4)}{32}}{\dfrac{d_1}{2}} = \frac{S_s\,\pi\,(d_1^4 - d_2^4)}{16\,d_1}.$$

Hence, if the shafts are to be equally strong,

$$\frac{S_s \pi d^3}{16} = \frac{S_s \pi (d_1^4 - d_2^4)}{16 d_1},$$

$$\text{or} \quad d^3 = \frac{d_1^4 - d_2^4}{d_1} = d_1^3 \left(1 - \frac{d_2^4}{d_1^4}\right).$$

$$\text{Let } \frac{d_2}{d_1} = m. \quad \text{Then, } d^3 = d_1^3 (1 - m^4),$$

$$\text{or} \quad d_1 = d \sqrt[3]{\frac{1}{1 - m^4}}. \qquad \textbf{(256.)}$$

EXAMPLE.—Suppose we wish to find the diameter of a hollow shaft which will have the same strength as a solid shaft 8 inches in diameter, assuming the hole to be one-half the diameter.

SOLUTION.—Applying formula **256,** $m = \frac{1}{2}$, and

$d_1 = d \sqrt[3]{\frac{1}{1 - m^4}} = 8 \sqrt[3]{\frac{1}{1 - (\frac{1}{2})^4}} = 8.174$, say $8\frac{3}{16}''$ outside diameter, and $4\frac{3}{32}''$ inside diameter. Ans.

Hollow steel shafts are much used for marine engines.

SHAFT COUPLINGS.

1994. Couplings are used to connect the ends of shafts, and are of three kinds: 1. Fast or permanent couplings. 2. Loose couplings, or clutches, by means of which shafts may be connected or disconnected at pleasure. 3. Friction clutches, which are loose couplings that hold by friction.

1995. Box, or **Muff, Couplings.**—This coupling, as shown in Fig. 671, consists of a cast-iron cylinder which fits

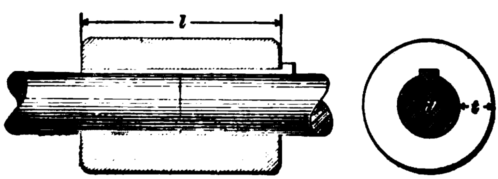


FIG. 671.

over the ends of the shaft. The two ends are prevented from moving relatively to each other by the sunk key. The key-way is cut half into the box and half into the shaft

ends. Quite commonly the ends of the shafts are enlarged to allow the key-way to be cut without weakening the shaft.

The key may be proportioned by the formulas already given. For the other dimensions take

$$\left.\begin{aligned} l &= 2\tfrac{1}{2}\,d + 2''. \\ t &= 4\;\;d + .5''. \end{aligned}\right\} \qquad \textbf{(257.)}$$

EXAMPLE.—Find the dimensions of a muff coupling for a shaft $2\frac{1}{2}''$ in diameter.

SOLUTION.—For the key we use formula **233.**

$$\left.\begin{aligned} b &= \tfrac{1}{4}\,d = \tfrac{1}{4} \times 2\tfrac{1}{2} = \tfrac{5}{8}''. \\ t &= \tfrac{1}{6}\,d = \tfrac{1}{6} \times 2\tfrac{1}{2} = \tfrac{7}{16}''. \end{aligned}\right\} \text{Ans.}$$

For the muff, we use formula **257.**

$$\left.\begin{aligned} l &= 2\tfrac{1}{2}\,d + 2'' = 2\tfrac{1}{2} \times 2\tfrac{1}{2} + 2'' = 8\tfrac{1}{4}''. \\ t &= .4\,d + .5'' = .4 \times 2\tfrac{1}{2} + .5'' = 1\tfrac{1}{2}''. \end{aligned}\right\} \text{Ans.}$$

1996. A clamp coupling is shown in Fig. 672. The faces for the joint are first planed off, the holes are drilled, and then the two halves are bolted together with pieces of

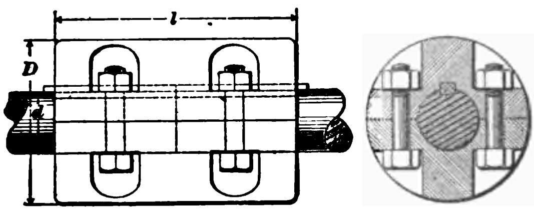


FIG. 672.

paper between them, after which the coupling is bored out to the exact size of the shaft. The pieces of paper upon being removed leave a slight space between the halves, and the coupling when bolted to the shaft grips it firmly.

This form of coupling is very easily removed or put on; it has no projecting parts, and may be used as a driving pulley, if desired. The key is straight, and fits only at the sides.

The following are the proportions used in practice;

d = diameter of shaft;

Diameter of coupling $D = 2\frac{1}{2}\,d + \frac{1}{2}''$;

Length of coupling $l = 4\,d$.

The diameter of the bolts may be $\frac{5}{8}''$ for shafts under $2\frac{1}{2}''$ in diameter; $\frac{3}{4}''$ for $2\frac{1}{2}''$ shaft, and $\frac{7}{8}''$ for larger shafts.

For shafts up to 3 inches in diameter use 4 bolts; for larger shafts use 6 bolts.

1997. The **flange coupling** is shown in Fig. 673. Cast-iron flanges are keyed to the ends of the shafts. To insure a perfect joint the flange is usually faced in the lathe after being keyed to the shaft. The two flanges are then brought face to face and bolted together. Sometimes the ends of the shafts are enlarged to allow for the key-way.

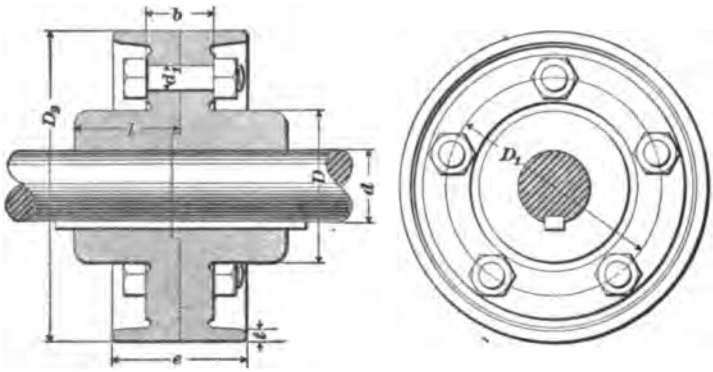


FIG. 673.

To prevent the possibility of the shafts getting out of line, the end of one may enter the flange of the other, as shown.

The following proportions may be used for the form of flange coupling shown in Fig. 673:

$$d = \text{diameter of shaft.}$$

$$\left.\begin{array}{rl}
D & = 1\tfrac{3}{4}d + 1''; \\
\text{Diameter of bolt circle } D_1 & = 2\tfrac{1}{2}d + 2''; \\
l & = 1\tfrac{1}{8}d + 1''; \\
\text{Number of bolts } n & = 3 + \dfrac{d}{2}; \\
\text{(Take the nearest even number.)} & \\
\text{Diameter of bolts } d_1 & = \dfrac{d}{n} + \tfrac{1}{4}''; \\
\text{Diameter of coupling } D_2 & = 1.4\,D_1; \\
b & = \tfrac{1}{2}d + \tfrac{5}{8}''; \\
e & = 2b; \\
t & = \tfrac{1}{8}d.
\end{array}\right\} \quad (258.)$$

The key may be proportioned by the formulas already given for keys.

1998. Solid flange couplings, shown in Fig. 674, are used largely to connect the sections of propeller shafts.

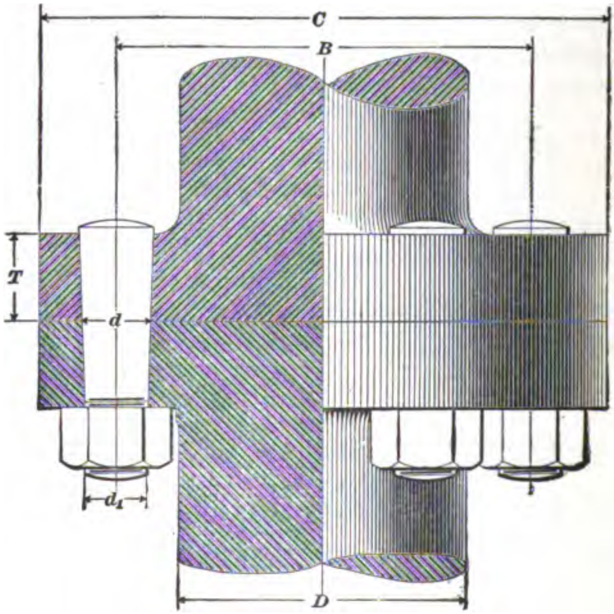


FIG. 674.

The flanges are forged on the ends of the shafts, and are connected by tapered bolts.

Let n = number of bolts uniting flanges;
d = diameter of the bolts at the joint formed by the flanges;
D = diameter of shaft;
B = diameter of bolt circle.

The shearing resistance of the bolts is $\frac{\pi d^2}{4} n S_s$, and the moment of this resistance about the center of shaft is

$$\frac{\pi d^2}{4} n S_s \times \frac{B}{2} = \frac{\pi d^2 n S_s B}{8}.$$

Now, it was shown in the derivation of formula **253** that the moment of resistance of the shaft when subjected to twisting is $\frac{\pi D^3}{16} S_s$. If the bolts and shafts are of equal strength,

$$\frac{\pi d^2 n S_s B}{8} = \frac{\pi D^3 S_s}{16},$$

$$\text{or} \qquad d^2 n B = \frac{D^3}{2}.$$

Approximately, $B = 1.6\ D$. Using this value, we obtain

$$1.6\ d^2 n D = \frac{D^3}{2}, \text{ or } d = \frac{D}{\sqrt{3.2\ n}}.$$

But more nearly, $B = D + 2\ d$.

Substituting for d its value just found,

$$B = D\left(1 + \frac{2}{\sqrt{3.2\ n}}\right).$$

$$\text{Whence, } d^2 n D\left(1 + \frac{2}{\sqrt{3.2\ n}}\right) = \frac{D^3}{2},$$

$$\text{or} \quad d = D\sqrt{\frac{\sqrt{3.2\ n}}{2\ n(\sqrt{3.2\ n} + 2)}} = k D \qquad \textbf{(259.)}$$

The values of $k = \sqrt{\frac{\sqrt{3.2\ n}}{2\ n(\sqrt{3.2\ n} + 2)}}$ for different values of n are as follows:

TABLE 50.

$n =$	3	4	5	6	7	8	9	10
$k =$	.318	.283	.258	.239	.224	.212	.201	.192

The following are the proportions of the other parts of the coupling:

$$\left.\begin{aligned} n &= \tfrac{1}{8}D + 2; \\ B &= D + 2d; \\ C &= D + 4\tfrac{1}{2}d; \\ T &= \frac{2D+1}{7}; \\ d_1 &= \tfrac{7}{8}d + \tfrac{1}{8}''. \end{aligned}\right\} \quad (260.)$$

The bolts are tapered $\frac{5}{8}$ inch per foot of length.

1999. Seller's Cone Coupling.—This coupling is shown in Fig. 675. It consists of an outer box or muff

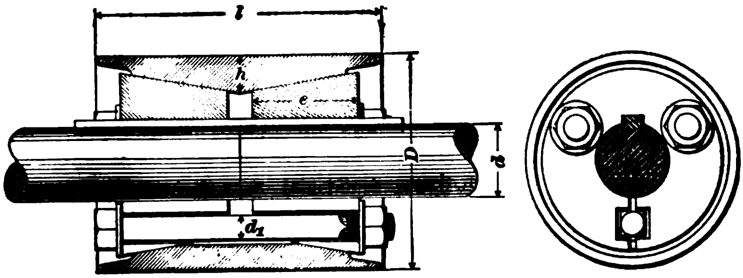


FIG. 675.

which is cylindrical externally, but has the form of a double truncated cone on the inside. Within the muff are placed two slotted sleeves, which are turned on the outside to fit the muff, and also bored out to fit the shaft. These sleeves are pulled together by three bolts, and as they are drawn farther into the muff, they grip it and the shafts firmly.

The bolt holes pass through both the sleeves and muff, and are square in cross-section. The friction between the sleeves and shaft is generally sufficient to prevent slipping, but to be on the safe side, the sleeves are usually keyed to the shaft. The key should have no taper, and fit at the sides only; its proportions may be obtained by formula **233.**

The other proportions may be taken as follows:

$$d = \text{diameter of shaft};$$

$$\left.\begin{aligned} l = 4d; \quad d_1 &= \tfrac{1}{3}d; \\ D = 3d; \quad e &= 1\tfrac{1}{2}d; \\ h &= \tfrac{1}{2}d. \end{aligned}\right\} \quad (261.)$$

The conical sleeves may be tapered 4 inches per foot of length. In putting up lines of shafting, the couplings should be placed, if possible, near bearings, and on the side of the bearing farthest from the driving point.

2000. Universal Joints, or Flexible Couplings.—The principal forms of these joints have already been described in Arts. **1458, 1459,** and **1460.**

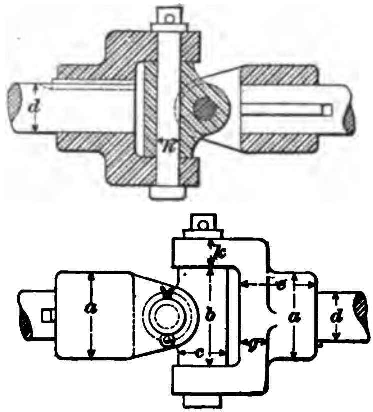


FIG. 676.

In Fig. 676 is shown a common form of these joints which, when constructed of wrought iron, may have the following proportions:

$$
\left.
\begin{array}{ll}
d = \text{diameter of shaft}; & \\
a = 1.8\ d; & e = 1.6\ d; \\
b = 2\ d; & g = .6\ d; \\
c = d; & h = .5\ d; \\
\multicolumn{2}{c}{k = .6\ d.}
\end{array}
\right\} \qquad (262.)
$$

2001. Loose Couplings.—These couplings are used

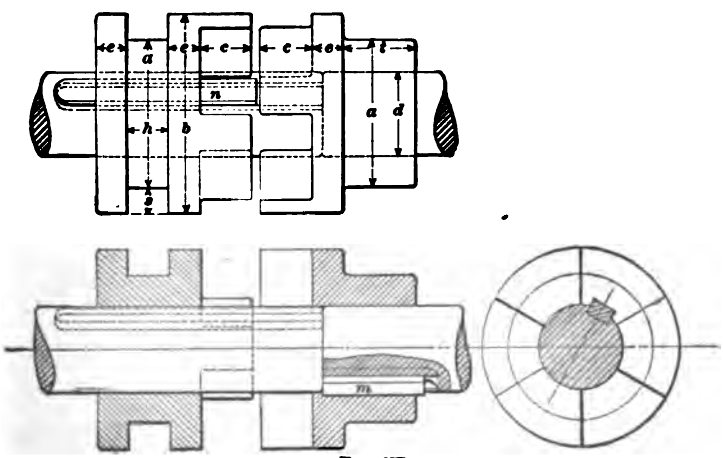


FIG. 677.

when the shafts are to be alternately connected and disconnected. For large slow-moving shafts, the *claw coupling* shown in Fig. 677 may be used. This coupling somewhat resembles the flange coupling, Fig. 673, except that the flanges, instead of being bolted together, are provided with a set of lugs *c* which fit into each other. One flange is permanently fastened to the shaft by a sunk key *m*, while the other is fastened to its shaft by a feather key *n*, and may be moved back and forth, thus throwing the coupling, or clutch, in or out of gear. The movement of the clutch is effected by a forked lever fitting into the recess *h*.

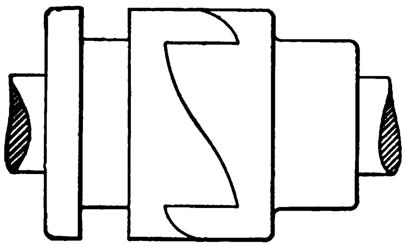


FIG. 678.

The lugs or claws may be given in the form shown in Fig. 678, in which case the couplings are more easily put in gear, but will drive in only one direction.

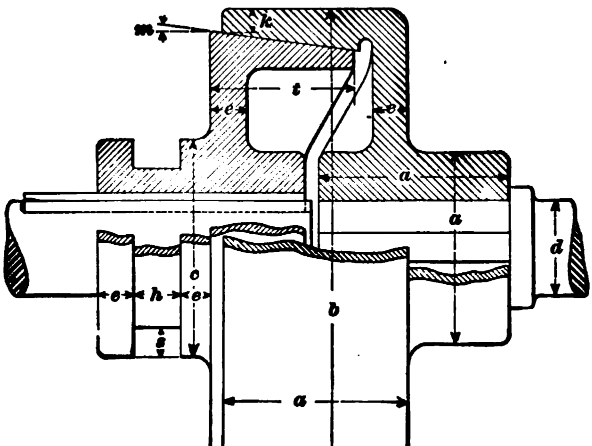


FIG. 679.

Cast-iron claw couplings may have the following proportions:

$$\left.\begin{array}{ll} d = \text{diameter of shaft}; & \\ a = 1\tfrac{3}{4}\,d; & e = \tfrac{3}{8}\,d; \\ b = 2\tfrac{3}{8}\,d; & h = \tfrac{1}{2}\,d; \\ c = \tfrac{5}{8}\,d; & s = \tfrac{5}{16}\,d; \\ \multicolumn{2}{c}{t = \tfrac{7}{8}\,d.} \end{array}\right\} \qquad (263.)$$

2002. Friction couplings, or **clutches,** are used as loose or disengaging couplings on shafts running at high speeds. They are often used to couple wheels or pulleys to shafts. The form shown in Fig. 679 is simple in construction, but it is open to the objection that it is hard to put in gear. Besides, the horizontal component of the pressure between the conical surfaces causes an end thrust on the shaft.

The average diameter of the conical part may be from 4 to 8 times the shaft diameter, according to the amount of power to be transmitted. The angle m of the cone may be from 4° to 10°. The other proportions are as follows:

$$\left.\begin{array}{ll} d = \text{diameter of shaft}; & \\ a = 2\,d; & e = \tfrac{3}{8}\,d; \\ b = 4 \text{ to } 8\,d; & h = \tfrac{1}{2}\,d; \\ c = 2\tfrac{1}{4}\,d; & s = \tfrac{5}{16}\,d; \\ t = 1\tfrac{1}{2}\,d; & k = \tfrac{1}{4}\,d. \end{array}\right\} \qquad (264.)$$

A better form of friction clutch is shown in Fig. 680. The shaft n carries a flange or cylinder A, and the shaft m

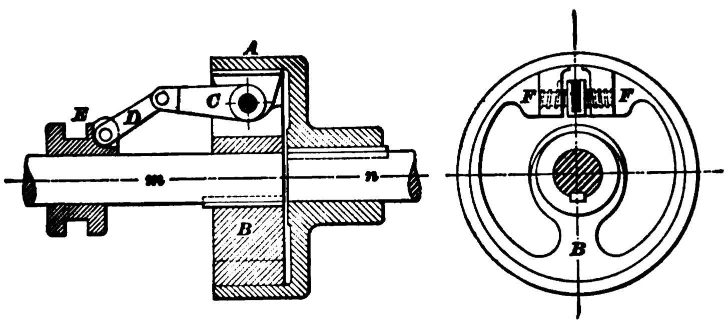


FIG. 680.

has keyed to it a ring B. The ring is split and fits inside

the flange or cylinder *A*. The split ends are connected by a screw with right and left hand threads. The screw is turned by the lever *C*, which is connected by the link *D* to the sleeve *E*. When the sleeve is pushed towards the clutch, the rotation of the screw throws the ends *F*, *F* of the ring *B* apart, and thereby causes the ring *B* to grip the flange *A* tightly. A clutch of this form is easy to operate, and produces no end thrust on the shaft.

Another form of clutch differs from the preceding in having the friction band *B* on the outside of the flange. The principal proportions of these clutches are about the same as given above for Fig. 679.

2003. Shifting Gear for Clutches.—A clutch is usually put in or out of gear by means of a forked lever, the

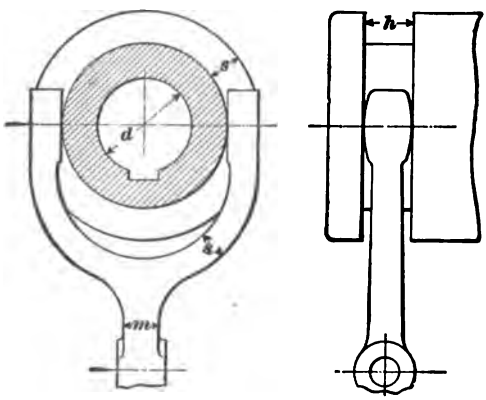


FIG. 681.

prongs of which fit into the groove cut in the sliding part of the clutch. The lever is usually worked by hand, though for large clutches the end of the lever may be moved by a screw and hand wheel. The ordinary design of the forked end of the lever is shown in Fig. 681. To increase the wearing surface a strap may be used, as shown in Fig. 682. The strap completely fills the groove, and is often made of brass.

The dimensions h and s are the width and depth of the

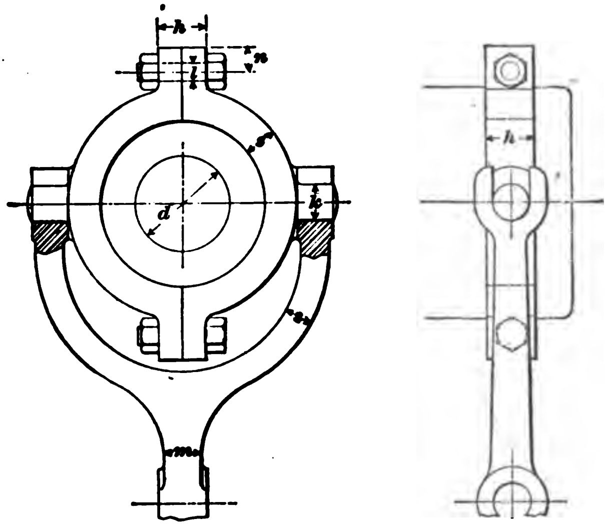


FIG. 682.

groove, respectively, which are to be determined by the rules already given for the proportions of the clutch.

Proportions for clutch lever:

$$\left.\begin{array}{ll} d = \text{diameter of shaft;} & \\ h = \frac{1}{2}d; & s = \frac{5}{16}d; \\ k = \frac{3}{8}d; & m = \frac{3}{8}d; \\ l = \frac{3}{16}d; & n = \frac{1}{4}d. \end{array}\right\} \qquad (265.)$$

MACHINE DESIGN.

(CONTINUED.)

BEARINGS.

2004. Solid Journal Bearings.—The simplest form of bearing for a journal is merely a hole in the frame which supports the rotating piece. Such a bearing is shown in Fig. 683. Motion endwise is prevented by two collars,

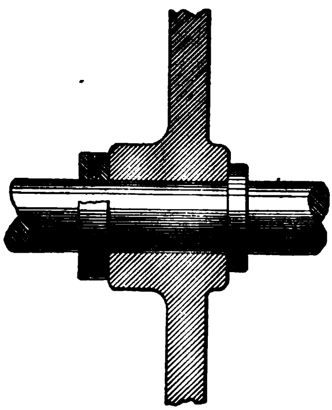


FIG. 683.

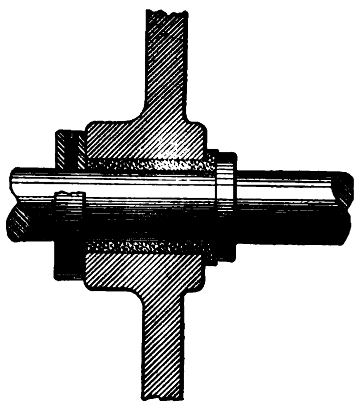


FIG. 684.

one of which may be forged on the shaft, and the other made separately and held in place by a set-screw or pin. A boss is cast upon the frame, as shown in the figure, in order to give the journal the necessary length and bearing area.

Such a bearing has no means of adjusting to take up the wear; for this reason it is better to use the form of solid bearing shown in Fig. 684. Here the hole is bored out larger than the journal, and lined with a bushing made of

brass or other metal. The wear thus comes on the bushing, which can easily be replaced.

2005. Divided Bearings.—In many cases solid bearings are undesirable, and in others it will be impossible to use them. The bearing is then divided, and the parts held

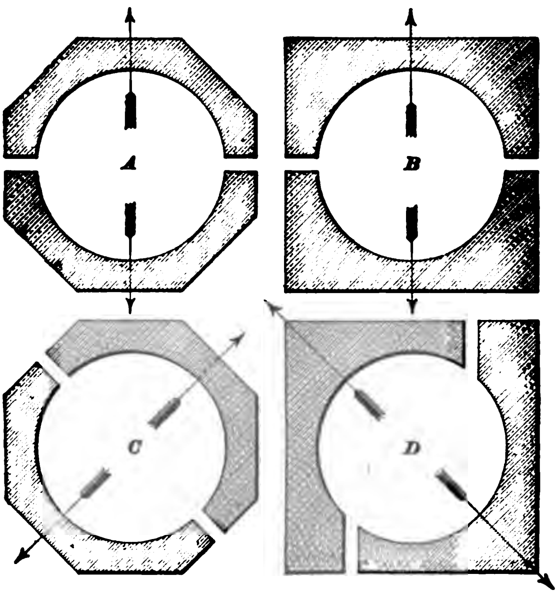


FIG. 685.

together by bolts. When this is done, the parts of the bushing are called *seats* or *steps*. The division of the bearing permits its adjustment for wear.

The load on a bearing usually acts constantly in one direction, and the bearing should be divided so that the line of division is perpendicular, or nearly so, to the direction of the load. Fig. 685 shows various methods of dividing the seats. In *A* and *B* the direction of the load is vertical; consequently, the seats are divided horizontally.

Naturally, the wear will come on the top and bottom of the seats, and the hole will become oval, with the long

diameter vertical. The seats are then screwed closer together until the hole regains its original circular form. The necessity of making the division perpendicular to the direction of the load is thus apparent. In C the direction of the load is oblique, and the adjustment perpendicular to it; in D the direction of the load is oblique also, but the seats are divided to allow a horizontal adjustment.

A simple form of divided bearing is shown in Fig. 686. It is of cast iron, with no separate seats, but with means of

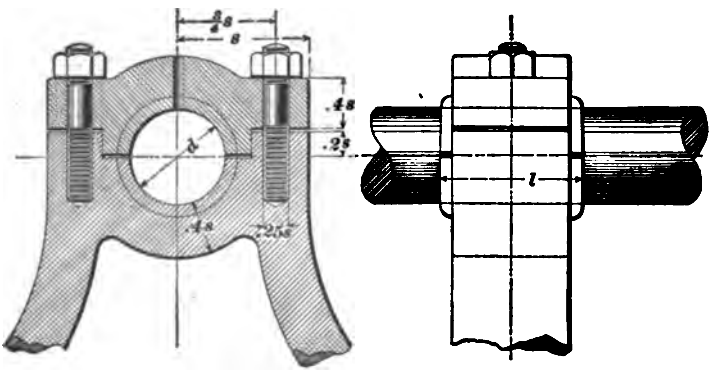


FIG. 686.

adjustment. The bearing forms part of the frame of the machine. This form of bearing is used only on cheap work. Sometimes, however, it has a recess, or groove, cast in it, which is filled with babbitt metal, upon which the journal rests.

The proportional unit for this bearing is

$$s = 1.15\, d + .4.$$

The dimensions l and d of the bearing are the same as those for a journal; the other proportions are given in terms of the unit s, and are shown in the figure.

2006. Seats, or Steps.—Carefully made bearings are always lined with so-called **seats,** or **steps,** which are made of brass, gun-metal, phosphor-bronze, white metal or babbitt metal, although other alloys are also used. When

made of a metal resembling brass in color, seats are often called **brasses.** The seats when worn out may be easily replaced, and being made of softer material than the journal, the latter wears but very little.

An ordinary form of seat is shown in Fig. 687. A part of the seat at each end is made octagonal in cross-section. This part is fitted into an octagonal recess in the **pillow-block,** or **pedestal,** which holds it, and the seats are thus prevented from turning. Sometimes the octagonal parts

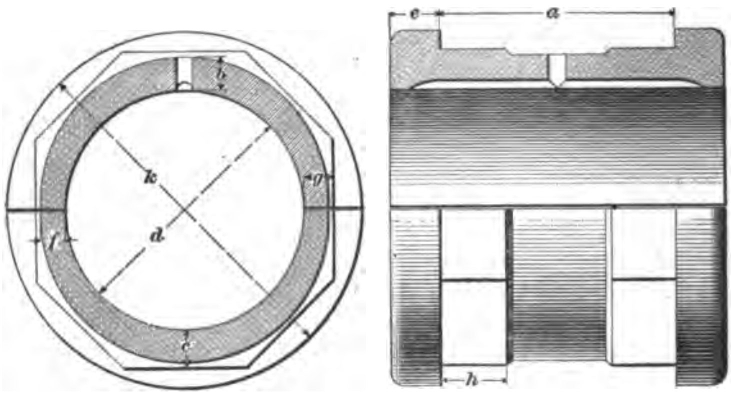


FIG. 687.

are dispensed with, and the seats are turned in a lathe, the pillow-blocks, or pedestals, being bored to receive them. It is then necessary to provide the seats with a lug, or pin, to prevent them from turning.

These seats may be made according to the following proportions, in which d, the diameter of the journal, is the proportional unit:

$b = \frac{1}{10}d + \frac{3}{16}''$.	$g = \frac{1}{8}d$.
$c = \frac{1}{8}d + \frac{3}{16}''$.	$h = \frac{1}{4}d$.
$e = \frac{1}{8}d + \frac{1}{8}''$.	$k = 1\frac{1}{2}d$.
$f = \frac{1}{10}d$.	$d =$ diameter of journal.

The dimension a should be made to fit the pillow-block for which the seats are intended.

Seats should be well supplied with grooves and channels, so that oil may be conducted to every part of the journal.

It is a good plan to place a groove parallel to the axis of the journal in the upper half of the seat, where it may be in direct communication with the oil holes. Then, the advancing side of the journal will always carry a thin film of oil along with it. The grooves may be from $\frac{1}{8}''$ to $\frac{1}{4}''$ wide, according to the size of the journal.

2007. Lining for Seats.—Seats for large bearings are often lined with babbitt metal, or antifriction metal. It has been found by experience that a bearing will run

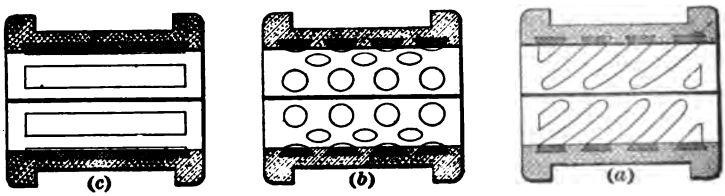


FIG. 688.

cooler when so lined, probably because the babbitt metal, being softer, accommodates itself to the journal more readily than the more rigid gun-metal.

Some of the common methods of lining the seats are shown in Fig. 688.

At (*a*), the babbitt metal is shown cast into shallow helical grooves; at (*b*), into a series of round holes, and at (*c*), into shallow rectangular grooves. Consequently, the journal rests partly on the brass and partly on the babbitt metal.

In cheap work, the seats are frequently made entirely of babbitt metal. A mandrel, the exact size of the journal, is placed inside the bearing, and the melted babbitt metal is poured around it. In better work, a smaller mandrel is used. After the metal hardens it is hammered in; the bearing is then bored out to the exact size of the journal.

PEDESTALS.

2008. The names **pedestal, pillow-block, bearing,** and **journal-box,** are used indiscriminately. They are all a form of bearing, and mean a support for a rotating piece.

2009. A form of journal-box frequently used for small shafts is shown in Fig. 689. It consists of two parts: (1) the box which supports the journal, and (2) the cap which is screwed down to the box. In this journal-box the seats are of babbitt, or, as it is commonly expressed, the box is babbitted. The cap is held in place by what are called cap-screws—an invariable method in small pedestals.

2010. The proportioning of a pedestal is largely a matter of experience. Few or none of the parts are calculated for strength.

All the proportions of the pedestals which follow are based on the diameter of the journal d as the unit; the length of the seats is the same as that of the journal.

For the journal-box shown in Fig. 689, the following proportions may be used for sizes of journals from $\frac{1}{2}''$ to $2''$ diameter, inclusive. The diameter of shaft $d =$ the unit.

$a = 2.25\,d.$	$l = .08\,d.$
$b = 1.75\,d.$	$m = .25\,d + .1875''.$
$c = d.$	$n = .5\,d.$
$e = .375\,d.$	$o = .625''$ (constant).
$f = .08\,d + .0625''.$	$p = 1.5\,d.$
$g = 1.75\,d.$	$q = 1.333\,d.$
$h = 2.45\,d.$	$r = .08\,d.$
$i = .3\,d.$	$s = .125''$ (constant).
$j = .33\,d.$	$t = .16\,d.$
$k = .25\,d + .125''.$	$u = 1.333\,d.$

$$v = .125\,d.$$

2011. In Fig. 690 is shown a common form of pedestal which is used for somewhat larger journals than the one shown in Fig. 689.

It consists of (1) a foundation plate which is bolted to the foundation on which the pedestal rests; the plate is essential when the pedestal rests on brickwork or masonry, but may be dispensed with when the pedestal rests on the frame of the machine; (2) the block which carries the seats and supports the journal; (3) the cap which is screwed down over

the seats. The bolt holes in both foundation plate and block are oblong, so that the pedestal may be readily adjusted.

The following proportions may be used for this kind of pedestal, with journals from 2" to 6", inclusive. An oil cup

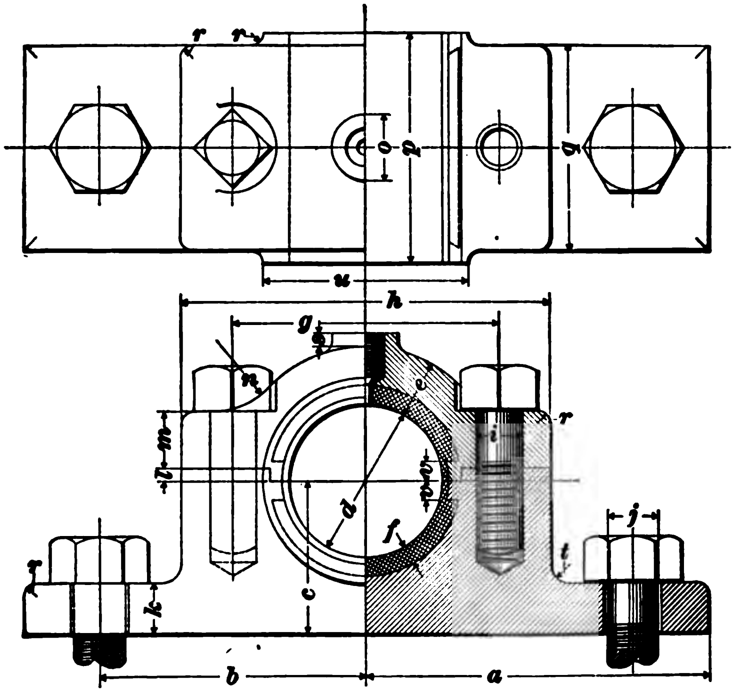
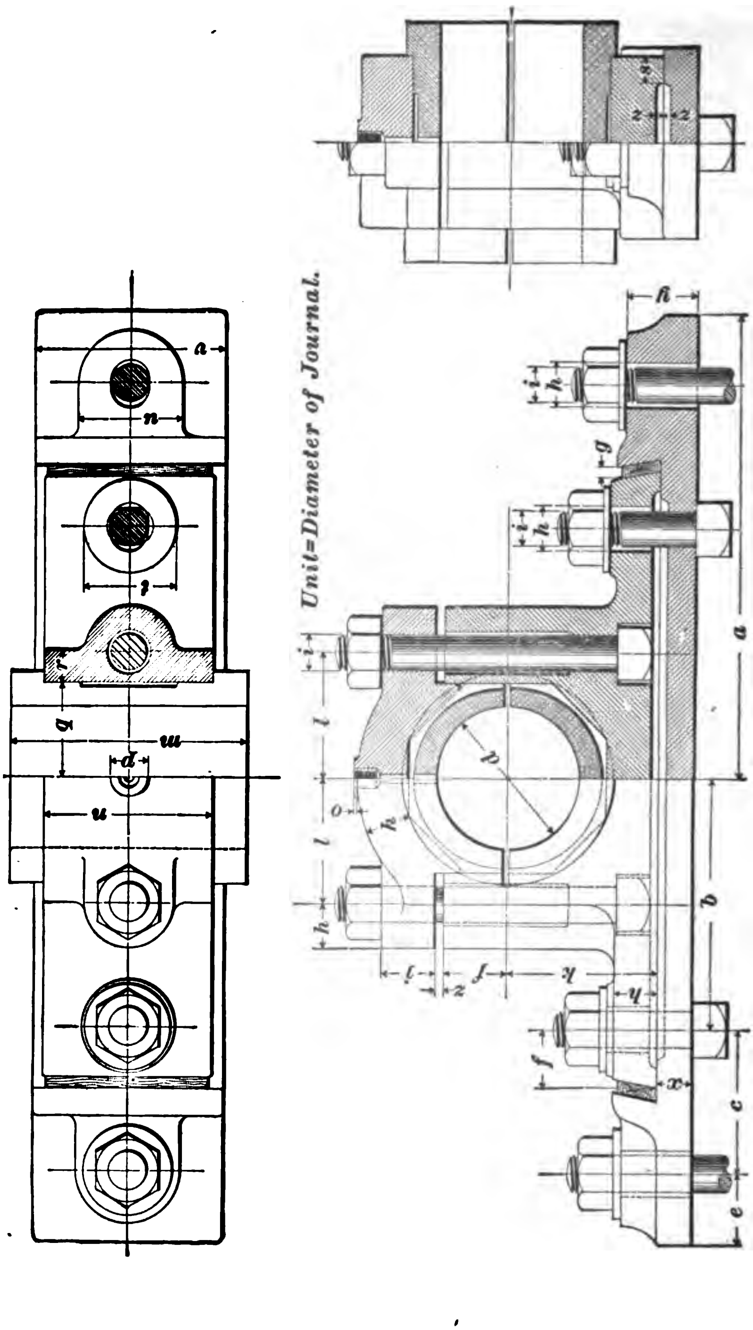


FIG. 689.

having a $\frac{1}{4}$" pipe tap shank may be used on pedestals for journals having diameters from 3" to 4", and $\frac{3}{8}$" pipe tap shank for larger sizes up to 6" diameter.

NOTE.—The shanks of oil cups and grease cups bought in the market are made with a $\frac{1}{8}$", $\frac{1}{4}$", $\frac{3}{8}$", or $\frac{1}{2}$" pipe thread. The amount of oil or grease the cup holds when filled is usually expressed in ounces.



Unit=Diameter of Journal.

FIG. 690.

The diameter of journal $d =$ the unit;

$$
\begin{aligned}
a &= 3.25\,d. \\
b &= 1.75\,d. \\
c &= d. \\
e &= .5\,d. \\
f &= .4375\,d. \\
g &= .09\,d. \\
h &= .3125\,d. \\
i &= .25\,d. \\
j &= .375\,d. \\
k &= 1.0625\,d. \\
l &= .875\,d. \\
m &= 1.75\,d. \\
n &= 1.25\,d. \\
o &= .125'' \text{ (constant).} \\
p &= .875'' \text{ (constant).} \\
q &= .625\,d. \\
r &= .25\,d. \\
s &= .1875\,d. \\
t &= .65\,d. \\
u &= .75\,d. \\
v &= 1.375\,d. \\
x &= .25\,d. \\
y &= .5\,d. \\
z &= .0625\,d.
\end{aligned}
$$

2012. Crank-Shaft Pedestals. — The load on a crank-shaft bearing is due partly to the weight of the shaft and fly-wheel, and partly to the alternate push and pull of the connecting-rod. The direction of the resultant of these two forces is, therefore, more or less oblique, and, consequently, the pedestal is often divided obliquely.

2013. A pedestal of this kind is shown in Fig. 691, which may be proportioned by taking the diameter d of the journal as the unit, and using the following proportions:

The line $A\ B$ is at an angle of 45° to the base line.

This pedestal, as shown, is babbitted. The babbitt is held

in place and prevented from turning with the journal by pro viding the surfaces with which the babbitt comes in contact

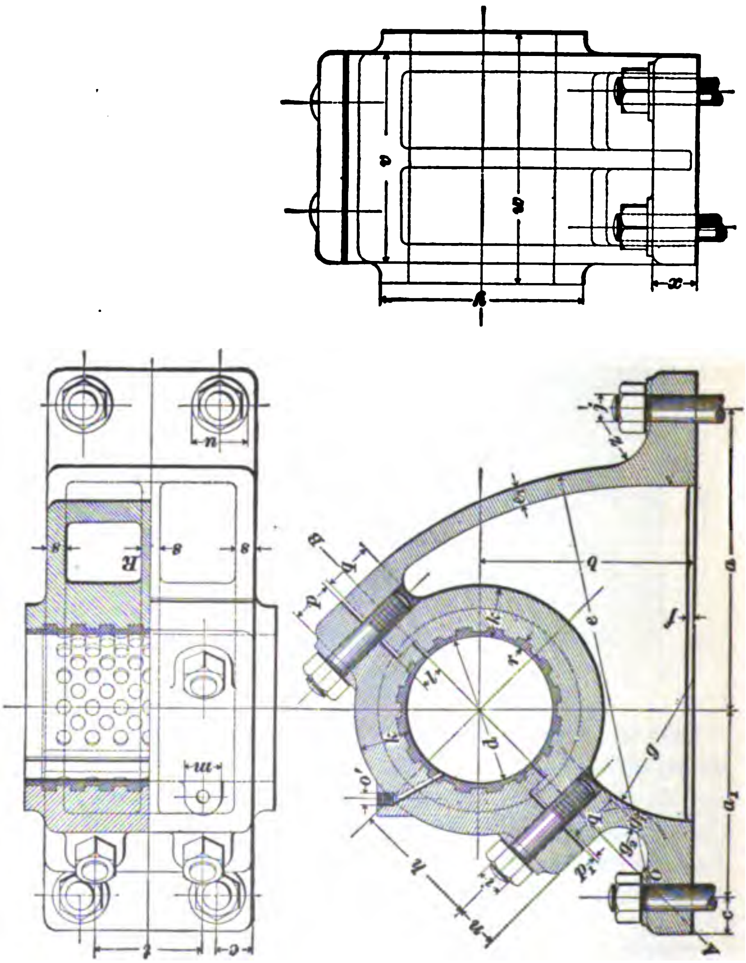


FIG. 691.

with round projections as shown. The projections are about one inch in diameter for the larger sizes of pedestals.

$a = 2d + 1''$.
$a_1 = 1.2d + 1''$.
$b = 1.5d$.
$c = .22d + .5''$.
$e_1 = .1d + .25''$.
$f = .5''$ (constant).
$g_1 = .1d + .25''$.
$h = .9d$.
$i = .15d + .25''$.
$j = .15d + .375''$.
$k = .3d + .625''$.
$l = .1d + .375''$.
$m = .25d$.
$n = .3d$.
$p = .25d + .25''$.
$p_1 = .04d$.
$q = .35d$.
$r = .02d + .3125''$.
$s = .1d + .25''$.
$t = .75d$.
$u = .27d + 1.125''$.
$v = 1.45d$.
$w = 1.75d$.
$x = .25d + .625''$.
$y = 1.3d + 1''$.
$z = .25d + 1''$.

To find the radius e draw a line parallel to the **base line,** and at a distance x above the base line. The point of intersection O of this line with the line $A\ B$ is the center of the arc having the radius e. The radius g_1 is found by trial. The center of the arc must lie on the line $A\ B$. With a radius $g_1 + g_1$ describe an arc from the same center. Draw a line parallel to the face of the bearing at a distance from it determined by q. The radius g is to be found by trial, the center being on the base line, and the arc tangent to the line determined by q and to the arc determined by $g_1 + g_1$. The rib R may be used on all pedestals above 6" diameter of journal. o' may be tapped with a ¼" pipe tap for 3" to 4"

journal, $\frac{3}{8}''$ pipe tap up to 6″ journal, $\frac{1}{2}''$ up to 9″ journal, and $\frac{3}{4}''$ for all sizes above 9″.

Two oil cups may be used on all bearings above 8″ diameter of journal. This style of pedestal may be used for journals from 3″ up to 15″ diameter, inclusive.

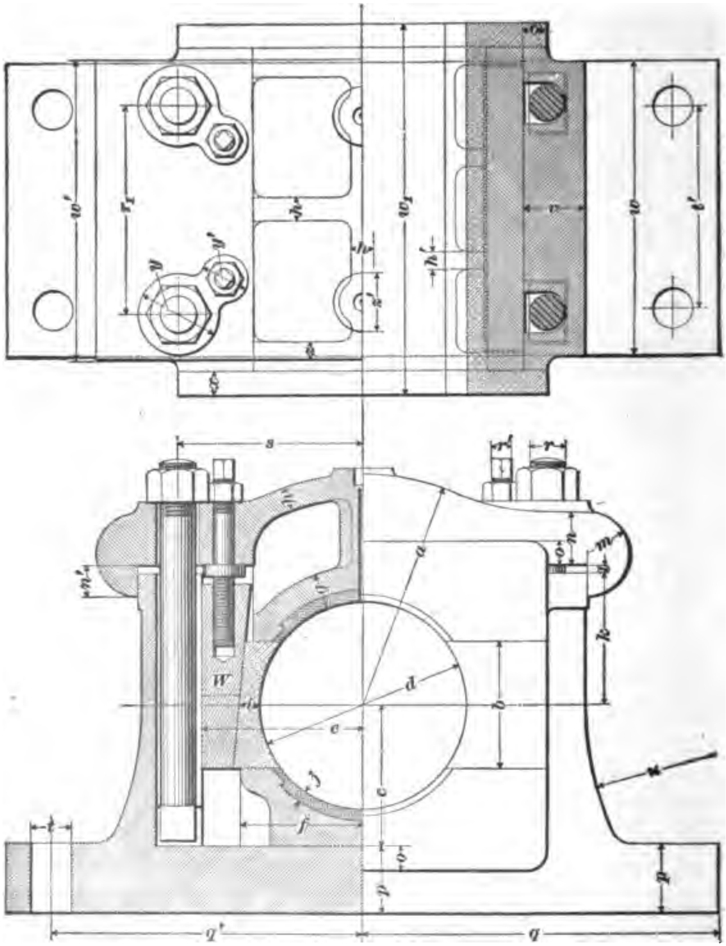


FIG. 692.

2014. Fig. 692 shows a pedestal suitable for the crank-shaft of a horizontal engine with journals from 8″ to 20″ in

diameter. The block may be complete in itself, as shown in the figure, but more often it forms part of the engine bed.

The seats are in three parts, and may be adjusted horizontally by means of the wedges W. The lower seat may be raised by placing packing pieces under it. To obtain its dimensions, use the following proportions, which are based on the unit $d =$ the diameter of the crank-shaft journal:

$a = d + 1''$.	$q' = 1.5\,d$.
$b = .5\,d + 1''$.	$r = .15\,d$.
$c = .66\,d$.	$r' = .1\,d$.
$e = .825\,d - .25''$	$r_1 = d$.
$f = .6\,d$.	$s = .9\,d$.
$g = .1\,d + .5625''$.	$t = .15\,d + .375''$.
$h = .1\,d + .25''$.	$t' = .9\,d$.
$h' = .08\,d$.	$u = 1.5\,d$.
$i = .11\,d$.	$v = .25\,d + .375''$.
$j = .625''$ (constant).	$w = 1.45\,d$.
$k = .5\,d + 1.25''$.	$w' = 1.47\,d$.
$l = .375''$ (constant).	$w_1 = 1.75\,d$.
$m = .175\,d + .3125''$.	$x = .1\,d$.
$n = .25\,d + .25''$.	$y = .3\,d + .75''$.
$n' = .1\,d + .375''$.	$y' = .2\,d + .5''$.
$o = 1''$ (constant).	$z = .09\,d$.
$p = .25\,d + .625''$.	$z' = 2.5''$ (constant).
$q = 1.75\,d$.	

Taper of adjusting wedge, 1 : 10.

Further details of the bottom seat and the cap are shown in Fig. 693, in which the unit is the same as in Fig. 692, and the proportions are as follows:

$a = 1''$ (constant).	$c = .08\,d$.
$b = 1.65\,d - .5''$.	$d = .1\,d$.

2015. The foundation, or bed casting, is shown in Fig. 694, and has dimensions to suit the pedestal shown in Fig. 692. Its proportions are as follows, the diameter of the crank-shaft journal d being taken as the unit:

$a = 2.45\,d + 7.25''$.
$b = 2.3\,d + 5.25''$.
$c = .5\,d + 3.5''$.
$e = 3.5\,d + 2''$.
$f = .25\,d + .5''$.
$g = .25\,d + 1.75''$.
$h = .25\,d + 2.25''$.
$n = 1.55\,d + 2.5''$.
$o = .25\,d + 2''$.
$o' = .25\,d + .5''$.
$o'' = .5\,d + 4.5''$.
$p = .08\,d$.
$q = 1.5\,d$.
$r = .15\,d + .375''$.

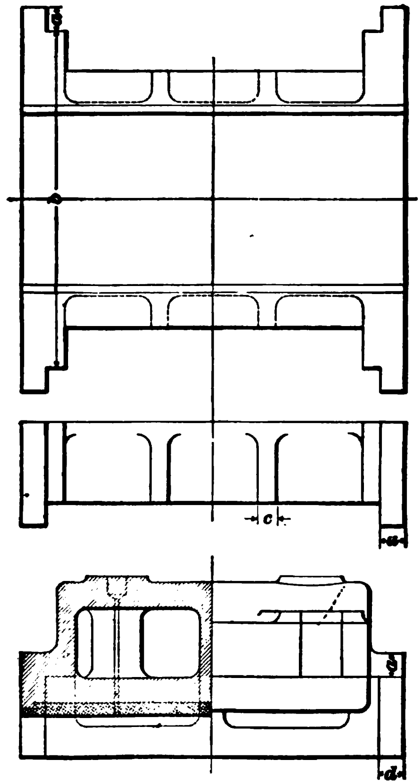


FIG. 693.

$i = .05\,d + .5''$.
$j = .05\,d + 1.125''$.
$k = .05\,d + .75''$.
$l = .25\,d + .75''$.
$m = .4\,d$.
$m' = .6\,d$.
$s = .15\,d + .375''$.
$t = .9\,d$.
$u = .15\,d + .875''$.
$v = .2\,d$.
$w = 1.5\,d$.
$x = 1.65\,d$.

2016. Ball-and-Socket Bearings.—These bearings, now largely used in the United States, were first introduced by William Sellers & Co., of Philadelphia. They have very long steps made of *cast iron* bored to fit the journal. In some cases, however, the steps are cast with a recess in which a babbitt lining is poured to form a bearing surface for the journal. The steps have a spherical enlargement at the center, which fits into corresponding hollows in the block and cap, thus making a ball-and-socket joint, which leaves

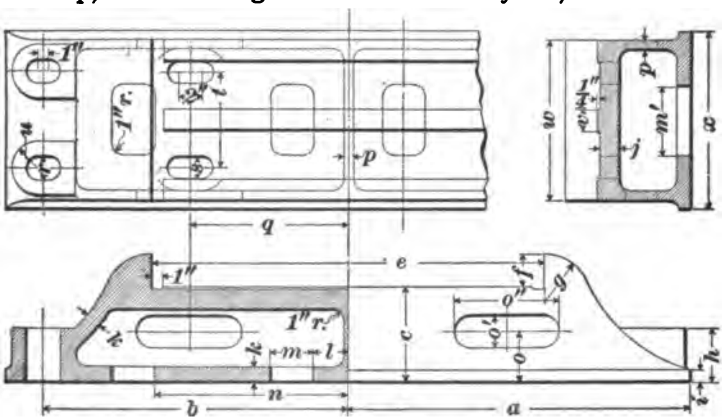


FIG. 694.

the bearing free to move slightly in any direction to conform to an inequality or want of alinement in the shaft. When rigid bearings are used, they must necessarily be short on account of unavoidable deflections of the shaft due to belt pull, thrust of gearing, etc. Pivoted bearings, on the contrary, by reason of their flexibility, may be made long, thereby giving a large wearing surface and increased durability.

2017. Fig. 695 shows a pedestal with ball-and-socket bearing. The steps are of cast iron, and have a length of four times the diameter of the journal.

Ordinarily the journal is lubricated through the oil hole in the center of the cap, but the upper step also has two cups, filled with a mixture of tallow and oil which melts at about 100° F. If the bearing becomes heated this mixture melts, and thus helps in the lubrication.

The dimensions of this bearing are obtained from the following proportions, which are based on the diameter of the journal d as the unit:

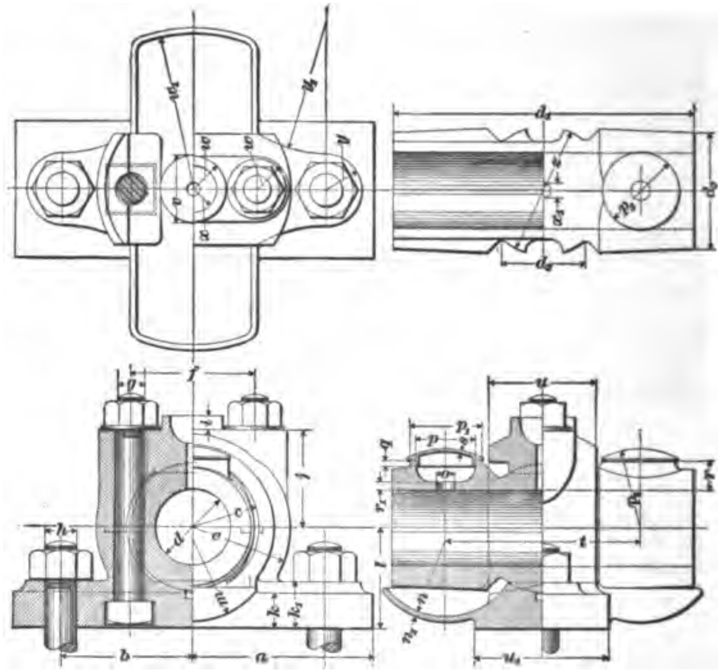


FIG. 695.

$a = 2.375\,d.$
$b = 1.75\,d.$
$c = .875\,d.$
$d_1 = 4\,d.$
$d_2 = 1.1\,d.$
$d_3 = 1.25\,d + .5''.$
$e = 1.3\,d.$
$f = 1.65\,d.$
$g = .2\,d + .25''.$
$h = .2\,d + .375''.$
$i = .1875\,d.$
$j = 1.27\,d.$
$k = .3\,d + .25''.$
$k_1 = .4\,d + .375''.$
$l = 1.33\,d.$
$m = d + .125''.$
$n = d.$
$n_1 = .125''$ (constant).
$o = .25\,d.$
$p = .8\,d.$
$p_1 = .95\,d.$
$p_2 = d.$
$p_3 = d.$
$q = .07\,d.$
$r = .4\,d.$
$r_1 = .1\,d.$

$s = .0625''$ (constant).	$w_1 = 2.06\,d$.
$t = 2.5626\,d$.	$x = .18\,d + 2.5''$.
$u = 1.45\,d$.	$x_1 = .1875\,d$.
$u_1 = 1.8\,d$.	$y = .25\,d + .375''$.
$v = .875\,d$.	y_1 to be found by trial.
$w = .2\,d + .375''$.	$z = 1.5\,d + .3125''$.

WALL BRACKETS AND HANGERS.

2018. Wall Brackets, or Post Hangers.—A shaft must sometimes be supported by a bearing fixed to a wall or pillar. In such a case the bearing is generally supported by a **wall bracket.** It will readily be seen that the bracket is a cantilever with practically a uniform load, due to its own weight, and a concentrated load, due to the weight of the shaft; hence, the bending moment diagram resembles that shown in Fig. 332, Art. **1391.**

Now, in order that a cantilever should be equally strong at all sections, the resisting moment, $\frac{SI}{c}$, of that section should be proportional to the bending moment at that section. Suppose the bracket in question to be a plate of a constant thickness b. Then, at any section, the bending moment $M = \frac{SI}{c} = \frac{S \frac{1}{12} b h^2}{\frac{1}{2} h} = \frac{1}{6} S b h^2$.

Since $\frac{1}{6} S b$ remains constant for all sections, the h^2 is proportional to the bending moment; or, in other words, the height of a cantilever at any section should be proportional to the square root of the bending moment at that section. It can be shown mathematically that, supposing the cantilever to have a constant width, it should have a parabolic form when the load is concentrated at the end, and it should have a triangular form when the load is uniformly distributed. In practice, a bracket or other machine part of the cantilever class is given a form which is often neither exactly parabolic nor triangular, but approximates closely to one or the other, according to the taste and judgment of the designer.

2019. A good design for a wall bracket is shown in Fig. 696. The bearing proper resembles that of the pedestal, Fig. 695. The spherical enlargement at the center

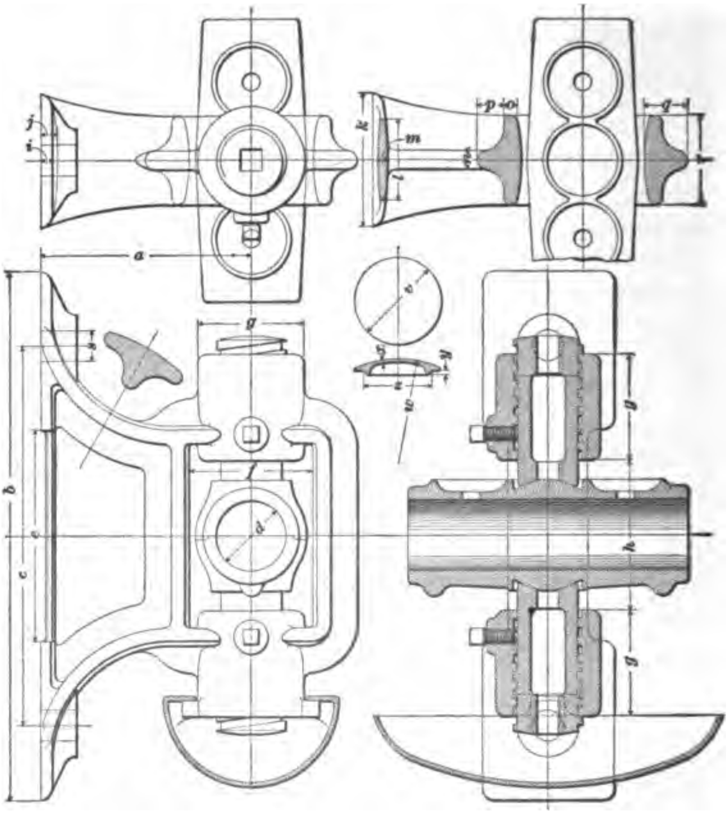


FIG. 696.

of the steps is held between the ends of two hollow cast-iron plugs, which are threaded at their outer ends, and screw into bosses cast on the bracket.

The bearing may be raised or lowered by screwing the plugs up or down. In order to turn the plugs the hole at the end is made square to receive a key. When the plugs are in the desired position they are locked by the set-screws.

The length of the bearing is 4 times the shaft diameter; the other dimensions are to be obtained from the figure and by the following proportions:

Diameter of shaft $d =$ unit;

$a = 3d.$	$k = 1.875d.$
$b = 7.5d.$	$l = 1.125d.$
$c = 5.375d.$	$m = .125d.$
$e = 3d.$	$n = .25d.$
$f = 1.75d.$	$o = .1875d.$
$g = 1.5d.$	$p = .375d.$
$h = 2.125d.$	$q = .625d.$
$i = .16d.$	$r = 1.25d.$
$j = .25d.$	$s = .25d + .5''.$

Proportions of oil or grease cup covers:

$v = 1.125d + .1875''.$	$y = .0625d.$
$w = 1.5d.$	$z = d - .0625''.$
$x = .125''$ (constant).	

The proportions of boxes, bosses, plugs, drip pans, etc., are to be made the same as those of the shaft hanger shown in Fig. 697.

2020. Hangers.—A hanger is used when a shaft bearing is to be suspended from the ceiling. Fig. 697 shows a form of a hanger made by a leading manufacturing company.

The bearing is the same as that of the wall bracket, Fig. 696. The frame of the hanger is divided, and the parts connected by bolts. With such a form the shaft may be more easily removed than when the hanger frame is a solid piece.

The units for determining the leading dimensions of a shaft hanger are the diameter d of the shaft and the drop D of the hanger.

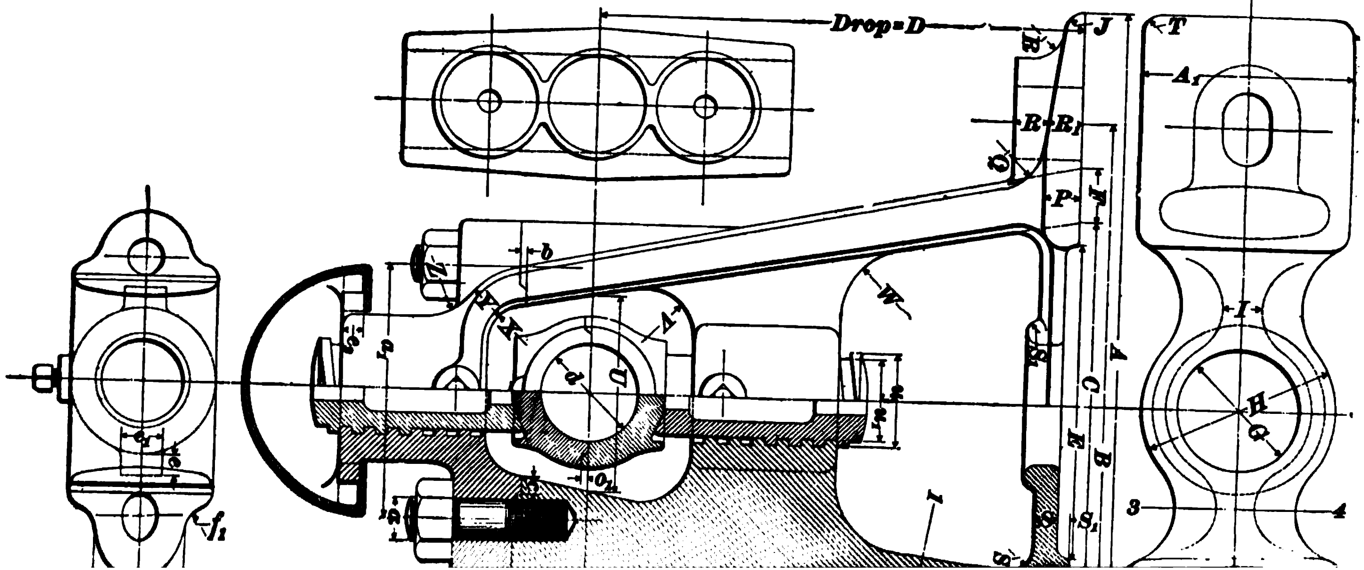
The following proportions are suitable for shafts ranging from $1\frac{1}{2}''$ up to $4\frac{1}{2}''$ in diameter:

$A = 6d + .45D.$	$E = 2d + .25D.$
$A_1 = 2d + .03D.$	$F = .5d + .01D.$
$B = 4d + .35D.$	$F_1 = 1.5d + .05D.$
$C = 2d + .3D.$	$G = 1.25d.$

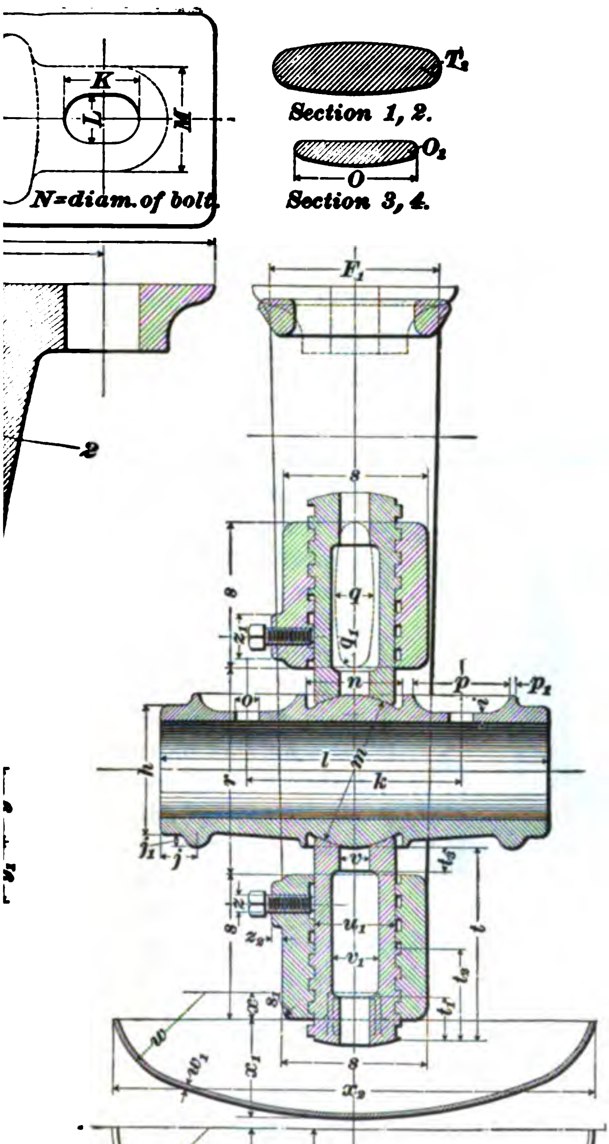
$H = .2\,d.$
$I = .4\,d.$
$J = .125\,d + .01\,D.$
$K = .5\,d + .5''.$
$L = .25\,d + .5''.$
$M = .75\,d + .6875''.$
$N = .25\,d + .375''.$
$O = 1.25\,d.$
$O_1 = .094\,d + .002\,D.$
$P = .375\,d + .008\,D.$
$Q = .375\,d + .008\,D.$
R and R_1 (see note).
$S = .25\,d + .005\,D.$
$S_1 = .125\,d + .003\,D.$
$T = .125\,d + .01\,D.$
$T_1 =$ (see note).
$U = 2\,d.$
$V = .5\,d.$
$W = .75\,d.$
$X = .375\,d.$
$Y = .25\,d + .125''.$
$Z = .625\,d.$
$a = .15\,d + .375''.$
$a_1 = 2.4\,d + .3125''.$
$b = .08\,d.$
$c = .125\,d + .0625''.$
$e = .2\,d.$
$e_1 = .4\,d.$
$e_2 = .2\,d.$
$f = .375\,d + 1''.$
$f_1 = .09\,d + .25''.$
$g = .75\,d.$
$g_1 = 1.3125\,d + .125''.$
$h = 1.25\,d + .1875''.$
$i = .1\,d.$
$j = .25\,d + .25''.$
$j_1 = .125\,d + .0625''.$
$k = 2.2\,d.$
$l = 4\,d.$
$m = 1.4\,d + .375''.$
$n = d.$
$o = .25\,d.$
$o_1 = .0625\,d.$
$p = d.$
$p_1 = .0625\,d.$
$q = .4\,d.$
$q_1 = .15\,d.$
$r = 2.125\,d.$
$s = 1.5\,d.$
$s_1 = .125\,d.$
$t = 2\,d.$
$t_1 = .5\,d.$
$t_2 = d.$
$t_3 = .25\,d.$
$u = .95\,d.$
$u_1 = .85\,d.$
$v = .25\,d + .125''.$
$v_1 = .5\,d.$
$w = d.$
$w_1 = .125''$ (constant).
$x = .25\,d.$
$x_1 = d.$
$x_2 = 4\,d + 2''.$
$y = 1.25\,d.$
$y_1 = .75\,d + .0625''.$
$y_2 = .4\,d + .0625''.$
$z = .06\,d + .25''.$
$z_1 = .12\,d + .75''.$
$z_2 = .3125''$ (constant).

Thread of plugs, $.5''$ pitch for all sizes.

NOTE.—To find R_1, draw the arc J; also, draw the arc Q tangent to P; then, draw a straight line tangent to these arcs, and R_1 will be



F



N=diam. of bolt.
K
L
M
T_2
Section 1, 2.
O_2
O
Section 3, 4.
F_1
2
8
q
q_1
z_1
n
p
p_1
o
h
l
m
r
k
j_1
j
v
d_2
z
u_1
v_1
z_2
t
t_2
t_1
s_1
x
w
w_1
x_1
x_2
y_1
y_2
y

697.

the distance along the center line determined by B included between this tangent and the upper face of the hanger. Having found R_1, make R equal to it.

The radius T_1 is made equal to ⅜ the thickness at the middle.

The steps of the ball-and-socket bearings shown in Figs. 695, 696, and 697 are of cast iron, and are bored to fit the journal without lining or brasses. The ball, and the recesses in the ends of the plugs, into which the ball is fitted, should be faced. The screw threads on the plugs, Figs. 696 and 697, may be cast on the plugs or turned, the latter being preferable. It is customary to use two threads per inch for all sizes of plugs.

2021. Pivot or Foot-step Bearings.—These bearings are used to support the ends of vertical shafts. An ordinary pivot bearing is shown in Fig: 698. The end of the pivot rotates on a disk A, which may be of steel, brass, or bronze. The brass bush B prevents the pivot from moving sidewise. The end of the pivot should be of steel, and it may be flat on the end or slightly cup shaped. The proportions are given in terms of the diameter d of the pivot as a unit.

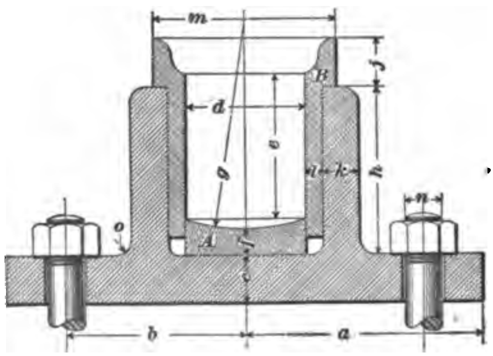


FIG. 698.

Its proportions are as follows:

$a = 2d.$	$j = .4d.$
$b = 1.5d.$	$k = .3d.$
$c = .25d + .375''.$	$l = .15d.$
$e = 1.25d.$	$m = 1.5d.$
$f = .2d + .125''.$	$n = .2d + .25''.$
$g = 1.75d.$	$o = .15d.$
$h = 1.4d.$	

2022. Pivots with Loose Disks.—When pivots are required to run at great speed, the relative speed of the surfaces in contact may be reduced by placing a number of loose disks between the pivot and the foot-step, as shown in Fig. 699.

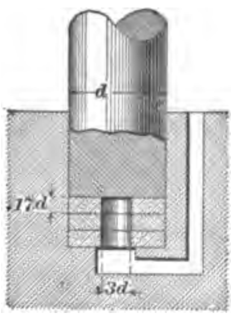


FIG. 699.

If $a =$ the number of disks and $n =$ the revolutions per minute of the pivot, then the relative speed of any of the surfaces in contact is $\frac{n}{a+1}$ revolutions per minute. Suppose, for example, a pivot runs at 3,000 revolutions per minute, and it is desired to reduce the speed between adjacent surfaces to 750 revolutions per minute; then, $750 = \frac{3,000}{a+1}$, or $a = 3$, the number of disks required. The pivot and disks may be lubricated by a channel, as shown in the figure. The disks may be made of gun-metal or phosphor-bronze. Proportions are given in the figure.

TOOTH GEARING.

SPUR AND BEVEL GEARS.

NOTE.—Before reading this section, the student should carefully review the portion of Applied Mechanics which relates to gear teeth.

2023. Materials of Gearing.—Gearing is ordinarily made of cast iron. If great strength is required steel may be used. Gears which are called upon to resist shocks may be made of gun-metal or phosphor-bronze. Fast-running gears are sometimes made with *wooden cogs*, or of *rawhide*, or *fiber*, instead of *metal*.

Gear-wheels are either cast with teeth and all complete, or with a blank rim in which the tooth spaces are afterwards cut with a milling cutter. Gears with cast teeth are less expensive; those with cut teeth are more accurate. Gear-cutting machines are now made, however, which can easily

cut teeth on both spur and bevel gears of the largest size; hence, cut gears are quite generally used, except on very large or rough work.

STRENGTH OF GEAR TEETH.

2024. A gear tooth may be considered as a cantilever whose length is l, depth, t, and breadth, b, see Fig. 700.

Let p = the pressure acting at the pitch line, and, as an extreme case, let the total pressure come on the edge of one tooth, as shown in the figure. (Usually the pressure will be divided among two or three teeth.)

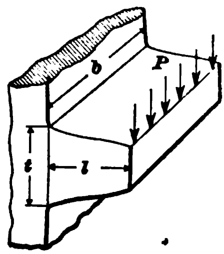


FIG. 700.

Then, the bending moment $= p\,l$.

The resisting moment $= \dfrac{S I}{c} = S\dfrac{\frac{1}{12} b t^2}{\frac{1}{2} t} = \dfrac{S b t^2}{6}$.

Therefore, $$p\,l = \frac{S b t^2}{6}. \qquad (a)$$

The thickness t at the root of the tooth may be taken equal to $\frac{1}{2}\,C$, and $l = .7\,C$ from the proportions given in Table 34, Art. **1555.** C = the circular pitch.

Substituting these values in equation (a), we obtain

$$.7\,C p = \frac{S b\,(\frac{1}{2}\,C)^2}{6} = \frac{S b C^2}{24}.$$

Or, $$b\,C = 16.8\frac{p}{S}. \qquad \textbf{(266.)}$$

The value of the working stress S depends upon the velocity of the pitch circle of the gear, since, as the speed increases, the teeth become more liable to shocks.

Let v = velocity of a point in the pitch circle in feet per minute. Then, if v is equal to or less than 100, take S = 4,200 lb. per sq. in. for cast iron.

If v is greater than 100 feet per minute, the safe stress S may be obtained from the following formula:

$$S = \frac{9,600,000}{v + 2,160}. \qquad \textbf{(267.)}$$

To find the pitch C from formula **266,** the relation between b and C must be known. The following proportions are those generally used in practice:

For wheels moving slowly or intermittently, as in hoisting apparatus

$$b = 2\,C \text{ to } 2\tfrac{1}{2}\,C.$$

For more rapidly moving cast gears, for example, transmitting gears,

$$b = 2\tfrac{1}{2}\,C \text{ to } 3\,C.$$

For gears moving more rapidly, and with cut teeth,

$$b = 3\,C \text{ to } 3\tfrac{1}{2}\,C.$$

For very rapidly moving gears with small pitch,

$$b = 3\tfrac{1}{2}\,C \text{ to } 4\,C.$$

EXAMPLE.—A cast gear 3 feet in diameter makes 40 revolutions per minute, and transmits 16 horsepower. Determine the circular pitch of the teeth, and number of teeth.

SOLUTION.—R = radius of gear = 18 inches.

From formula **231,** Art. **1963,**

$$p = \frac{63{,}025\,H}{n\,R} = \frac{63{,}025 \times 16}{40 \times 18} = 1{,}400 \text{ lb.}$$

From formula **267,**

$$S = \frac{9{,}600{,}000}{v + 2{,}160} = \frac{9{,}600{,}000}{3\,\pi \times 40 + 2{,}160} = 3{,}784 \text{ lb. per sq. in.}$$

Hence, from formula **266,**

$$b\,C = 16.8\,\frac{p}{S} = \frac{16.8 \times 1{,}400}{3{,}784} = 6.2.$$

Assume $b = 2\tfrac{1}{2}\,C$. Then, $b\,C = 2\tfrac{1}{2}\,C^2 = 6.2$ sq. in., or $C = 1.575$ in.

Taking $C = 1.5708$ in., the diametral pitch is exactly 2, and the number of teeth $36 \times 2 = 72$. Ans.

Hence, $C = 1.5708$. Ans.

$$P = \frac{\pi}{C} = 2.$$

Number of teeth $N = D\,P = 36 \times 2 = 72$. Ans.

Breadth of tooth $b = 2\tfrac{1}{2}\,C = 3.93''$, say $3\tfrac{15}{16}''$.

In calculating cast-steel gears, or gears with wooden cogs, the above method is applicable, but the value of S should be multiplied by $3\tfrac{1}{2}$ for steel teeth, and by .6 for wooden cogs.

2025. Wear of Teeth.—Though gear teeth may be amply strong, they may, if poorly designed, wear rapidly. To ensure durability, the dimensions of the tooth should fulfil the following conditions:

Let p = pressure acting at pitch line;
b = breadth of tooth;
n = number of revolutions per minute;
H = horsepower transmitted;
D = diameter of gear in inches.

Then, $$\frac{pn}{b} = 28{,}000, \text{ or } b = 4\tfrac{1}{2}\frac{H}{D}. \qquad \textbf{(268.)}$$

In the example above, we have $4\frac{1}{2}\frac{H}{D} = \frac{4\frac{1}{2} \times 16}{36} = 2$ in., and since the breadth calculated for strength is $3\frac{1}{16}''$, the gear will wear well. It is usually necessary to apply formula **268** to pinions of small diameter.

2026. Shrouded Gear Teeth.—The teeth of a gear-wheel are said to be **shrouded** when the rim is made wider

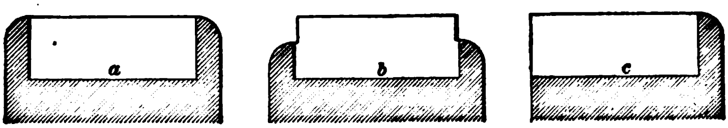


Fig. 701.

than the tooth, and carried outwards so as to unite the ends of the teeth.

Three methods of shrouding are shown in Fig. 701. At *a* the shrouding is carried on both sides to the end of the tooth. In this case it is evident that only one of the pair of wheels gearing together can be shrouded. At *b* the shrouding is carried to the pitch line, and, consequently, both wheels of the pair may be shrouded. At *c* the shrouding is carried up on one side only.

Pinions with few teeth are most benefited by shrouding, since the teeth in that case are weak at the roots, and also because more wear comes on the pinion.

PROPORTIONS OF SPUR AND BEVEL GEAR WHEELS.

2027. Rims.—Various rim sections are shown in Fig. 702. It is good practice to make the rim thickness c equal to the tooth thickness on the pitch circle for gears

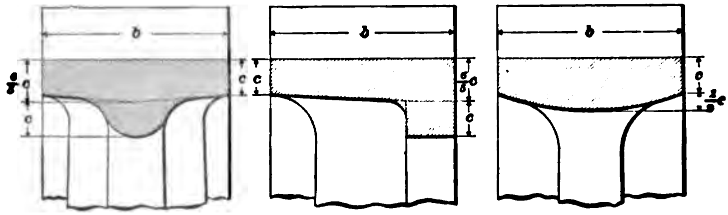


FIG. 702.

whose circumferential pitch is greater than $1\frac{1}{2}$ inches. For gears with small pitch, we may take

$$c = .4\,C + \tfrac{1}{8}'' = \frac{5}{4\,P} + \tfrac{1}{8}''. \qquad \textbf{(269.)}$$

EXAMPLE.—Calculate the thickness of the rim of a gear-wheel whose diametral pitch is 4.

SOLUTION.—By formula **269**,

$$c = \frac{5}{4\,P} + \tfrac{1}{8}'' = \tfrac{5}{16} + \tfrac{1}{8} = \tfrac{7}{16}''. \quad \text{Ans.}$$

2028. The rim of a bevel wheel may have the proportions shown in Fig. 703, where c is to be computed by the rules given in the last article.

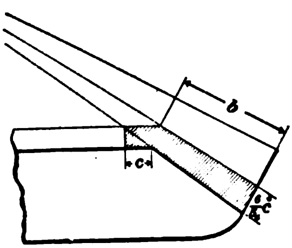


FIG. 703.

2029. When wooden cogs are mortised into a gear-wheel, the rim may have the proportions shown in Fig. 704.

Usually only one of the wheels in a pair that gear together is furnished with wooden teeth, or cogs, in which case the wooden cogs may be made $1\frac{1}{2}$ times as thick as the iron teeth meshing with them; that is,

for the wooden cogs, $t = .6\,C$;
for the iron teeth, $t = .4\,C$, or less.

Fig. 704 shows two common methods of fastening the cogs to the rim.

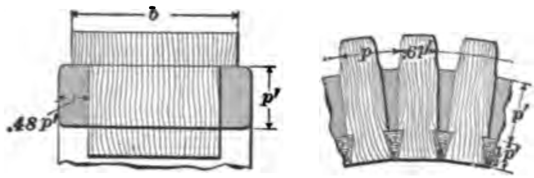


FIG. 704.

2030. Arms of Wheels.—The form of the cross-section of the arms of a gear-wheel depends upon the form

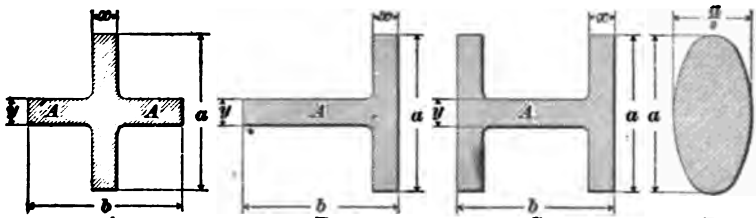


FIG. 705.

of rim used. The forms in ordinary use are shown in Fig. 705. The cross-shaped form A is the one mostly used for spur gears; and the section shown at B, for bevel gears. The section shown at C is good for heavy gears, while the oval form D is a neat form for light gears.

In all of the above forms the dimension a lies in the plane of the wheel, and the dimension b at right angles to this plane, or parallel to the axis of the wheel.

In calculating the strength of the arm, the ribs A, A are not taken into account, as they are intended to give the arm lateral stiffness, and not to add strength.

Let s = number of arms in wheel;
a = width of arm, measured at center of wheel;
x = thickness of arm;
S_1 = allowable safe stress in arm;
R = radius of wheel in inches;
p = pressure between gears at pitch line.

We may consider each arm as fixed at the center of the wheel, and free at the end, which, though not strictly true, is on the safe side. Then, the total bending moment is pR inch-pounds, and the bending moment on each arm is $\frac{pR}{z}$. The moment of resistance is

$$\frac{S_1 I}{c} = S_1 \frac{\frac{1}{12} a^2 x}{\frac{1}{2} a} = \frac{1}{6} S_1 a^2 x.$$

Hence, $$\frac{pR}{z} = \frac{1}{6} S_1 a^2 x.$$

To avoid strains and breakage in casting, all parts of the wheel should be of nearly the same thickness; therefore, the thickness of the arm may be made about equal to the thickness of the tooth or of the rim.

That is, $$x = \frac{1}{2} C.$$

But, $$C = \frac{16.8 p}{b S}. \quad \text{(See formula 266.)}$$

Hence, $$x = \frac{1}{2} C = \frac{8.4 p}{b S};$$

or, $$\frac{pR}{z} = \frac{1.4 S_1 p a^2}{b S};$$

whence, $$a^2 = \frac{p R b S}{1.4 S_1 p z} = \frac{R b S}{1.4 S_1 z}.$$

Owing to the initial strain set up in the contraction of the casting, S_1 should have a low value, say 3,000 lb. S may have the value given above; that is, 4,200 lb.

Then, $$a^2 = \frac{R b \times 4{,}200}{1.4 \times 3{,}000 \times z} = \frac{R b}{z};$$

or, $$\left.\begin{array}{l} a = \sqrt{\dfrac{R b}{z}} \\ x = \frac{1}{2} C. \end{array}\right\} \quad \textbf{(270.)}$$

For a case in which the proportions given by formula **270** do not give an arm whose appearance is satisfactory, the value of x may be varied a little in either direction, and a new value of a may then be calculated so as to give the required moment of resistance.

EXAMPLE.—A gear-wheel $2\frac{1}{2}$ feet in diameter, with six arms, has a pitch of 2 inches, and the breadth of tooth is 5 inches. Find the dimensions of the arm.

SOLUTION.—Use formula **270.**

$$a = \sqrt{\frac{R\,b}{z}} = \sqrt{\frac{15 \times 5}{6}} = \sqrt{12\frac{1}{2}} = 3\frac{1}{2}'', \text{ nearly. Ans.}$$

$$x = \tfrac{1}{2}C = 1''. \text{ Ans.}$$

For arms of oval or elliptical cross-section, the moment of resistance is $\frac{S_1 I}{c}$, which equals $\frac{S_1 \frac{\pi a^3 (\frac{1}{2}a)}{64}}{\frac{1}{2}a}$. (See Table of Moments of Inertia).

Hence, $$\frac{pR}{z} = \frac{S_1 \pi a^3}{64}.$$

Combining this equation with formula **266,** we obtain, after reducing,

$$\left.\begin{aligned} a &= 1.2\sqrt[3]{\frac{b\,C\,R}{z}}. \\ \text{Or,}\quad a &= 1.75\sqrt[3]{\frac{b\,R}{Pz}}. \end{aligned}\right\} \qquad \textbf{(271.)}$$

EXAMPLE.—A gear-wheel 12 inches in diameter has 48 teeth and 4 arms. The rim is $2\frac{1}{2}$ inches wide. Supposing the arms to be of elliptical cross-section, find their dimensions, the thickness being half the breadth.

SOLUTION.—Using formula **271,**

$$a = 1.75\sqrt[3]{\frac{b\,R}{P\,z}} = 1.75\sqrt[3]{\frac{2\frac{1}{2} \times 6}{\frac{11}{8} \times 4}} = 1.70''.$$

$$a = 1\frac{23}{32}''. \text{ Ans.}$$

$$\tfrac{1}{2}a = \tfrac{7}{8}'', \text{ nearly. Ans.}$$

Formulas **270** and **271** give the width of arm measured at the center of wheel. The arm is tapered from center to circumference. For small gears the taper may be $\frac{1}{32}$ on each side; for larger gears, $\frac{1}{48}$ on each side. The thickness x remains constant, but in the elliptical form the arm is tapered in both width and thickness, so that the latter is constantly equal to half the former.

The average thickness of the stiffening ribs A, Fig. 705, may be .4 C. The ribs are tapered slightly to allow the pattern to be easily drawn from the mold. At the hub the

width of the rib may be b, the same as the length of the tooth. The rib is tapered so that at the rim the width is $\frac{3}{4}b$ to $\frac{7}{8}b$.

The number of arms to be used in a given gear-wheel is largely a matter of judgment. Reuleaux gives the following formula:

$$z = .55\sqrt[4]{N^2 C}, \qquad \textbf{(272.)}$$

where z is the number of arms; N, the number of teeth, and C, the circular pitch.

EXAMPLE.—How many arms should be given a gear 4 feet in diameter, diametral pitch $1\frac{1}{2}$?

SOLUTION.—$N = 4 \times 12 \times 1\frac{1}{2} = 72$.

$$C = \frac{\pi}{P} = \frac{\pi}{1.5} = 2.09''.$$

Hence, from formula **272**,

$$z = .55\sqrt[4]{72^2 \times 2.09} = 5.61.$$

Therefore, use 6 arms. Ans.

If the formula gives an odd number of arms, the nearest even number may be used, if desired.

2031. Hubs or Naves of Gear-Wheels.—The thickness of the hub is often made equal to the radius of the shaft on which the wheel is placed. If the shaft is enlarged for the wheel seat, the thickness of the hub is made equal to the radius of the main portion of the shaft. Then, if d represents the diameter of the shaft, $1.2\,d =$ the diameter of the enlarged portion, and $\frac{1}{2}d$ is the thickness of the hub.

The above rule is very generally used, and gives good proportions when the wheel transmits the full power of the shaft. If, however, a gear transmits only a fraction of the power of the shaft, the hub thickness may be calculated from the following formula, which may also be used for finding the thickness of the hub of any gear-wheel:

$$w = \frac{1}{3}\sqrt[3]{b\,C\,R}, \qquad \textbf{(273.)}$$

where w is the thickness of the hub, and b, C, and R the same as in the previous formulas.

The length of the hub varies from b to $1.4\,b$.

The hubs of large and heavy wheels may be split as shown in Fig. 706. This relieves the hub from the initial strains due to unequal contraction in cooling.

Strips of metal are placed in the slots, and the segments

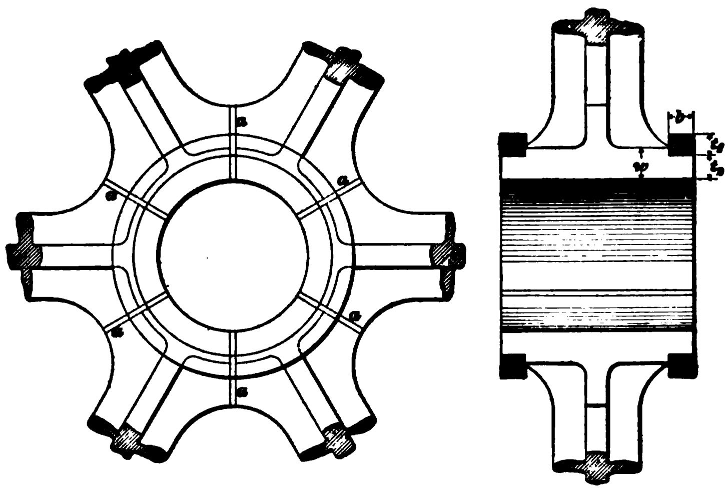


FIG. 706.

are held together by iron or steel bands shrunk on. The proportions are as follows:

$$t_1 = \tfrac{3}{8}\,w;\ t_2 = \tfrac{1}{4}\,w;\ b = \tfrac{7}{12}\,w.$$

2032. Built-Up Wheels.—For convenience in casting, and also for convenience in transportation, large and heavy wheels are made in sections, which are assembled and bolted together. The division of the wheel may be made in various ways. The hub and arms may be cast separately, and the rim in segments; the hub and arms may be cast together, and the rim in segments; or, finally, each division may include a portion of the hub, an arm, and a segment of the rim. Fig. 707 shows one method of bolting the rim, segments, and arm.

Fig. 708 shows a method in which the rim and hub of the gear are parted and held together by bolts.

EXAMPLE.—Compute the leading dimensions for a spur wheel to work under the following conditions:

Diameter of pitch circle = 5 ft. 6 in.

Revolutions per minute = 45.

Horsepower transmitted = 240.

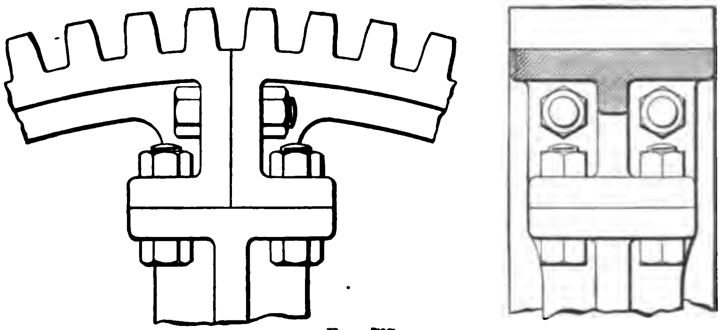


FIG. 707.

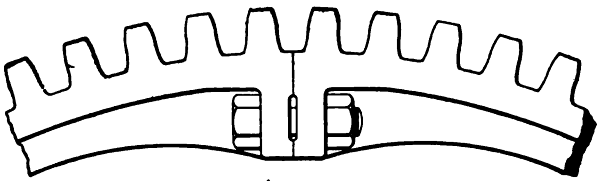


FIG. 708.

SOLUTION.—The velocity of a point in the pitch circle is $\pi D n = 3.1416 \times 5\frac{1}{2} \times 45 = 778$ ft. per minute.

Hence, from formula **267,**

$$S = \frac{9,600,000}{778 + 2,160} = 3,270 \text{ lb. per sq. in., in round numbers.}$$

From formula **231,** Art. 1963,

$$p = \frac{63,025\,H}{R\,n} = \frac{63,025 \times 240}{33 \times 45} = 10,186 \text{ lb.}$$

Now, from formula **266,**

$$b\,C = 16.8\,\frac{p}{S} = \frac{16.8 \times 10,186}{3,270} = 52.3 \text{ sq. in.}$$

$$\text{Assume } b = 3\,C;$$

$$\text{Then, } 3\,C^2 = 52.3;$$

$$C = 4.17''.$$

$$\text{Number of teeth} = \frac{\text{circumference of wheel}}{4.17} = \frac{3.1416 \times 66}{4.17} = 49.7.$$

Taking 50 for the number of teeth, we have

$$C = \frac{3.1416 \times 66}{50} = 4.147 \text{ inches.}$$

Breadth of face $= b = 3C = 3 \times 4.147 = 12.44 = 12\frac{7}{16}''$. Ans.
Thickness of tooth $= t = .475\,C = .475 \times 4.147 = 1.97''$ Ans.
Height above pitch line $= .3\,C = .3 \times 4.147 = 1.24''$. Ans.
Depth below pitch line $= .4\,C = .4 \times 4.147 = 1.66''$. Ans.

For the size of the shaft we may use formula **123,** Art. **1415.**

$$d = 4.92\sqrt[4]{\frac{H}{n}} = 4.92\sqrt[4]{\frac{240}{45}} = 7\tfrac{1}{2}'', \text{ nearly. Ans.}$$

The enlargement for the wheel seat will then have a diameter of

$$1.2\,d = 1.2 \times 7.5 = 9''. \text{ Ans.}$$

For the thickness of the hub we use formula **273.**

$$w = \tfrac{1}{2}\sqrt[3]{b\,C\,R} = \tfrac{1}{2}\sqrt[3]{12.44 \times 4.147 \times 33} = 4'', \text{ nearly. Ans.}$$

The length of hub may be

$$1.4\,b = 1.4 \times 12.44 = 17\tfrac{7}{16}'', \text{ nearly. Ans.}$$

For the number of arms, formula **272** gives

$$z = .55\sqrt[4]{N^2\,C} = .55\sqrt[4]{50^2 \times 4.147} = 5.5.$$

Therefore, use 6 arms. Ans.

For the width of arm at the center of the wheel, formula **270** gives

$$a = \sqrt{\frac{R\,b}{z}} = \sqrt{\frac{33 \times 12.44}{6}} = 8.27, \text{ say } 8\tfrac{1}{4}''. \text{ Ans.}$$

Supposing the arms to taper $\frac{1}{16}$ on each side, the width of arm at pitch line is

$$8\tfrac{1}{4} - (2 \times 33 \times \tfrac{1}{32}) = 6\tfrac{1}{8}''. \text{ Ans.}$$

The thickness of the arm is $\frac{1}{2}C = \frac{1}{2} \times 4.147 = 2.07 = 2\frac{1}{16}''$. Ans.
The thickness of the stiffening rib is $.4\,C = .4 \times 4.147 = 1.66 = 1\frac{11}{16}''$. Ans.

2033. Proportions of Bevel Gears.—The rules and formulas used in designing spur gears are equally applicable to bevel gears. For the circular pitch C, used in spur-gear formulas, should be substituted the mean circular pitch of the bevel gear; that is, the circular pitch measured at the *middle* of the tooth face.

Denote this mean circular pitch by the symbol C_m. Then, the bevel wheel may be designed by formulas **266** to **273** by placing C_m for C, and P_m for P.

Then,
$$b\,C_m = 16.8\,\frac{p}{S}. \qquad \textbf{(266a.)}$$

$$c = .4\,C_m + \tfrac{1}{8}'' = \frac{5}{4P} + \tfrac{1}{8}''. \qquad \textbf{(269a.)}$$

$$\left.\begin{aligned} a &= \sqrt{\frac{R\,b}{z}}. \\ x &= \tfrac{1}{2}\,C_m. \end{aligned}\right\} \qquad \textbf{(270a.)}$$

$$\left.\begin{aligned} a &= 1.2\sqrt[3]{\frac{b\,C_m\,R}{z}}. \\ a &= 1.75\sqrt[3]{\frac{b\,R}{P_m\,z}}. \end{aligned}\right\} \qquad \textbf{(271a.)}$$

$$z = .55\,\sqrt[4]{N^2\,C_m}. \qquad \textbf{(272a.)}$$

$$w = \tfrac{1}{3}\sqrt[3]{b\,C_m\,R}. \qquad \textbf{(273a.)}$$

WORM GEARING.

2034. Construction of Worm Gearing.—Worm

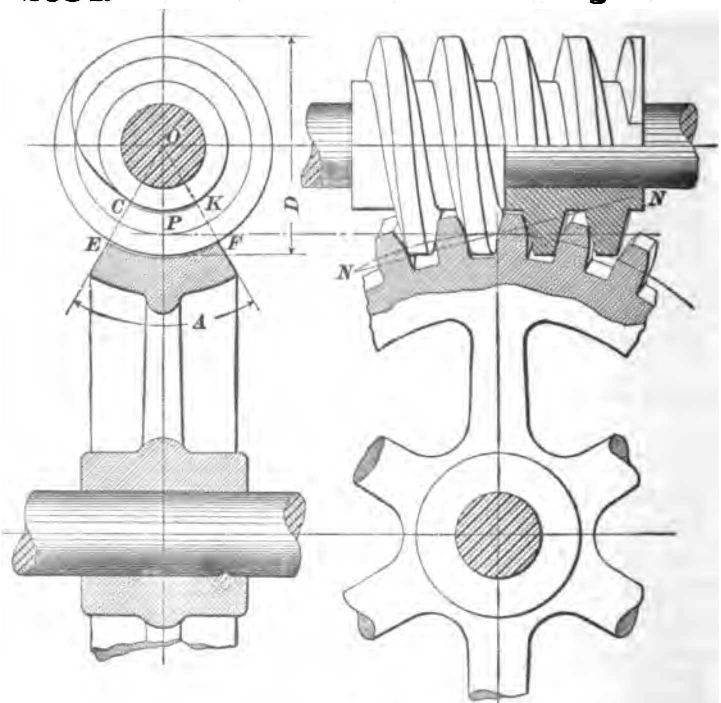


FIG. 709.

wheels may be constructed in various ways. The wheel may be similar to a spur wheel, except that the teeth make an

angle with the axis of the wheel equal to that of the threads of the worm measured at the pitch diameter. (See Fig. 709.)

For careful and accurate work, the wheel is cut by a special milling tool called a **hob.** This hob is a steel worm which has been cut in a lathe, notched to form a milling cutter, and then hardened and tempered. The teeth cut by this hob must evidently have the correct form to gear with a worm having the same pitch as the hob. When the wheel teeth are constructed by this method, there is much closer contact between the worm and the wheel. Therefore, the durability and efficiency of the mechanism are largely increased.

2035. Friction of Worm Gearing.—Since the threads of the worm slide on the teeth of the wheel, there is considerable work expended in overcoming friction. It, therefore, becomes necessary to consider the form of worm and wheel which will work with least friction.

Let R_1 = radius of pitch circle of wheel;
R = radius of pitch circle of worm;
p = the axial pitch of the worm;
p_1 = circular pitch of wheel, and divided axial pitch of worm;
N = number of threads in worm;
N_1 = number of teeth in wheel;
P = resistance to turning at circumference of worm-wheel;
Q = actual force acting at pitch line of worm necessary to turn worm;
Q' = force required to turn worm if there were no friction between it and wheel;
e = efficiency of worm gearing;
a = angle between tangent to worm thread and any plane perpendicular to axis of worm.

The efficiency of the gear is the ratio of the useful work to the total work. Assuming the coefficient of friction to be about .15 or .16, the efficiency may be obtained by the following formula:

$$e = \frac{Q'}{Q} = \frac{Np_1}{Np_1 + R}. \qquad (274.)$$

Hence, for maximum efficiency, the radius R of the pitch circle of the worm should be made as small as possible, and the number of threads should be as large as possible.

EXAMPLE.—If the radius of pitch circle of worm is 3 times the divided axial pitch p_1, and if the worm has 2 threads, what is its efficiency?

SOLUTION.—From formula **274**,

$$e = \frac{N p_1}{N p_1 + R} = \frac{2 p_1}{2 p_1 + 3 p_1} = .40 = 40 \text{ per cent. Ans.}$$

If the radius R were only $2 p_1$, the efficiency would be

$$e = \frac{2 p_1}{2 p_1 + 2 p_1} = 50 \text{ per cent.}$$

On the other hand, if the number of threads were 1, the efficiency would only be

$$e = \frac{p_1}{p_1 + 3 p_1} = 25 \text{ per cent.}$$

The leading problem, therefore, in designing a worm gear is to make the ratio $\frac{R}{p_1}$ as small as possible, and at the same time allow the worm shaft a sufficient diameter for strength.

The twisting moment acting on the worm shaft is

$$Q R = \frac{Q' R}{e}.$$

From the principle of work we have $2 \pi R Q' = P p$.

Hence,
$$\frac{Q' R}{e} = \frac{P p}{2 \pi e},$$

which is the twisting moment to be used in calculating the worm shaft.

EXAMPLE.—The circular pitch of a worm-wheel is $.7854 = \frac{\pi}{4}$; it has 40 teeth, and transmits 2 horsepower at 30 revolutions per minute. The worm has 1 thread, and the radius of its pitch circle is 2 inches. Calculate the twisting moment on the shaft, and the size of the shaft, if made of wrought iron.

SOLUTION.—Diametral pitch of wheel $= \frac{\pi}{\frac{\pi}{4}} = 4$.

Diameter of wheel $= \frac{40}{4} = 10''$.

Radius of wheel $= R_1 = 5''$.

From formula **231**, Art. **1963**,

$$P = \frac{63{,}025\,H}{R_1\,n} = \frac{63{,}025 \times 2}{5 \times 30} = 840 \text{ lb., nearly.}$$

From formula **274**,

$$e = \frac{N p_1}{N p_1 + R} = \frac{.7854}{.7854 + 2} = 28 \text{ per cent.}$$

Hence, twisting moment $= \dfrac{P p}{2\pi e} = \dfrac{840 \times \frac{\pi}{4}}{2\pi \times .28} = 375$ in.-lb. Ans.

Using formula **123**, Art. **1415**,

diameter of shaft $d = .31\,\sqrt[3]{375} = 1.364$, say $1\frac{3}{8}''$. Ans.

The pitch of the worm-wheel may be calculated from the formula for the pitch of spur gears.

$$b p_1 = 16.8\frac{P}{S}.$$

2036. The rim and arms of the worm-wheel may be given the same proportions as the rim and arms of a spur wheel of the same diameter and pitch, except that the width of the face of the wheel is generally made equal to double the pitch.

The rim may have the forms shown in Fig. 710. The

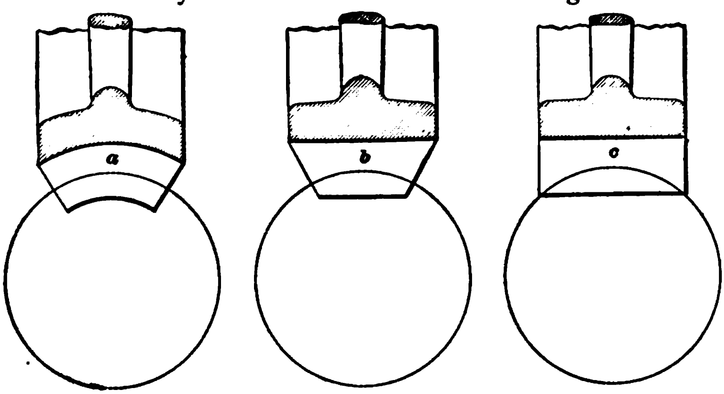


FIG. 710.

form *a* is best, as there is line contact between the thread and tooth.

The ratio $\dfrac{R}{p_1}$ may vary from 1 to 4 or 6. When the worm is cut on the shaft, the lower values may be used. When, however, the worm is cast and keyed on the shaft, the

ratio must be made greater. As has been shown, the smaller this ratio the greater the efficiency. The number of threads in the worm varies from 1 to 4, depending upon the velocity ratio to be transmitted.

Letting N and N' represent the number of threads on worm and teeth on wheel, respectively, and w the velocity ratio, we have

$$w = \frac{N'}{N}.$$

Usually it is undesirable to give the worm-wheel less than 30 teeth; hence, if the desired velocity ratio is less than 30, the worm must have more than one thread. For example, if the worm shaft is to make 20 revolutions while the wheel shaft makes one revolution, we may have either $N' = 40$ and $N = 2$, or $N' = 60$ and $N = 3$.

The length of the worm is from $3p$ to $6p$, usually $4p$; that is, there are usually 4 turns, as shown in Fig. 709.

FRICTION GEARING.

2037. Friction gearing may be used for the transmission of small powers; it should not be used for heavy work.

In this style of gearing the smooth faces of a pair of wheels are pressed together, and one drives the other by means of the friction between the surfaces in contact.

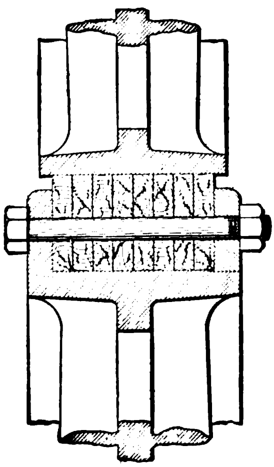


FIG. 711.

If the resistance to turning offered by the driven wheel is less than the frictional resistance between the surfaces of the two wheels, motion will be transmitted; otherwise the surfaces will slide over each other, and the gearing will not work.

It is the usual practice to face the rim of one of the wheels with leather, wood, or paper. Fig. 711 shows a friction wheel thus faced with wood.

As shown, the grain of the wood lies along the working surface. When leather or paper is used, the edges of the layers are used as the driving surface. The pulley which is covered with wood or paper must be the driver; the driven pulley is usually of cast iron.

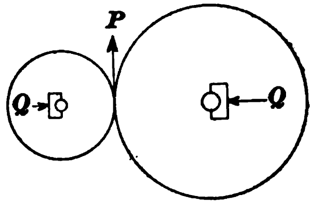


FIG. 712.

Let P (see Fig. 712) = driving force at circumference of friction wheels;

Q = force acting on the bearings to press the wheels together;

f = coefficient of friction between surfaces.

Then, $$Qf = P, \text{ or } Q = \frac{P}{f}. \qquad \textbf{(275.)}$$

The coefficient of friction f has about the following values:

Metal on metal................f = .15 to .25.
Wood on metal ....f = .25 to .40.
Paper on metal................f = .20.
Leather on metal.............f = .25 to .30.

The power transmitted by a pair of friction gears can be easily calculated. The following example shows the method:

EXAMPLE.—Suppose a friction wheel faced with leather to drive another $2\frac{1}{2}$ feet in diameter at 300 revolutions per minute. The force pressing the wheels together is 450 pounds. Assuming f = .3, calculate the horsepower transmitted.

SOLUTION.—By formula **275**, $P = Qf = 450 \times .3 = 135$ lb.

$$R = \text{radius of wheel in inches} = \frac{2\frac{1}{2} \times 12}{2} = 15''.$$

From formula **231**, Art. **1963**,

$$H = \frac{PRN}{63{,}025} = \frac{135 \times 15 \times 300}{63{,}025} = 9.64 \text{ horsepower. Ans.}$$

The calculations for bevel friction gears are more complicated, but may be easily made.

The proportions of the rim, hub, and arm of a friction gear-wheel may be made the same as those of a toothed gear-wheel transmitting the same power.

FLEXIBLE GEARING.

BELT GEARING.

BELT MATERIALS.

2038. Leather.—The material mostly used for belts is leather tanned from ox hides. The leather is about $\frac{5}{16}$ inch thick, and is obtained in strips up to 5 feet in length. Belts are made of any required length by joining these strips together.

Single belts are made of one thickness of leather; **double belts** from two thicknesses of leather.

Cotton may be used for belts which are exposed to dampness. Cotton belts can be made very wide, and without the many joints necessary in leather belts. The necessary thickness is obtained by sewing together from 4 to 10 plies of cotton duck. Cotton belting is cheaper and stronger than leather belting, but probably less durable.

Rubber belts are made by cementing together plies of cotton duck with india rubber. Rubber belts are more adhesive than leather belts, and, hence, have greater driving capacity. Rubber belts are considered to be the best to use in damp situations.

Dimensions of Belts.—Practical formulas for computing the dimensions of belts are given in Arts. **1483** to **1489** inclusive.

BELT PULLEYS.

2039. Fig. 713 shows a **solid,** and Fig. 714 a **split, cast-iron belt pulley.** The general form of these pulleys corresponds very closely to the best modern American practice. The split pulley has the advantage of being more easily put on the shaft, especially when the shaft is in position or has other pulleys already on it.

When the amount of power to be transmitted by a pulley is small, it may be fastened to the shaft by set screws. Split pulleys are also made so that the bolts through the hub will serve as a clamp to draw the hub tight enough on the shaft to prevent slipping with small loads. When the amount of

power to be transmitted is considerable, pulleys should be fastened with keys, and in some cases both keys and set-screws are provided.

FIG. 713.

FIG. 714.

In the rules and formulas for dimensions of pulleys, the following symbols will be used to represent the various dimensions:

b = breadth of belt running on pulley;
B = breadth of pulley rim;
d = diameter of shaft on which pulley is keyed;
t = thickness of pulley rim at edge;
a = width of arm at center of pulley;
w = thickness of hub of pulley;
l = length of hub of pulley;
n = number of arms;
D = diameter of pulley;
R = radius of pulley;
s = swell at center of pulley rim.

All dimensions to be in inches.

NOTE.—To obtain a, the arms are supposed to extend through the hub to the center of the pulley. (See Fig. 720.)

2040. Rim of Pulley.—The rim is usually of the form shown in Fig. 715. If the rim is crowning, the curve may be struck with a radius of from $3B$ to $5B$, in which

case $s = \frac{B}{20}$ to $\frac{B}{40}$, about. The width B is made from $\frac{9}{8}b$ to $\frac{5}{4}b$; i. e., $B = \frac{9}{8}b$ to $\frac{5}{4}b$.

For the thickness t, the following formula gives results which agree well with the practice of good shops:

$$t = \frac{B + D}{200} + \tfrac{1}{16}''. \qquad \textbf{(276.)}$$

For double belts, $\frac{1}{8}''$ may be added to the thickness obtained by the above formula.

2041. Flange Pulleys.—When there is a liability of the belt frequently slipping, caused by fluctuations in the

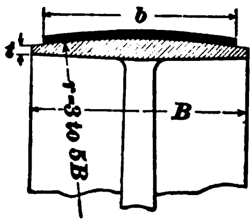


FIG. 715.

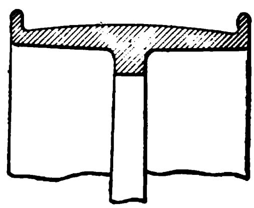


FIG. 716.

power transmitted, the pulley rim may be cast with flanges, as shown in Fig. 716. See also Figs. 381 and 382, Art. **1500.**

2042. Arms of Pulleys.—The arms are usually of oval section, as shown in Fig. 717, A and B. It is customary to make the thickness $\frac{1}{2}$ the width.

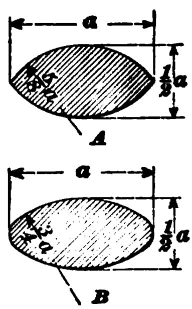


FIG. 717.

The arms are generally straight, as in Figs. 713 and 714, though curved arms are occasionally used.

The number of arms to be used is largely a matter of judgment, but in practice, for all sizes of pulleys and engine band-wheels, under 10 feet in diameter, by far the greater number are made with 6 arms; 8 or 10 arms are sometimes used for pulleys above 6 feet in diameter, and for very small sizes, 4 arms are sufficient.

To calculate the width a of the arm, we may assume, as in gear-wheels, that the bending moment is $p R$, and that, consequently, the bending moment for each arm is $\frac{p R}{n}$.

The resisting moment of the section shown in Fig. 717 is practically $.04\, a^3 S$, where S is the allowable safe stress. If we allow 2,500 lb. per sq. in. as the safe stress, we have

$$\frac{p R}{n} = 100\, a^3, \text{ or } a = \sqrt[3]{\frac{p R}{100\, n}}.$$

$p = T_1 - T_2$, the difference of the belt tension is usually not known exactly. Its maximum value can, however, easily be found, as follows:

For single belts $\frac{7}{32}''$ thick, an average tension of 320 lb. per sq. in. of section may be allowed. Then,

$$T_1 = \tfrac{7}{32} \times 320 = 70 \text{ lb. per inch of width of belt.}$$

Usually P is not more than $\frac{1}{3}$ of T_1, and its ***maximum*** value may be taken at say 50 lb. per inch of width. For double belts, take $P = 100$ lb. per inch of width.

Then, $p = 50\, B$ for single belts.
$p = 100\, B$ for double belts.

Substituting these values of p in the above equation,

$$\left.\begin{aligned} a &= \sqrt[3]{\frac{B R}{2 n}}, \text{ for single belts.} \\ a &= \sqrt[3]{\frac{B R}{n}}, \text{ for double belts.} \end{aligned}\right\} \qquad \textbf{(277.)}$$

The taper in the width of the arms towards the rim may be made $\frac{1}{2}''$ per foot, and the thickness at the rim $\frac{1}{2}$ the width.

EXAMPLE.—Calculate the size of the arms of a 6-arm pulley, 30 inches in diameter, with a 6-inch face.

SOLUTION.—For single belt, we have, from formula **277**,

$$a = \sqrt[3]{\frac{B R}{2 n}} = \sqrt[3]{\frac{6 \times 15}{2 \times 6}} = 2 \text{ in., nearly. Ans.}$$

At the rim the width is $2'' - (\frac{1}{2} \times 1\frac{1}{4}) = 2'' - \frac{5}{8}'' = 1\frac{3}{8}''$. Ans.
Thickness at center $= 2 \times \frac{1}{2} = 1''$. Ans.
Thickness at the rim $= 1\frac{3}{8}'' \times \frac{1}{2} = \frac{11}{16}''$. Ans.

For very wide pulleys it is sometimes desirable to use two sets of arms, as shown in Fig. 718.

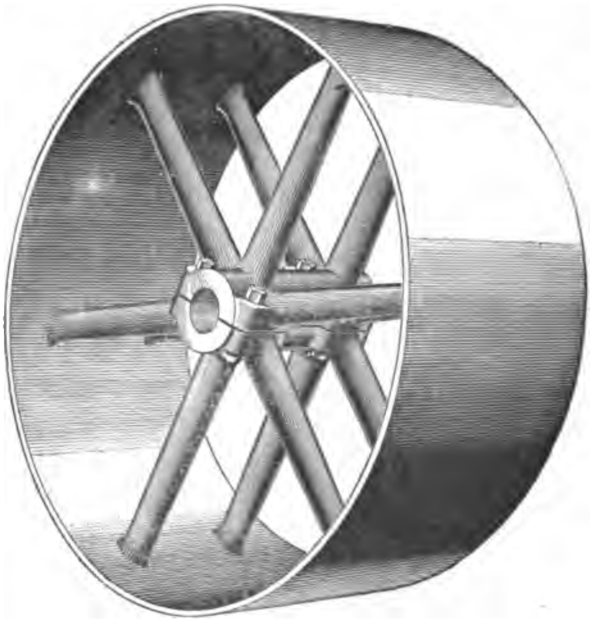


FIG. 718.

To calculate the size of the arms in this case, the pulley may be considered as made of two pulleys each of width $\frac{1}{2} B$. Then, find the dimensions of the arms as before, and multiply these dimensions by $\sqrt[3]{\frac{1}{2}} = .8$, nearly.

2043. Hub of Pulley.—The thickness of the hub may be obtained from the following formula:

$$w = \frac{(B + R)}{32} + \tfrac{1}{8}''. \qquad \textbf{(278.)}$$

The length of the hub may be, $l = \frac{2}{3} B$ to B.

The key-way may be calculated by the rules given in Art. **1965**, etc.

2044. Loose Pulleys.—Pulleys which run loose on a shaft should have longer hubs than those keyed fast; the hubs may also be lined with brass bushings, if desired.

Where a fast and a loose pulley are placed together on a shaft, as shown in Fig. 719, the length l of the hub of the loose pulley may be $l = 1.2\,B$, and that of the fixed pulley $l' = .8\,B$. The thickness of the hub of the loose pulley may be less than that of the fast pulley on account of its increased length.

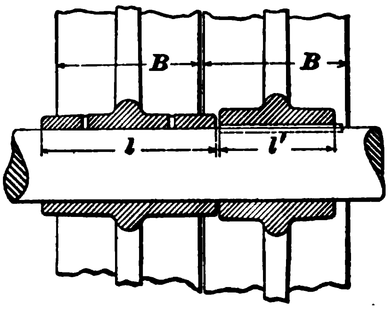


FIG. 719.

Some arrangement must be provided for oiling the loose pulley; generally one or two oil holes are drilled through the hub.

2045. For split pulleys the size of the bolts at rim and hub may be determined as follows:

Let A = area of cross-section of rim;
A' = area of cross-section of hub along the line of division;
S = net section of bolt or bolts at rim;
S' = net section of bolt or bolts at hub.

Then,
$$\left.\begin{aligned} S &= \frac{A}{4} + \tfrac{1}{4} \text{ square inch.} \\ S' &= \frac{A'}{4} \text{ square inch.} \end{aligned}\right\} \qquad \textbf{(279.)}$$

EXAMPLE.—The hub of a split pulley is 4 inches long and $1\frac{1}{4}$ inches thick. If the hub is held by 4 bolts, what should be the diameter of the bolts?

SOLUTION.—$A' = 4 \times 1\frac{1}{4} = 5$ sq. in.
By formula **279**,
$$S' = \frac{A'}{4} = \tfrac{5}{4} \text{ sq. in.} = \text{net section of 4 bolts.}$$
$$\tfrac{5}{4} \div 4 = \tfrac{5}{16} \text{ sq. in.} = \text{net section of 1 bolt.}$$
Hence, from Table 43, Art. **1926**, the diameter of bolt is $\frac{3}{4}$ inch. Ans.

2046. Wrought-iron pulleys are coming into extensive use, and possess important advantages over those made of cast iron. They are lighter and stronger, and are free

from the initial stresses to which cast-iron pulleys are liable. Owing to the stresses due to centrifugal force, cast-iron pulleys can not be safely run at very high speeds; wrought-iron pulleys, however, may be used at almost any reasonable speed, because of the greater tenacity of the material of which they are made. Wrought-iron pulleys are usually *split*.

2047. Maximum Speed of Cast-Iron Pulleys.—When a pulley rotates, each portion of the rim tends to fly outwards in the direction of a tangent to the rim. This tendency is due to the *centrifugal force*, so called.

The centrifugal force on each element of the rim acts radially outwards, and the pulley under its action is analogous to a section of a steam boiler under the pressure of steam. The rim tends to break at two sections which are at the opposite ends of a diameter.

Table 51 gives values of the stress per square inch produced in a pulley rim by centrifugal force for various velocities:

TABLE 51.

Velocity of rim in feet per second........	60	80	100	150	200
Velocity of rim in feet per minute........	3,600	4,800	6,000	9,000	12,000
Stress in rim in pounds per sq. in.:					
Cast iron...........	351	624	975	2,194	3,900
Wrought iron.......	378	672	1,050	2,362	4,200

To the stress given in the above table must be added the initial stresses due to contraction in cooling, and the stress caused by the belt pull.

It is usually considered unsafe to run a cast-iron pulley, gear-wheel, or fly-wheel at a higher rim speed than 100 feet per second. Since the centrifugal force increases in direct proportion to the cross-section of the rim, it is evident that it is useless to try to provide against it by putting more material in the rim.

TABLE 52.

Diameter.	Face.	Rim. A	Rim. B	Arm. C	Arm. D	Hub. E	Hub. F	Boss. G	Boss. H	Boss. I
6″	4	1/8	3/16	3/4	7/16	3	3/8	1/2	1	3/4
	6	....	..	"	"	3 1/2	1/2	"	"	"
	8	....	..	"	"	"	"	"	"	"
	10	....	..	"	"	4	"	"	"	"
	12	....	..	"	"	"	"	"	"	"
8	4	1/8	3/16	13/16	7/16	3	3/8	1/2	1	3/4
	6	....	..	....	....	3 1/2	1/2	..	..	..
	8	5/32	1/4	1 1/16	9/16	4 1/2	..	...	..	..
	10	....	..	....	....	5 1/2	..	..	..	..
	12	....	..	....	....	..	..	..	..	..
10	4	1/8	3/16	15/16	9/16	3	1/2	1/2	1	3/4
	6	5/32	1/4	1 1/16	....	3 1/2	..	..	..	..
	8	....	..	....	....	4 1/2	..	..	..	..
	10	....	..	1 5/16	5/8	5 1/2	5/8	5/8	1 1/4	5/8
	12	....	..	....	....	..	..	..	..	..
12	4	5/32	1/4	1	7/16	3 1/4	1/2	1/2	1	3/4
	6	....	..	1 1/4	1/2	4	..	..	..	..
	8	....	..	....	....	5	5/8	5/8	1 1/4	5/8
	10	3/16 +	5/16	1 1/2	3/4	5 1/2	..	..	..	..
	12	....	..	....	....	6 1/2	..	..	..	..
14	4	5/32 +	1/4	1 1/8	1/2	3 1/2	1/2	1/2	1	3/4
	6	....	..	....	....	4 1/2	5/8	5/8	1 1/4	5/8
	8	3/16	5/16	1 5/16	9/16	5	..	..	..	..
	10	....	..	....	....	6	..	..	..	..
	12	...	..	1 11/16	13/16	6 1/2	..	..	..	..
16	4	5/32 +	1/4	1 3/8	9/16	3 1/2	1/2	1/2	1	3/4
	6	....	..	....	....	4 1/2	5/8	3/4	1 1/2	5/8
	8	3/16 +	5/16	1 7/16	5/8	5	..	..	..	..
	10	....	..	....	....	6	..	..	..	..
	12	7/32	11/32	....	....	6 1/2	3/4	..	..	..
	16	....	..	1 7/8	15/16	8 1/4	7/8	3/4	1 3/4	..
18	4	3/16	5/16	1 5/16	9/16	4	5/8	5/8	1 1/4	5/8
	6	....	..	....	....	4 1/2	..	..	..	..
	8	7/32	11/32	1 1/2	11/16	5 1/2	3/4	..	..	..
	10	....	..	....	....	6	..	..	..	..
	12	....	..	....	....	7 1/2	7/8	3/4	1 3/8	..
	16	1/4	3/8	2 1/4	1 1/4	8	..	..	..	..
	20	....	..	....	....	9	..	..	..	..
20	4	3/16 +	5/16	1 3/8	5/8	4	5/8	5/8	1 1/4	5/8
	6	....	..	....	....	4 1/2	..	..	..	..
	8	....	..	....	....	5	3/4	..	..	..
	10	7/32	11/32	1 5/8	3/4	6	..	..	..	..
	12	....	..	....	....	7	..	..	..	..
	16	9/32	7/16	2 1/4	1 1/8	8	7/8	3/4	1 3/4	..
	20	....	..	....	....	10	1	..	..	..

TABLE 52.—*Continued.*

Diameter.	Face.	Rim. A	Rim. B	Arm. C	Arm. D	Hub. E	Hub. F	Boss. G	Boss. H	Boss. I
22	4	$\frac{3}{16}$	$\frac{5}{16}$	$1\frac{1}{2}$	$\frac{5}{8}$	4	$\frac{5}{8}$	$\frac{5}{8}$	$1\frac{1}{4}$	$\frac{5}{8}$
	6	....	..	....	....	$4\frac{1}{2}$	$\frac{3}{4}$	..	..	..
	8	....	..	....	....	5	....	..	..	..
	10	$\frac{7}{32}+$	$\frac{11}{32}$	$1\frac{3}{4}$	$\frac{13}{16}$	$6\frac{1}{2}$	....	..	..	..
	12	....	..	....	....	....	$\frac{7}{8}$	$\frac{3}{4}$	$1\frac{3}{8}$	..
	16	....	..	....	....	$8\frac{3}{4}$	1	..	..	..
	20	$\frac{9}{32}+$	$\frac{7}{16}$	$2\frac{1}{2}$	$1\frac{1}{4}$	11	$1\frac{1}{8}$	$\frac{7}{8}$	$1\frac{1}{2}$	..
24	4	$\frac{7}{32}$	$\frac{11}{32}$	$1\frac{9}{16}$	$\frac{11}{16}$	4	$\frac{5}{8}$	$\frac{5}{8}$	$1\frac{1}{4}$	$\frac{5}{8}$
	6	....	..	....	....	$4\frac{3}{4}$	$\frac{3}{4}$	..	..	..
	8	....	..	....	....	$5\frac{1}{2}$	....	..	..	..
	10	$\frac{1}{4}$	$\frac{3}{8}$	$1\frac{7}{8}$	....	7	$\frac{7}{8}$	$\frac{3}{4}$	$1\frac{3}{8}$	..
	12	....	..	....	....	....	....	..	..	..
	16	....	..	....	....	$9\frac{1}{2}$	1	..	..	..
	20	$\frac{5}{16}$	$\frac{15}{32}$	$2\frac{3}{4}$	$1\frac{3}{8}$	....	$1\frac{1}{8}$	$\frac{7}{8}$	$1\frac{1}{2}$	..
	24	....	..	....	....	11	....	..	..	..
26	4	$\frac{7}{32}$	$\frac{11}{32}$	$1\frac{11}{32}$	$\frac{3}{4}$	$4\frac{1}{4}$	$\frac{3}{4}$	$\frac{5}{8}$	$1\frac{1}{4}$	$\frac{5}{8}$
	6	....	..	....	....	5	....	..	..	..
	8	....	..	....	....	6	$\frac{7}{8}$	$\frac{3}{4}$	$1\frac{3}{8}$	..
	10	$\frac{1}{4}$	$\frac{3}{8}$	2	$\frac{7}{8}$	7	....	..	..	..
	12	....	..	....	....	$7\frac{1}{2}$	....	..	..	..
	16	....	..	....	....	10	$1\frac{1}{8}$	$\frac{7}{8}$	$1\frac{1}{2}$	..
	20	$\frac{5}{16}+$	$\frac{15}{32}$	$2\frac{15}{16}$	$1\frac{7}{16}$	$10\frac{1}{2}$	....	..	..	..
	24	....	..	....	....	11	....	..	..	..
28	4	$\frac{7}{32}+$	$\frac{15}{32}$	$1\frac{3}{4}$	$\frac{3}{4}$	$4\frac{1}{2}$	$\frac{3}{4}$	$\frac{5}{8}$	$1\frac{1}{4}$	$\frac{5}{8}$
	6	....	..	....	....	$5\frac{1}{2}$	$\frac{7}{8}$	$\frac{3}{4}$	$1\frac{3}{8}$	..
	8	....	..	....	....	7	....	..	..	..
	10	....	..	....	....	$7\frac{1}{2}$	....	..	..	..
	12	$\frac{1}{4}+$	$\frac{3}{8}$	$2\frac{1}{8}$	$\frac{13}{16}$	8	1	..	..	..
	16	....	..	....	....	10	....	..	..	..
	20	$\frac{5}{16}$	$\frac{15}{32}$	....	....	11	$1\frac{1}{8}$	$\frac{7}{8}$	$1\frac{1}{2}$	..
	24	$\frac{11}{32}$	$\frac{1}{2}$	$3\frac{1}{2}$	$1\frac{1}{2}$	....	....	..	..	..
30	4	$\frac{7}{32}+$	$\frac{11}{32}$	$1\frac{7}{8}$	$\frac{13}{16}$	$4\frac{1}{2}$	$\frac{3}{4}$	$\frac{5}{8}$	$1\frac{1}{4}$	$\frac{5}{8}$
	6	....	..	....	....	$5\frac{1}{2}$	$\frac{7}{8}$	..	..	..
	8	....	..	....	....	$6\frac{1}{4}$	....	$\frac{3}{4}$	$1\frac{3}{8}$	..
	10	....	..	....	....	$6\frac{1}{2}$	....	..	..	..
	12	$\frac{9}{32}$	$\frac{7}{16}$	$2\frac{1}{4}$	1	8	1	..	..	..
	16	....	..	....	....	$8\frac{1}{2}$	....	..	..	..
	20	....	..	....	....	$11\frac{1}{2}$	$1\frac{1}{4}$	$\frac{7}{8}$	$1\frac{1}{2}$	..
	24	$\frac{3}{8}$	$\frac{9}{16}$	$3\frac{5}{16}$	$1\frac{5}{8}$	13	....	..	..	..
32	4	$\frac{1}{4}+$	$\frac{3}{8}$	$2\frac{1}{8}$	$\frac{15}{16}$	$4\frac{1}{2}$	$\frac{7}{8}$	$\frac{3}{4}$	$1\frac{3}{8}$	$\frac{5}{8}$
	6	....	..	....	....	$5\frac{1}{2}$	....	..	..	..
	8	....	..	....	....	$6\frac{1}{2}$	1	..	..	..
	10	....	..	....	....	$7\frac{1}{2}$	....	..	..	..
	12	$\frac{5}{16}$	$\frac{13}{32}$	$2\frac{7}{16}$	$1\frac{1}{16}$	8	$1\frac{1}{8}$	$\frac{7}{8}$	$1\frac{1}{2}$	..
	16	....	..	....	....	$9\frac{1}{2}$	....	..	..	..
	20	....	..	....	....	11	$1\frac{1}{4}$	..	..	..
	24	....	..	....	....	13	....	..	..	..

TABLE 52.—*Continued.*

Diameter.	Face.	Rim. A	Rim. B	Arm. C	Arm. D	Hub. E	Hub. F	Boss. G	Boss. H	Boss. I
34	4	1/4 +	3/8	2 1/8	15/16	4 1/2	7/8	3/4	1 3/8	3/8
	6	....	..	....	....	5 1/2	..	..	..	..
	8	....	..	....	....	6 1/2	1	..	..	..
	10	....	..	....	....	7 1/4	..	..	..	..
	12	5/16	15/32	2 7/16	1 1/16	7 3/4	1 1/8	7/8	1 1/2	..
	16	....	..	....	....	9 1/2	..	..	..	..
	20	....	..	....	....	12	1 1/4	..	..	..
	24	....	..	....	....	13	..	..	..	..
36	4	1/4 +	3/8	2 3/16	15/16	4 1/2	7/8	3/4	1 3/8	3/8
	6	....	..	....	....	5 1/2	..	..	..	..
	8	....	..	....	....	6 3/4	..	..	..	..
	10	....	..	....	....	7 1/4	..	..	..	..
	12	5/16	15/32	....	....	7 3/4	1 1/8	7/8	1 1/2	..
	16	....	..	2 9/16	1 1/8	10 1/4	1 1/4	..	..	..
	20	....	..	....	....	12	..	..	..	..
	24	....	..	....	....	13 1/2	1 3/8	1	1 3/4	1/2
40	8	5/16	15/32	2 5/16	1	6 3/4	1	3/4	1 3/8	3/8
	12	....	..	....	....	7 3/4	1 1/8	7/8	1 1/2	..
	16	11/32	1/2	2 3/4	1 1/4	10	1 1/4	..	..	..
	20	....	..	....	....	11 1/2	1 3/8	1	1 3/4	1/2
	24	....	..	....	....	15 1/4	1 1/2	..	..	..
44	8	9/32	7/16	2 1/2	1 1/4	6 3/4	1 1/8	7/8	1 1/2	3/8
	12	....	..	....	....	8	..	..	..	..
	16	11/32	1/2	3	1 5/16	10	1 1/4	..	..	..
	20	....	..	....	....	12	1 3/8	1	1 3/4	1/2
	24	....	..	3 1/2	....	15	1 1/2	..	..	..
48	8	9/32 +	7/16	2 3/4	1 3/4	7 1/2	1 1/8	7/8	1 1/2	3/8
	12	....	..	....	....	8 3/4	1 1/4	..	..	..
	16	3/8	9/16	3 1/4	1 7/16	10	1 3/8	1	1 3/4	1/2
	20	....	..	....	....	12	..	..	..	..
	24	....	..	....	....	15	1 1/2	..	..	..
54	12	5/16 +	15/32	3	1 5/16	9 3/4	1 3/8	1	1 3/4	1/2
	16	....	..	....	....	11 1/4	1 1/2	..	..	..
	20	13/32	19/32	3 3/8	1 3/8	....	..	..	..	..
	24	....	..	....	....	15	1 3/4	1 1/4	2	..
60	12	11/32	1/2	3 5/16	1 7/16	10	1 3/8	1	1 3/4	1/2
	16	....	..	....	....	11 1/4	1 1/2	..	..	..
	20	7/16	5/8	3 13/16	1 3/4	12 1/2	1 3/4	1 1/4	2	..
	24	....	..	....	....	15	..	..	..	..
66	12	11/32	1/2	3 9/16	1 9/16	10	1 1/2	1	1 3/4	1/2
	16	....	..	....	....	11 1/2	1 5/8	1 1/4	2	..
	20	1/2	3/4	4 1/4	1 15/16	13 1/2	1 7/8	..	..	..
	24	....	..	....	....	15	..	..	..	..
72	12	3/8	9/16	3 7/8	1 11/16	10 1/2	1 5/8	1 1/4	2	1/2
	16	....	..	....	....	12 1/2	1 3/4	..	..	..
	20	9/16	13/16	4 1/2	2 1/16	13 1/2	1 7/8	..	..	..
	24	....	..	....	....	15	2	..	..	..

2048. Examples of Belt Pulleys.—Table 52 gives the dimensions of a set of cast-iron belt pulleys ranging from 6″ to 72″ in diameter, as made by a well-known manufac-

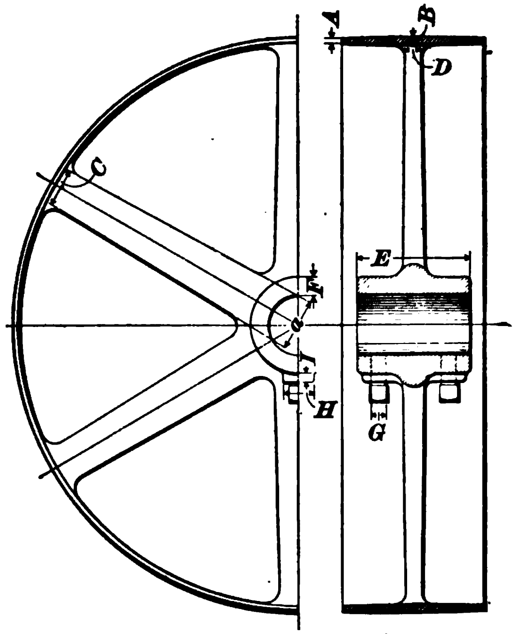


FIG. 720.

turing company. These pulleys are so designed that the number of patterns may be kept within reasonable limits, and at the same time have the dimensions correspond as nearly as possible with well-established rules.

The letters over the columns of dimensions given in the table correspond to the letters in Fig. 720.

In all cases the number of arms is 6, and the arms increase in size towards the hub with a taper of $\frac{1}{4}$″ per foot.

2049. Counterbalance.—Pulleys that run at high speeds must be carefully balanced, i. e., the center of gravity of the pulley must correspond with the center of the shaft, otherwise there will be a heavy stress on the shaft and bearings. Since it is seldom possible to make the pulley

exactly symmetrical, the difference in weight of the heavy side is compensated for by weights riveted to the inside of the rim on the light side.

ROPE BELTING.

2050. There is a growing tendency towards the substitution of hemp and cotton ropes for belting and line shaft-

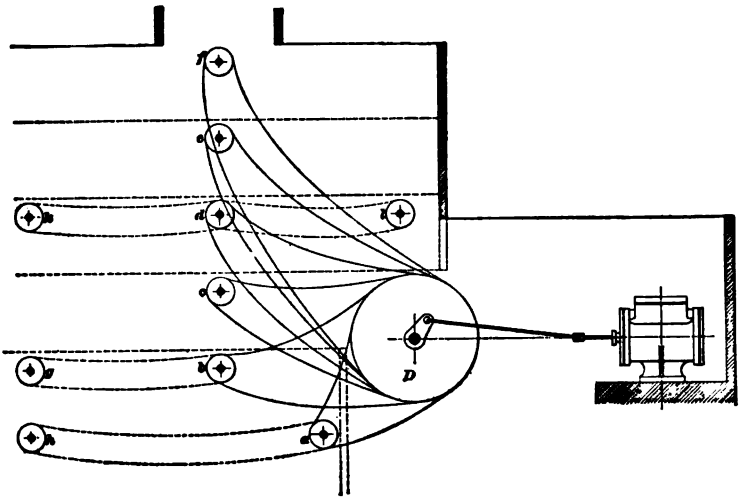


FIG. 721.

ing as a means of transmitting power in large factories and shops. The advantages claimed for the rope-driving system are :

1. Economy ; for a rope system is cheaper to install than either leather belting or shafting.

2. In the rope system there is less loss of power by slipping.

3. Flexibility ; that is, the ease with which the power is transmitted to any distance, and in any direction.

2051. There are two systems of rope transmission in common use. In the first, the transmission is effected by several parallel independent ropes which pass around the fly-wheel of the engine and the pulley or pulleys to be driven.

Each rope is made quite taut at first, but stretches until it slips, after which it is re-spliced. A good example of a rope transmission of this character is shown in outline in Fig. 721.

The fly-wheel *D* carries 35 parallel ropes which distribute power to the pulleys *a*, *b*, *c*, *d*, *e*, and *f*, located on the five floors of the mill. The ropes are distributed as follows: *a*, 4 ropes; *b* and *c*, 5 ropes each; *d*, *e*, and *f*, 7 ropes each. A secondary system of ropes drives the pulleys *g*, *h*, *k*, and *l*.

2052. In the second system of rope transmission, a single rope is carried around the pulley as many times as is

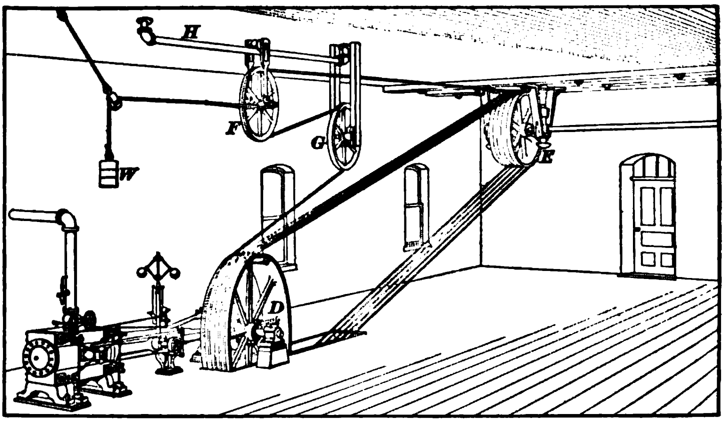


FIG. 722.

necessary to produce the required power, and the necessary tension is obtained by passing a loop of the rope around a weighted pulley. An example of this system is shown in Fig. 722.

The rope is wrapped continuously around the fly-wheel *D* and the driven pulley *E*. From the last groove of *E* the rope is led over the idlers *F* and *G*, which are set at such an angle as to lead it back to the first groove in *D*. The weight *W* is attached to the pulley *F* which is movable along the rod *H*. The movement of the pulley *F*, therefore, takes up the stretch of the rope, and keeps it always at the same tension. Rope pulleys may be attached to the shaft of the

pulley E, and the power received by E may thus be transmitted to any desired points.

The first of the above systems of transmission is used chiefly in Europe; the second, in the United States.

2053. Ropes.—The ropes used in rope transmission are either of hemp, manila, or cotton. Manila ropes are mostly used in this country. They are of three strands, hawser laid, and may be from $\frac{1}{2}$ inch to 2 inches in diameter.

The weight of ordinary manila, or cotton, rope is about $.3\,D^2$ pounds per foot of length, where D represents the diameter of the rope in inches. Letting $w =$ the weight per foot of length,

$$w = .3\,D^2. \qquad \textbf{(280.)}$$

The breaking strength of the rope varies from 7,000 to 12,000 lb. per sq. in. of cross-section. The average value may be taken as 7,000 D^2, when D is the diameter of rope.

For a continuous transmission, it has been determined by experiment that the best results are obtained when the tension in the driving side of the rope is about $\frac{1}{35}$ of the breaking strength. That is,

$$T_1 = \text{tension in tight side} = \frac{7{,}000\,D^2}{35} = 200\,D^2.$$

2054. Power Transmitted by Ropes.—The ropes run in V-shaped grooves (see Fig. 724), and the coefficient of friction is, of course, greater than on a smooth surface. The coefficient for grooves with sides at an angle of 45° may be taken at from .25 to .33. The ratio $\frac{T_1}{T_2}$ will vary from $1\frac{1}{2}$ to 3, depending upon the arc of contact and coefficient of friction.

The horsepower that can be transmitted by a single rope running under favorable conditions is given by the formula

$$H = \frac{v\,D^2}{825}\left(200 - \frac{v^2}{107.2}\right). \qquad \textbf{(281.)}$$

The horsepower transmitted by ropes of different diameters running at different velocities may be calculated from formula **281,** and plotted on cross-section paper. The

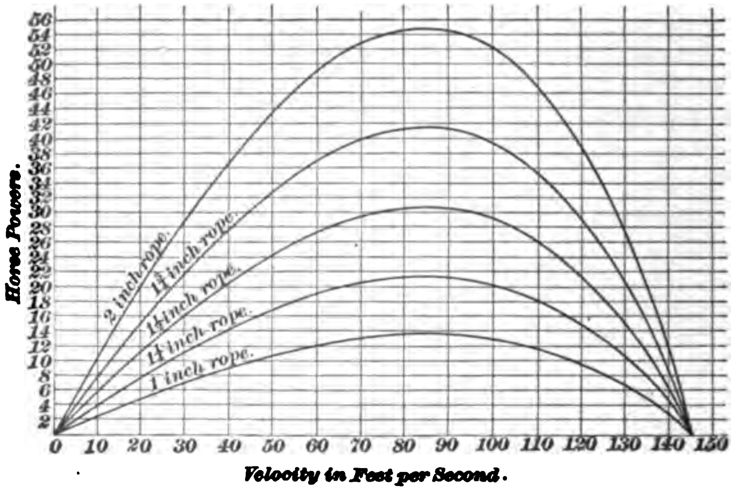


FIG. 723.

accompanying diagram (Fig. 723) shows the horsepowers transmitted by 1, $1\frac{1}{4}$, $1\frac{1}{2}$, $1\frac{3}{4}$, and 2 inch ropes for various velocities. The horizontal distances represent velocities in feet per second, and the vertical ordinates the horsepower transmitted by a single rope.

The diagram shows that the maximum power is obtained at a speed of about 84 feet per second. For higher velocities, the centrifugal force becomes so great that the power is decreased, and when the speed reaches 145 feet per second, the centrifugal force just balances the tension, so that no power at all is transmitted. Consequently, a rope should not run faster than about 5,000 feet per minute, and it is preferable, on the score of durability, to limit the velocity to 3,500 feet per minute.

EXAMPLE.—A rope fly-wheel is 26 feet in diameter, and makes 55 revolutions per minute. The wheel is grooved for 35 turns of $1\frac{1}{4}$-inch rope. What horsepower may be transmitted ?

SOLUTION.—Velocity per second =

$$v = \frac{26 \times \pi \times 55}{60} = \frac{4,492}{60} = 74.9 \text{ feet.}$$

Then, from formula **281**,

$$H = \frac{v D^2}{825}\left(200 - \frac{v^2}{107.2}\right) = \frac{74.9 \times (1\frac{1}{2})^2}{825}\left(200 - \frac{(74.9)^2}{107.2}\right) = 30.16,$$

the horsepower transmitted by one rope or turn. Then, $30.16 \times 35 = 1,055.6$ = horsepower transmitted by the 35 ropes. Ans.

Practically the same result may be obtained by referring to the diagram. It will be seen that for $1\frac{1}{2}$-inch rope running 74.9 feet per second, the horsepower per rope is slightly over 30; hence, the total horsepower is $30 \times 35 = 1,050$, nearly.

EXAMPLE.—How many times should a $1\frac{3}{4}$-inch rope be wrapped around a grooved wheel in order to transmit 400 horsepower, the speed being 3,500 feet per minute?

SOLUTION.—3,500 ft. per min. $= \frac{3,500}{60} = 58\frac{1}{3}$ ft. per sec. Referring to the diagram, a $1\frac{3}{4}$-inch rope running at a speed of $58\frac{1}{3}$ ft. per sec. transmits $36\frac{3}{4}$ horsepower. Hence, the number of turns should be $\frac{400}{36\frac{3}{4}} = 11$, nearly. Ans.

2055. Pulleys for rope gearing differ from ordinary pulleys in having a grooved rim. The sides of the groove are inclined at an angle which may vary from 30° to 60°.

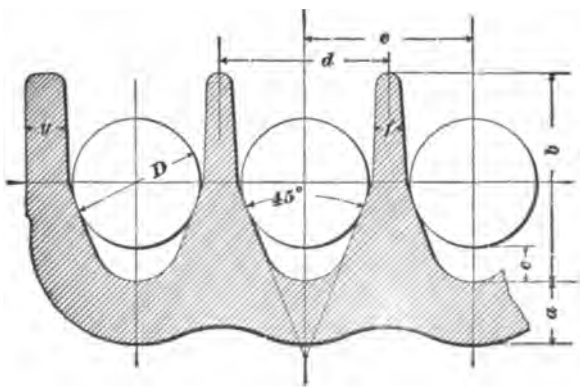


FIG. 724.

The weight of the rope wedges it into the angle of the groove, and, therefore, the more acute the angle the greater

is the coefficient of friction, and likewise the wear of the rope. The general practice, at present, is to make the angle between the sides 45°. The grooves are made circular at the bottom, and are polished or smoothed to avoid wearing the rope. A section of what is known as the English form of grooved rim is shown in Fig. 724. The following proportions may be used:

$D =$ diameter of rope in inches.

$a = \frac{1}{2} D.$	$e = \frac{4}{3} D.$
$b = 1\frac{5}{8} D.$	$f = \frac{1}{4} D.$
$c = \frac{1}{4} D.$	$g = f + \frac{1}{4}''.$
$d = \frac{4}{3} D.$	

A section of a grooved rim, in which the sides of the grooves are formed with circular arcs, is shown in Fig. 725.

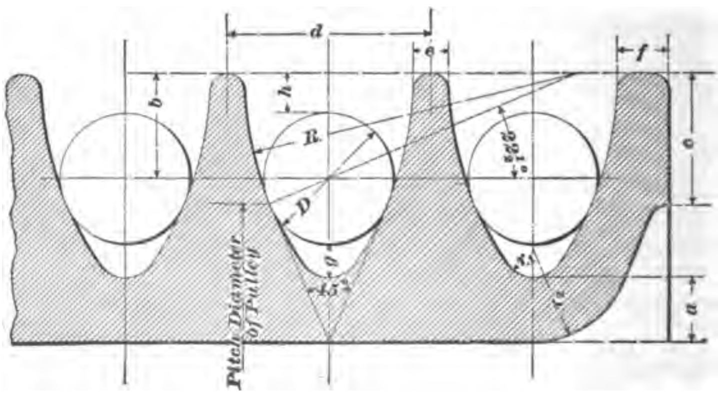


FIG. 725.

The proportions for this rim are as follows, using the diameter D of the rope as a unit:

$a = \frac{1}{2} D.$	$e = \frac{1}{4} D + \frac{1}{16}''.$
$b = \frac{3}{4} D + \frac{1}{16}''.$	$f = \frac{1}{4} D + \frac{3}{16}''.$
$c = D.$	$g = \frac{1}{3} D.$
$d = 1.6\, D.$	$h = \frac{1}{4} D + \frac{1}{16}''.$

r_1 and r_2 are to be found by trial; they should be of such lengths as to make the curves drawn by them tangent to the required lines.

The long radius R is determined by drawing a line through the center of the rope at an angle of $22\frac{1}{2}°$ with the horizontal, and producing it until it intersects a line drawn through the tops of the dividing ribs; then, with this point of intersection as a center, draw the curve forming the side of the groove tangent to the circumference of the rope.

The advantage claimed for this groove is that the rope will turn more freely in it, thus presenting new sets of fibers to the sides of the grooves, and increasing the life of the rope.

2056. Guide pulleys, idlers, and tension pulleys do not have V grooves, but the rope rests upon the bottom of a circular groove, as shown in Fig. 726.

2057. The diameter of a rope pulley should be at least 30 times the diameter of the rope. Good results are obtained when the diameters of pulleys and idlers on the driving side are 40 times, and those on the driven side 30 times the rope diameter. Idlers used simply to support a long span may have diameters as small as 18 rope diameters, without injuring the rope.

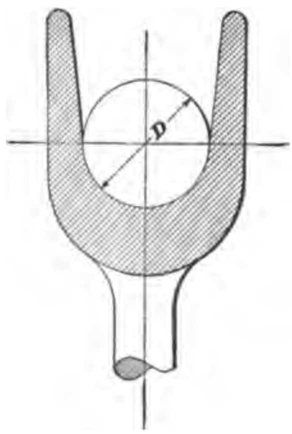


FIG. 726.

When possible, the lower side of the rope should be the driving side, for in that case the rope embraces a greater portion of the circumference of the pulley, and increases the arc of contact.

When the continuous system of rope transmission is used, the tension pulley should act on as large an amount of rope as possible. It is good practice to use a tension pulley and carriage for every 1,200 feet of rope, and have at least 10 per cent. of the rope subjected directly to the tension.

Aside from the grooved rim, rope pulleys are constructed the same as other pulleys. They may be cast solid, in halves

or in sections. The pulley grooves must be turned to exactly the same diameter; otherwise, the rope will be severely strained.

WIRE-ROPE GEARING.

2058. Telodynamic Transmission.—This name is applied to the method of transmitting power by means of wire ropes and pulleys. The method was introduced on the continent of Europe, in 1850, by C. F. Hirn, and has proven very successful and economical. Power may be transmitted great distances with very little loss.

The telodynamic transmission is very simple. It consists of driving and driven pulleys connected by a wire rope running at high velocity. When the distance is very great (sometimes several miles), relay pulleys are placed every 400 to 500 feet. The driving pulley then drives the first relay pulley, which in turn drives the second, and so on, there being a separate rope for each relay. A single rope, however, may be used over a distance of 1,000 to 2,000 feet by supporting it by guide pulleys, as shown in Fig. 727. The guide pulleys should be not more than 500 feet apart.

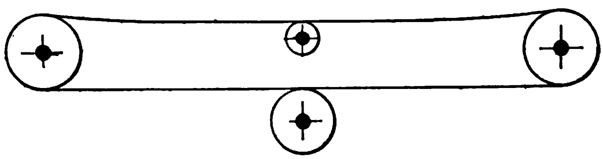


FIG. 727.

The diameter of the guide pulley in the driving side of the rope should be equal to that of the driving pulley, while the diameter of the pulley supporting the slack side may be half as great.

The pulleys are fixed to wood, iron, or masonry supports which are high enough to prevent the rope from dragging on the ground.

2059. The wire rope used for transmitting power is usually composed of 6 strands twisted around a hemp core. Each strand is composed of either 7 wires or 6 wires, and a

central hemp core. The number of strands, and wires to the strand, may be varied at pleasure.

Calling D the diameter of the rope, that is, the diameter of a ring that would just fit over the rope, and d the diameter of a single wire, we have for the ordinary rope of 42 wires,

$$D = 9\,d, \text{ or } d = \frac{D}{9}. \qquad \textbf{(282.)}$$

The weight of wire rope per foot equals

$$w = 1.43\,D^2, \text{ nearly.} \qquad \textbf{(283.)}$$

EXAMPLE.—What is the diameter of the wire composing a $\frac{5}{8}$-inch wire rope containing 42 wires, and what is its weight per foot?

SOLUTION.—$d = \frac{D}{9} = \frac{\frac{5}{8}}{9} = .07''$, nearly, or about No. 18 Brown & Sharpe's wire gauge. Ans.

Weight per foot $= 1.43\,D^2 = 1.43 \times (\frac{5}{8})^2 = .56$ lb. Ans.

The total stress allowable in a rope of iron wires may be taken at about 25,000 lb. per sq. in.; the stress in ropes of steel wire may be taken at 28,000 lb. per sq. in.

In calculating the cross-section of a rope, the sum of the cross-sections of the individual wires must be taken; thus, if n is the number of wires in the rope, the cross-section is

$$\frac{\pi}{4} d^2\,n, \text{ not } \frac{\pi}{4} D^2.$$

2060. Tensions in a Suspended Rope.—A perfectly flexible rope suspended between two points, as, for example, a wire rope suspended between two pulleys, hangs in a curve called the **catenary.** When, however, the deflection is not very great, as is usually the case in wire-rope transmission, the curve is very nearly a true parabola.

2061. In designing a wire-rope transmission, it is required to know the distance the rope will hang below the points of suspension, i. e., the **deflection** of the rope, and the tension in the rope at the pulleys.

In Fig. 728, let $A\ C\ B$ be a rope hanging from the points A and B, which are supposed to be at the same elevation;

then the tension at A or B is given by the formula

$$T = \frac{w\,a^2}{2\,h} + w\,h, \qquad (284.)$$

where w is the weight of the rope per foot of its length,

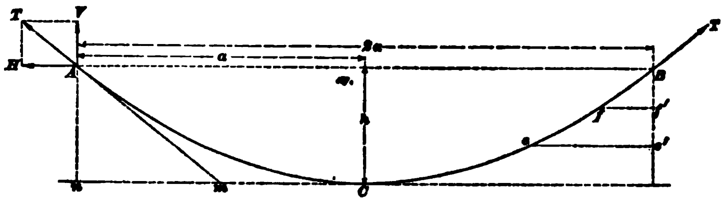


FIG. 728.

$a = \frac{1}{2}$ the distance in *feet* between the points of support, and h is the deflection in *feet* at the lowest point of the rope.

EXAMPLE.—The total distance between two rope pulleys is 400 ft., and the deflection at the center of the rope is 10 ft. Supposing the rope to weigh 1 lb. per ft., find the tension at the pulleys due to the weight of the rope.

SOLUTION.—$a = 400 \div 2 = 200$; $h = 10$; $w = 1$.

By formula **284**, we have

$$T = \frac{w\,a^2}{2\,h} + w\,h = \frac{1 \times (200)^2}{20} + 1 \times 10 = 2{,}010 \text{ lb. Ans.}$$

2062. If we solve formula **284** for h, we obtain

$$h = \frac{T}{2\,w} - \sqrt{\frac{T^2}{4\,w^2} - \frac{a^2}{2}}. \qquad (285.)$$

This formula may be used to find the deflection when the tension, weight of rope, and span are known.

2063. Stresses in a Wire Rope.—A wire rope when transmitting power is subjected to three different stresses:

1. The stress due to the direct longitudinal tension which depends upon the span, power transmitted, and weight of rope.
2. A stress caused by bending the wire around the convex portion of the pulley or sheave.
3. Stress due to centrifugal force.

The values of these stresses are expressed by the following formulas: The stress per square inch due to direct tension is

$$S_t = \frac{\frac{w a^2}{2h} + w h}{\frac{\pi}{4} d^2 n} = \frac{2 w(a^2 + 2 h^2)}{\pi d^2 n h}. \qquad (286.)$$

The stress per square inch due to bending around the pulley equals

$$S_b = \frac{E_1 d}{2R}, \qquad (287.)$$

where E_1 is the coefficient of elasticity of the material composing the wire (see Art. **1352**), d = diameter of wire composing the rope, and R = radius of pulley in inches.

The stress per square inch due to centrifugal force is

$$S_c = \frac{w v^2}{\frac{\pi}{4} g d^2 n} = \frac{4 w v^2}{\pi d^2 n g}, \qquad (288.)$$

where v = velocity of rope in feet per second, and $g = 32.16$.

The total stress per square inch is the sum of these stresses, and is equal to

$$S = S_t + S_b + S_c = \frac{2 w(a^2 + 2 h^2)}{\pi d^2 n h} + \frac{E_1 d}{2R} + \frac{4 w v^2}{\pi d^2 n g}. \quad (289.)$$

This total stress S should not exceed

25,000 lb. per sq. in. for wrought iron;
28,000 lb. per sq. in. for steel.

2064. Ratio of Tensions.—With long-rope transmissions the arc of contact does not vary much from 180°, and the coefficient of friction may be taken as about .22. With these conditions the tension T_1 on the driving side is about twice the tension T_2 on the slack side; i. e., $T_2 = \frac{1}{2} T_1$, therefore, the driving force $P = T_1 - T_2 = T_1 - \frac{1}{2} T_1 = \frac{1}{2} T_1 = T_2$.

The horsepower transmitted is

$$H = \frac{P v}{550} = \frac{P V}{33,000}.$$

One or two examples will serve to illustrate the use of the above equations and formulas.

EXAMPLE.—Power is transmitted by an iron wire rope containing 42 wires No. 15 B. & S. wire gauge, or .057 inch in diameter. The pulleys are 11 feet in diameter, and the rope runs at a speed of 4,800 feet per minute. The distance between pulleys is 400 feet. Find the horsepower transmitted, and the deflections of both tight and driving sides of rope.

SOLUTION.—The stress due to bending is

$$S_b = \frac{E_1 d}{2R}.$$

For iron E_1 is about 25,000,000. Hence,

$$S_b = \frac{25{,}000{,}000 \times .057}{2 \times \dfrac{11 \times 12}{2}} = 10{,}795 \text{ lb. per sq. in.}$$

Diameter of rope is, by formula **282**,

$$D = 9\,d = 9 \times .057 = .513''.$$

Weight of rope per foot is, by formula **283**,

$$w = 1.43\,D^2 = 1.43 \times .513^2 = .3764 \text{ lb.}$$

Stress due to centrifugal force is

$$S_c = \frac{4\,w\,v^2}{\pi\,d^2\,n\,g} = \frac{4 \times .3764 \times \left(\dfrac{4{,}800}{60}\right)^2}{3.1416 \times .057^2 \times 42 \times 32.16} = 699 \text{ lb. per sq. in.}$$

S_t can not be calculated by formula **286**, as the value of h is not known; but from formula **289**,

$$S_t = S - (S_b + S_c),$$

and since S must not exceed 25,000 for wrought iron,

$$S_t = 25{,}000 - (10{,}795 + 699) = 13{,}506 \text{ lb. per sq. in.}$$

$$T_1 = \text{maximum tension on driving side} = \frac{\pi}{4}\,d^2\,n\,S_t =$$

$$.7854 \times .057^2 \times 42 \times 13{,}506 = 1{,}448 \text{ lb.}$$

$$T_2 = \tfrac{1}{2}\,T_1 = 724 \text{ lb.};\ P = \text{driving force} = T_2 = 724 \text{ lb.}$$

$$H = \text{horsepower} = \frac{P\,V}{33{,}000} = \frac{724 \times 4{,}800}{33{,}000} = 105.3. \quad \text{Ans.}$$

For the deflection, we may use formula **285**,

$$h = \frac{T_1}{2\,w} - \sqrt{\frac{T_1^2}{4\,w^2} - \frac{a^2}{2}}.$$

The deflection of the driving side is, taking w as $\frac{3}{8}$,

$$h = \frac{1{,}448}{2 \times \frac{3}{8}} - \sqrt{\frac{1{,}448^2}{4 \times (\frac{3}{8})^2} - \frac{(200)^2}{2}} = 5 \text{ ft.} \quad \text{Ans.}$$

The deflection of the driven side is

$$h = \frac{724}{2 \times \frac{3}{8}} - \sqrt{\frac{724^2}{4 \times (\frac{3}{8})^2} - \frac{(200)^2}{2}} = 10.4 \text{ ft.} \quad \text{Ans.}$$

In order to solve the converse problem, that is, find the necessary size of a wire rope to safely transmit a given horsepower, some assumptions must be made. In the first place, the stress due to bending can not be found directly, since the diameter of the wire is unknown. We may, however, assume a ratio for $\frac{R}{d}$, and after finding the size of the wire required, the radius of the pulley is at once known. This ratio of $\frac{R}{d}$ varies from 600 to 1,400. When $\frac{R}{d} = 850$ for iron wire, the pulley diameter will be smaller than for any other ratio. The ratio should, however, be as great as the conditions will allow, for the larger the pulley for a given diameter of wire, the greater is the durability of the rope.

If the value of S_c, the stress due to the centrifugal force, be taken into account, it will complicate the solution very much; in fact, the only method of solution will be a cut-and-try method. As S_c is small in comparison with S_b or S_t for reasonable values of v, it may be neglected, and the diameter of the wire calculated as in the next example. If greater exactness is desired, substitute this value of d in formula **288**; calculate the value of S_c, and then re-calculate the value of d.

EXAMPLE.—Required, to find the diameter of the wires in a steel wire rope transmitting 200 horsepower at a velocity of 5,100 feet per minute; also, required, the diameter of the pulley and rope. The rope is to contain 42 wires.

SOLUTION.—The driving force equals

$$P = \frac{33,000\,H}{V} = \frac{33,000 \times 200}{5,100} = 1,294 \text{ lb.}$$

The tension on driving side equals

$$T_1 = 2\,P = 2 \times 1,294 = 2,588 \text{ lb.}$$

Assume the ratio $\frac{R}{d} = 900$; E_1 for steel $= 30,000,000$.

Then, the stress due to bending is

$$S_b = \frac{E_1\,d}{2\,R} = \frac{30,000,000}{2 \times 900} = 16,667 \text{ lb. per sq. in.}$$

Hence, $S_t = 28,000 - 16,667 = 11,333$ lb. per sq. in.

Cross-section of wires =

$$\frac{1}{4}\pi d^2 n = \frac{T_1}{S_t} = \frac{2,588}{11,333} = .2283 \text{ sq. in.}$$

$$d = \sqrt{\frac{.2283}{.7854 \times 42}} = .0832 \text{ in.}$$

The diameter of the rope is, therefore,

$$D = 9d = 9 \times .0832 = .7488, \text{ or say } \tfrac{3}{4} \text{ in.}$$

Assuming the diameter of rope to be $\frac{3}{4}'' = .75''$, as found, the weight per foot is by formula **283**,

$$w = 1.43\, D^2 = 1.43 \times .75^2 = .8044 \text{ lb.}$$

By formula **282**, $d = \frac{D}{9} = \frac{.75}{9} = .0833.$

Hence, by formula **288**,

$$S_o = \frac{4 \times .8044 \times \left(\frac{5,100}{60}\right)^2}{3.1416 \times .0833^2 \times 42 \times 32.16} = 790.$$

And, $S_t = 28,000 - (16,667 + 790) = 10,543$ lb. per sq. in.

Cross-section of wires =

$$\frac{1}{4}\pi d^2 n = \frac{T_1}{S_t} = \frac{2,588}{10,543} = .2455 \text{ sq. in.,}$$

and $$d = \sqrt{\frac{.2455}{.7854 \times 42}} = .0862. \quad \text{Ans.}$$

The diameter of the rope is, therefore,

$$.0862 \times 9 = .7758'', \text{ say } \tfrac{3}{4}'',$$

the same value as obtained when S_o was neglected. Ans.

Since $\frac{R}{d} = 900$, $R = 900\, d = 900 \times .0862 = 77.58''$, and the diameter of the pulley $= 2 \times 77.58 = 155'' = 12$ ft. 11″. Ans.

2065. Wire ropes are used also for hoisting and hauling loads in and about mines. The calculation of the size of a wire rope for any purpose is similar to that given above. If the rope is subjected to a straight pull, and is not bent around a sheave or drum, the cross-section may be found directly from the load and allowable safe stress; that is, cross-section $= \frac{\text{total load on rope}}{\text{safe stress per sq. in.}}$.

When the rope is bent around a pulley or drum, the stress due to bending, that is, $S_b = \frac{E_1 d}{2R}$, must be subtracted from the total safe stress.

Wire ropes used for hoisting are often made with 6 strands of 19 wires each, wound around a central hemp core, making in all $6 \times 19 = 114$ wires. This rope is more flexible than the regular transmission rope, and is, therefore, injured less in passing over small pulleys; but it will not stand as much wear when dragged over rough surfaces, as the wires of which it is composed are so much smaller.

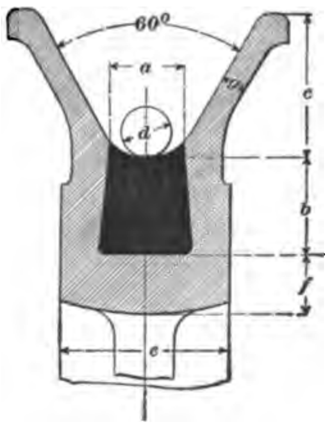


FIG. 729.

2066. Wire-Rope Pulleys.—The pulleys, or sheaves, used in wire-rope transmission are made of cast iron with a groove lined with rubber, gutta-percha, leather, or other similar substance, on which the rope runs. The grooves are made so wide that the rope rests on the rounded bottom instead of being wedged against the sides, as in the case of hemp or cotton rope.

The proportions of the pulley rim are shown in Fig. 729. They are as follows:

$$d = \text{diameter of rope};$$

$$a = d + \tfrac{1}{4}''. \qquad e = 2\,d + \tfrac{3}{4}''.$$
$$b = d + \tfrac{1}{2}''. \qquad f = \tfrac{1}{2}\,d + \tfrac{3}{8}''.$$
$$c = 3\,d. \qquad g = \tfrac{1}{4}\,d + \tfrac{1}{8}''.$$

The arms may be cross-shaped or oval; the latter form is preferable, as it offers less resistance to the air when the pulley is run at high speed. The size of the arms corresponds to those of belt pulleys transmitting the same force. For the number of arms Reuleaux gives the formula

$$n = 4 + \frac{R}{40\,D}, \qquad \textbf{(290.)}$$

where R = radius of pulley in inches and D = diameter of

rope. The diameter of the pulley is fixed by the diameter of the rope and the number of wires in a strand. For a rope with seven wires to the strand, the diameter of the pulley should not be less than 150 times the diameter of the rope; and for a rope with 19 wires to the strand, the proportion should not be less than 90 to 1.

Table 53 gives the breaking strength and the power transmitted by various sizes of ropes, as determined by practical experience:

TABLE 53.

POWER TRANSMITTED BY WIRE ROPES (42 WIRES).

Diameter of Ropes, Inches.	Diameter of Pulleys, Feet.	Revolutions per Minute.	Breaking Stress of Rope per Pound.	Horsepower Transmitted.	Velocity of Rope in Feet per Second.
$\frac{7}{16}$	5	100	4,260	8.6	26
$\frac{15}{32}$	6	100	5,660	13.4	31
$\frac{1}{2}$	7	100	8,200	21.1	36
$\frac{5}{8}$	8	100	11,600	27.5	42
$\frac{5}{8}$	8	120	11,600	33.0	50
$\frac{5}{8}$	9	100	11,600	51.9	47
$\frac{5}{8}$	9	120	11,600	62.2	56
$\frac{11}{16}$	10	100	15,200	73.0	52
$\frac{11}{16}$	10	120	15,200	87.6	62
$\frac{11}{16}$	10	140	15,200	102.2	73
$\frac{11}{16}$	12	100	15,200	116.7	63
$\frac{3}{4}$	12	120	17,600	148.9	75
$\frac{3}{4}$	12	140	17,600	173.7	87
$\frac{3}{4}$	14	100	17,600	185.0	73
$\frac{3}{4}$	14	120	17,600	222.0	87
$\frac{3}{4}$	15	120	17,600	300.0	94

NOTE.—The student may obtain much information concerning wire ropes from the trade catalogue of John A. Roebling's Sons Co., Trenton, N. J.

CHAINS.

2067. Chains may be used as simple fastenings or as belts for transmitting power. The ordinary, or open-link and the stud-link round iron chains are shown in Fig. 730. The links are made from round iron bars which are cut off at the proper length, bent, and welded. The links should be made as small as possible, both on account of strength and flexibility. Ordinary chain proportions are shown on the figure. They are as follows:

$$d = \text{diameter of iron};$$

$$\text{For open link} \begin{cases} a = 4\tfrac{1}{2}\,d \text{ to } 5\,d; \\ b = 3\tfrac{1}{2}\,d. \end{cases}$$

$$\text{For stud link} \begin{cases} a = 5\,d \text{ to } 6\,d; \\ b = 3\tfrac{1}{2}\,d \text{ to } 3\tfrac{3}{4}\,d; \\ c = .6\,d; \\ e = .7\,d. \end{cases}$$

Link chains which are used merely to support loads, as in suspension bridges, etc., have the links from 3 to 9 feet or more in length. As such chains do not belong properly to the subject of Machine Design, they will not be considered here.

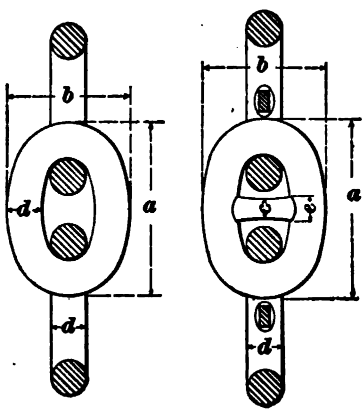


FIG. 730.

2068. Strength of Chains.—The strength of a chain is less than that of the iron composing it, on account of the weld, and also because of the presence of bending action.

Formulas **130** and **131**, Art. **1421**, may be used to find the safe load in ordinary cases. For crane chains which require a large factor of safety, Towne gives the following as the safe load:

$$P = 3.3d^2 \text{ tons} = 6{,}600\, d^2 \text{ lb.} \qquad \textbf{(291.)}$$

The *weight of chains* (open and stud link) may vary from $9\,d^2$ to $9\frac{1}{2}\,d^2$ pounds per foot.

2069. Chain Drums.—When a chain must be coiled, as in the case of cranes and derricks, a grooved drum may be used. The groove passes spirally around the drum, and is just wide enough to receive the edge of a link of the chain. The drum may have a diameter of from $24\,d$ to $30\,d$ or more; the length should be such that the total amount of chain may be coiled on in one layer; because, if one layer is wound over another, the chain is injured.

Instead of a drum, a wheel or sheave with pockets may be used. Such a wheel requires less space than the drum, and injures the chain less. A form of chain wheel largely

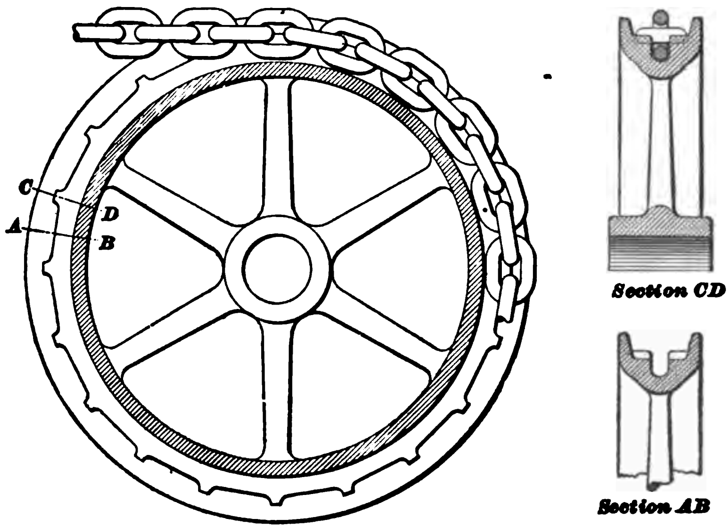


FIG. 731.

used for transmitting power, especially on cranes, chain blocks, etc., is shown in Fig. 731. The rim of the wheel is grooved for the links, and pockets are provided into which the links that lie parallel with the axis of the wheel rest.

The pitch of the pockets must, of course, be the same as the pitch of the links.

2070. Flat-link chains are used for driving machinery where very heavy resistances are to be overcome, as, for example, in wire-drawing machines, cranes, and dredging machines.

When the chain merely supports a load, it may have the form shown in Fig. 732. It consists of flat plates which are connected by pins. The pins are evidently in shear, and the plates are in direct tension. Since each of the two parallel plates carries one-half the load, it should have one-half the thickness of the single plate to which it is pinned.

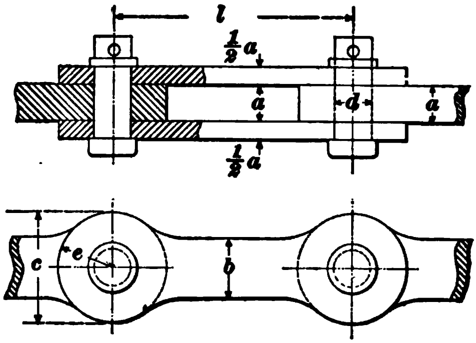


FIG. 732.

The links may be of any length desired, but the shortest convenient length is about $l = 3d$. The cross-section $= ab = \frac{P}{S_t}$, where P is the load and S_t the safe stress in tension. Taking both P and S_t in tons, we may assume $S_t = 5$ tons.

Then, $$ab = .2P.$$

The shearing section of a pin is $2 \times \frac{1}{4}\pi d^2 = \frac{1}{2}\pi d^2$. Therefore,

$$\tfrac{1}{2}\pi d^2 = \frac{P}{S_s},$$

where S_s = safe shearing stress.

Assuming $S_s = 4$ tons, we have

$$d = \sqrt{\frac{P}{2\pi}}.$$

The following proportions may be used in ordinary cases:

$$b = \tfrac{4}{3}d. \quad c = \tfrac{8}{3}d. \quad e = \tfrac{11}{8}d.$$

When the links are short, the width b may be the same throughout. The pin connecting the plates may be riveted over or secured by a washer and split pin.

A flat-link chain may be used for transmitting powers somewhat after the fashion of a belt. The chain passes over wheels provided with teeth which engage with the links of the chain. Such a wheel is known as a **sprocket wheel.** Examples of chains used in this way are met with in agricultural machinery, bicycles, coal-mining machines, dredges, etc.

The flat-link gearing chain, Fig. 733, consists of two series of flat links which are kept some distance apart by the pins

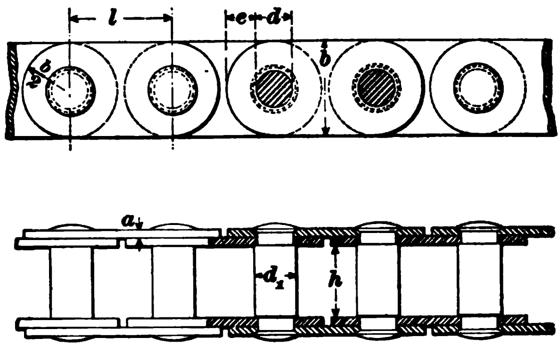


FIG. 733.

which connect them. These pins engage with the teeth of the sprocket wheel; they are enlarged between the series of plates so as to form a shoulder to prevent the plates from slipping, and also to give a greater wearing surface.

Let n = number of plates on one side of chain;
a and b = thickness and breadth of plates, respectively;
d = diameter of ends of pin;
d' = diameter of center of pin;
h = length of enlarged part of pin;
l = length of link between centers of pins;
P = total load on chain in pounds.

All dimensions in inches.

Then, the following formulas and proportions are generally used:

$$n = .13\sqrt[3]{P}; \qquad h = 1.7\,d + .5;$$

$$d = .0115\sqrt{\frac{P}{\sqrt{n}}}; \qquad l = 2.9\,d;$$

$$b = 2.5\,d; \qquad a = \frac{85\,d}{n + l};$$

$$d' = 1.2\,d; \qquad e = .85\,d.$$

EXAMPLE.—Calculate the dimensions of a plate-link gearing chain for a working load of 8,000 lb.

SOLUTION.—Number of plates on one side =

$$n = .18\sqrt[3]{P} = .13\sqrt[3]{8{,}000} = 2.6, \text{ say } 3;$$

$$d = .0115\sqrt{\frac{P}{\sqrt{n}}} = .0115\sqrt{\frac{8{,}000}{\sqrt{3}}} = .78, \text{ say } \tfrac{3}{4}'';$$

$$d' = 1.2\,d = 1.2 \times \tfrac{3}{4} = .9'', \text{ say } \tfrac{29}{32}'';$$

$$b = 2.5\,d = 2.5 \times \tfrac{3}{4} = 1\tfrac{7}{8}'';$$

$$h = 1.7\,d + .5 = 1.7 \times \tfrac{3}{4} + .5 = 1.775'';$$

$$l = 2.9\,d = 2.9 \times \tfrac{3}{4} = 2.175'';$$

$$a = \frac{.85\,d}{n + l} = \frac{.85 \times \frac{3}{4}}{3 + 2.175} = .12'', \text{ say } \tfrac{1}{8}'';$$

$$e = .85\,d = .85 \times \tfrac{3}{4} = .64''.$$

Various other forms of gearing chains are in common use. In designing machinery requiring the use of such chains it is customary to use some standard size made by a company engaged in the manufacture of chains.

It would be well for the student to send for the catalogue of some such company, and observe the proportions and sizes there adopted. Much good information may be obtained from the catalogue of The Link Belt Engineering Company, Chicago, Ill.

2071. Hooks.—Chains used on cranes and derricks must be provided with hooks for connecting to the load to be raised. The design of a crane hook must be made with care, since a break may cause loss of property and life.

A hook may be treated theoretically, as follows: The

maximum stress comes on the section *m n* of the hook. See Fig. 734. Let the center of gravity of this section be a distance *a* from the line of action of the load, and a distance *c* from the inside edge of the hook.

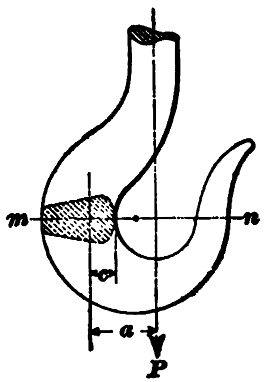


FIG. 734.

Let S_t = the stress in the section due to direct tension;
S_b = the stress due to bending;
A = area of section;
I = moment of inertia of section.

Then, for direct tension, $P = A\,S_t$, or $S_t = \frac{P}{A}$.

For bending, letting S_b = *safe* bending stress, the moment $= Pa = \frac{S_b I}{c}$ (see Art. **1398**), or $S_b = \frac{Pac}{I}$.

Now, $S_b + S_t$, equals total stress, must not exceed a safe value, say about 12,000 lb., or 6 tons per sq. in. Then, if we take P in tons,

$$S_b + S_t = 6 \text{ tons} = P\left(\frac{1}{A} + \frac{ac}{I}\right).$$

Suppose, for example, we assume the section of the hook to be a rectangle, of length b and width $\frac{2}{3}b$, and let the distance a, Fig. 734, equal the length b of the section; what should be the dimensions of section for a load of 3 tons?

We have

$$6 = P\left(\frac{1}{A} + \frac{ac}{I}\right) = 3\left(\frac{1}{b \times \frac{2}{3}b} + \frac{b \times \frac{1}{2}b}{\frac{1}{12}b^3 \times \frac{2}{3}b}\right),$$

$$\text{or } b^2 = \tfrac{21}{4}.$$

$$\text{Hence, } b = \sqrt{\frac{21}{4}} = 2.29, \text{ say } 2\tfrac{5}{16}'';$$

$$\text{and } \tfrac{2}{3}b = 1.53, \text{ say } 1\tfrac{17}{32}''.$$

The proportions shown in Fig. 735 are those used by a large crane manufacturing company. They are based on the diameter of the iron rod of which the hook is made. This diameter may be obtained by the following formula:

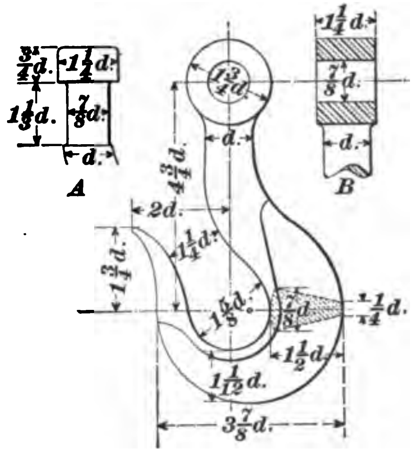


FIG. 735.

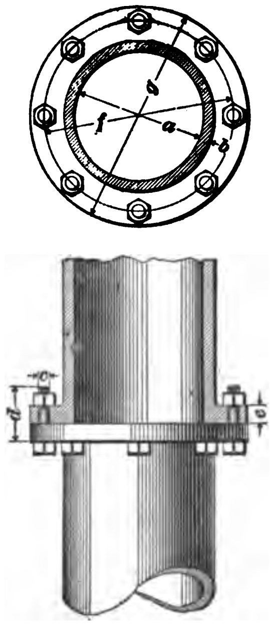


FIG. 736.

$d = \sqrt{P}$, where P is the load in tons.

The hook may have an eye, as shown in the figure, or a neck for a swivel, as shown at A.

PIPE FLANGES.

2072. The ends of cast-iron pipes, elbows, etc., are often connected by means of flanges and bolts.

Fig. 736 shows the method of flanging and bolting the ends of two cast-iron pipes. The dimensions of the flanges for the various sizes of pipes are given in the following table:

TABLE 54.

STANDARD PIPE FLANGES.

n = number of bolts.

a	*b*	*c*	*d*	*n*	*e*	*f*	*g*
2.0	.409	$\frac{5}{8}$	2.0	4	$\frac{5}{8}$	4.75	6.0
2.5	.429	$\frac{5}{8}$	2.25	4	$\frac{11}{16}$	5.25	7.0
3.0	.448	$\frac{5}{8}$	2.5	4	$\frac{3}{4}$	6.0	7.5
3.5	.466	$\frac{5}{8}$	2.5	4	$\frac{13}{16}$	6.5	8.5
4.0	.486	$\frac{3}{4}$	2.75	4	$\frac{15}{16}$	7.25	9.0
4.5	.498	$\frac{3}{4}$	3.0	8	$\frac{15}{16}$	7.75	9.25
5	.525	$\frac{3}{4}$	3.0	8	$\frac{15}{16}$	8.5	10.0
6	.563	$\frac{3}{4}$	3.0	8	1	9.625	11.0
7	.600	$\frac{3}{4}$	3.25	8	$1\frac{1}{16}$	10.75	12.5
8	.639	$\frac{3}{4}$	3.5	8	$1\frac{1}{8}$	11.75	13.5
9	.678	$\frac{3}{4}$	3.5	12	$1\frac{1}{8}$	13.0	15.0
10	.713	$\frac{7}{8}$	3.625	12	$1\frac{3}{16}$	14.25	16.0
12	.790	$\frac{7}{8}$	3.75	12	$1\frac{1}{4}$	16.5	19.0
14	.864	1	4.25	12	$1\frac{3}{8}$	18.75	21.0
15	.904	1	4.25	16	$1\frac{3}{8}$	20.0	22.25
16	.946	1	4.25	16	$1\frac{7}{16}$	21.25	23.5
18	1.020	$1\frac{1}{8}$	4.75	16	$1\frac{9}{16}$	22.75	25.0
20	1.090	$1\frac{1}{8}$	5.0	20	$1\frac{11}{16}$	25.0	27.5
22	1.180	$1\frac{1}{4}$	5.5	20	$1\frac{13}{16}$	27.25	29.5
24	1.250	$1\frac{1}{4}$	5.5	20	$1\frac{7}{8}$	29.5	32.0
26	1.300	$1\frac{1}{4}$	5.75	24	2	31.75	34.25
28	1.380	$1\frac{1}{4}$	6.0	28	$2\frac{1}{16}$	34.0	36.5
30	1.480	$1\frac{3}{8}$	6.25	28	$2\frac{1}{8}$	36.0	38.75
36	1.710	$1\frac{3}{8}$	6.5	32	$2\frac{3}{8}$	42.75	45.75
42	1.870	$1\frac{1}{2}$	7.25	36	$2\frac{5}{8}$	49.5	52.75
48	2.170	$1\frac{1}{2}$	7.75	44	$2\frac{3}{4}$	56.0	59.5

2073. The larger sizes of wrought-iron pipe, especially for high pressures, are best connected by means of flanges. Fig. 737 shows three styles of fastening these flanges to the pipe.

A is a cast-iron flange screwed to the end of the pipe by means of the standard pipe thread. It is much used, especially for sizes of pipe below 6″.

B is a wrought-iron or steel flange which is screwed to the end of the pipe, and the end of the pipe is then riveted over as shown at *a*.

C is a forged or rolled steel flange that is used for large pipes and high pressures. The pipe is fastened to the flange

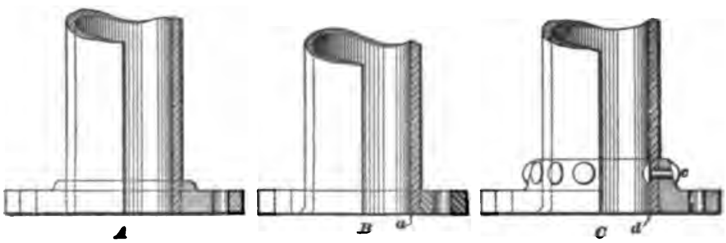


FIG. 737.

by rivets as shown at *c*, and the end is also calked or riveted over at *d* so as to form a tight joint.

The diameters of flanges and bolt circles, and number and sizes of bolts for wrought-iron pipe flanges, should be the same as given in Table 54; for, then, the flange for a pipe of a given size, whether cast or wrought iron, will fit any other pipe, valve, or fitting of the same size.

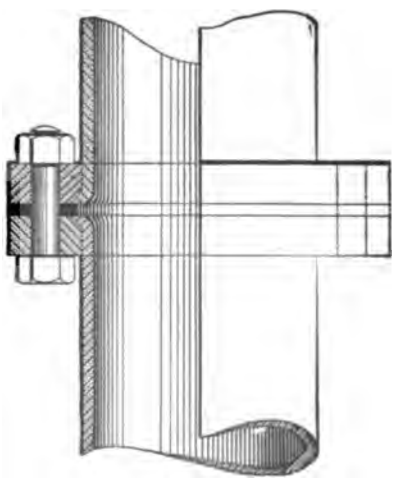


FIG. 738.

The thickness of the flanges may also be the same, although wrought-iron and steel flanges are usually made somewhat thinner.

2074. Gaskets.—In order to secure a tight joint, a thin strip of some soft material is placed between the flanges as shown in Fig. 738. The material most

commonly used is either sheet rubber or paper, cut to the size of the flange.

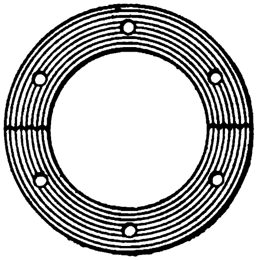


FIG. 739.

For steam pressures above 100 pounds per square inch, however, these materials are injured by the heat and often make trouble. To overcome these difficulties gaskets are sometimes made of lead or copper. Fig. 739 shows a gasket made of thin sheets of corrugated copper, as shown in the section.

MACHINE DESIGN.

(CONTINUED.)

STEAM ENGINE DESIGN.

PRELIMINARY DATA.

2075. The designer of an engine has for his preliminary data:

1. The type of engine desired; that is, simple or compound, horizontal or vertical, Corliss or marine, etc.
2. The horsepower to be developed.
3. Sometimes the boiler pressure.

He must determine:

1. Boiler pressure, if not already known.
2. Piston speed.
3. Point of cut-off.
4. Clearance.
5. Back pressure and compression.

For a simple engine, after the above data have been obtained, a theoretical indicator diagram may be drawn, and the mean effective pressure determined. Then the proportions of the cylinder can be calculated, and the design of the other parts readily follows. For a compound or triple expansion engine a more complicated process is necessary.

2076. The **boiler pressure** may be fixed beforehand if the engine is to have steam furnished by an existing boiler, or set of boilers, carrying a definite pressure. Theoretically, the boiler pressure should be made as high as the conditions will allow, since the thermal efficiency of the engine is

increased by increasing the range of pressures. There is, however, a practical limit to the pressure to be employed, and the following are about the pressures used for the types of engines indicated:

Simple	70 to 120 lb.
Compound	100 to 150 lb.
Triple	150 to 200 lb., or higher.
Locomotive	140 to 210 lb.

For condensing engines the average boiler pressure for a given type will probably be from 15 to 30 pounds lower than given above.

The initial pressure in the engine cylinder will be less than the boiler pressure on account of loss caused by resistance to flow through the steam pipe and connections. Ordinarily the loss may be taken at about 8 per cent. of the boiler pressure.

2077. The **piston speed** for the various types of engines has already been discussed in Art. **1270.**

The tendency of modern engine builders is towards increased piston speed and higher steam pressures.

2078. The **point of cut-off** should, on the score of economy, usually occur early in the stroke. This is not possible in the case of simple slide-valve engines, but may be accomplished with engines fitted with Corliss valve gear or expansion valves. For ordinary simple engines, Mr. C. E. Emery gives the following formula for the most economical cut-off:

Let k = real cut-off;
e = number of expansions;
p_i = absolute initial pressure in lb. per sq. in.

Then,
$$k = \frac{1}{e} = \frac{22}{22 + p_i}. \qquad \textbf{(292.)}$$

Formula **292** gives results rather too large for compound engines.

2079. The **clearance** may vary from $\frac{1}{2}$ to 3 per cent. in Corliss engines, and from 4 to 14 per cent. in high-speed engines.

The distance between the piston at end of stroke and the cylinder head, or the **piston clearance,** should be made as small as possible. On small stationary engines this distance may be $\frac{1}{4}$ inch, and it rarely exceeds $\frac{1}{2}$ inch on the largest marine engines. In some cases in actual practice, with a low-pressure cylinder 7 feet in diameter and a conical piston, this clearance is only $\frac{3}{8}$ inch.

2080. The **back pressure** should not exceed 16 or 17 lb. (absolute) in a well-designed non-condensing engine. For a condensing engine, the back pressure may be from 2 to 4 lb. per sq. in. The proper amount of compression can not be determined until the reciprocating parts are designed and their weight found. For slow-speed engines the compression is usually small. For high-speed engines the compression is usually large. For high-speed engines the pressure at the end of compression may be

$$p_o = \frac{p_i + 16}{2}, \qquad \textbf{(293.)}$$

where p_i is the absolute initial pressure, and p_o is the absolute pressure at the end of compression.

2081. In order to show the general method of procedure in the design of an engine, we will take the following example:

Determine the data for a simple high-speed automatic non-condensing engine which is to develop 120 indicated horsepower.

For an engine of this type the boiler pressure may be taken at 85 pounds, gauge. The initial cylinder pressure will then be about $85 - .08 \times 85 = 78.2 = 78.2 + 14.7 = 92.9$, say 93 pounds, absolute. As the engine is to be non-condensing, we may take the back pressure as 17 pounds. Assume a clearance in this case of 6 per cent. The cut-off from formula **292** equals

$$k = \frac{22}{22 + 93} = .192, \text{ say } \tfrac{1}{5}.$$

By formula **293**, the steam is compressed to

$$\frac{93 + 16}{2} = 54\tfrac{1}{2} \text{ lb., absolute.}$$

We now have sufficient data to draw a theoretical diagram;

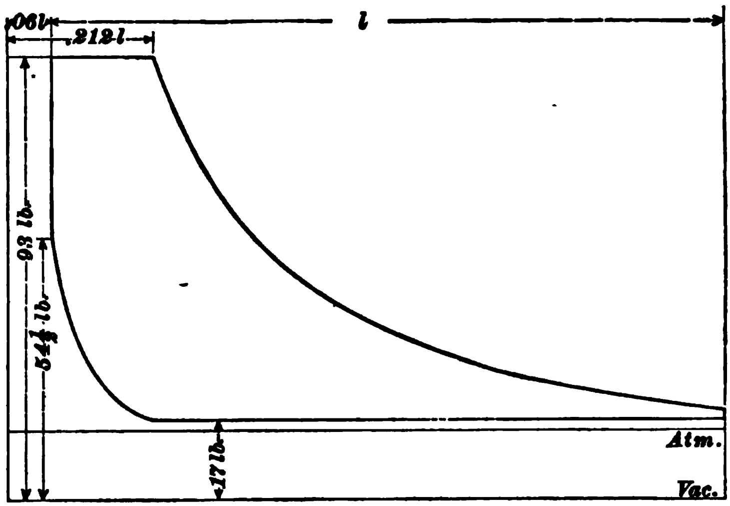


FIG. 740.

see Fig. 740. In drawing the diagram some convenient scale may be chosen for the pressures; say, 30 pounds per inch. Then, the height of the diagram above the vacuum line will be $93 \div 30 = 3.1$ in.

The length of the diagram is immaterial; assume for convenience that it is 4 in. Then, $l = 4$ in., and the clearance $= 4 \times .06 = .24$, say $\frac{1}{4}$ in. Now draw the diagram, assuming that both expansion and compression curves are equilateral hyperbolas (see Arts. **1161** and **1222**). By measurement or by calculation, the *mean effective pressure*, or the M. E. P., of the diagram, Fig. 740, is found to be 28.3 lb. per sq. in.

On account of cylinder condensation and other losses, the M. E. P. given by the theoretical card is never attained by the actual engine. To find the probable M. E. P. of the actual engine, the M. E. P. of the theoretical card must be multiplied by a factor, the magnitude of which depends upon

the style of engine. A good authority gives the following factors for the types of engines indicated:

Simple Engines.	Factor.
Special valve gear, engine jacketed...........	0.94.
Good ordinary valves, engine jacketed, large ports..................................	0.90 to .92.
Ordinary valve gear, unjacketed..............	0.80 to .85.

In the present case, we will assume the factor to be .85; then, the probable M. E. P. is $28.3 \times .85 = 24$ lb. per sq. in. nearly.

In order to be a high-speed engine, the piston speed should be at least 600 feet per minute. Suppose 700 feet per minute is assumed; then, the area A of the cylinder equals

$$A = \frac{33{,}000 \times \text{horsepower}}{\text{M. E. P.} \times \text{piston speed}} = \frac{33{,}000 \times 120}{24 \times 700} = 235.7 \text{ sq. in.}$$

Then, diameter of cylinder $= d = 17\frac{1}{4}$ inches.

For convenience, take $d = 17$ in. Then, $A = 227$ sq. in., and the piston speed will be $\frac{33{,}000 \times 120}{24 \times 227} = 727$ ft. per min.

The length of the stroke may be made 24 inches; hence, the number of revolutions will be $\frac{727 \times 6}{24} = 182$ per minute.

The diameter of the low-pressure cylinder of a compound or triple-expansion engine may be found in the above manner by assuming that all the work is to be done in the low-pressure cylinder. In this case the factor by which to multiply the theoretical M. E. P. to obtain the probable M. E. P. is from .7 to .8 for a compound and .6 to .7 for a triple-expansion engine. The ratio of the volumes of the high, intermediate, and low-pressure cylinders being determined (see Art. **1306**), the diameter of the high and intermediate cylinders may be found from that of the low-pressure cylinder.

CYLINDERS AND STEAM CHESTS.

2082. Fig. 741 is an example of a cylinder designed for a simple slide-valve engine. The front head A is cast solid with the cylinder. The method of fastening to the frame B is clearly shown.

The principal dimensions of this cylinder may be determined from the following proportions :

D = diameter of cylinder.
L = length of stroke + thickness of piston + twice the piston clearance.
C = length of stroke + distance from outer edge to outer edge of piston rings − $(.01\,D + .125'')$.
$a = 5.5\,i.$
$b = 4.2\,i.$
$c = i.$
$d = i.$
e' = the net area of a single cylinder head bolt whose nominal diameter is $e = \frac{A\,P}{4{,}000\,n}$,

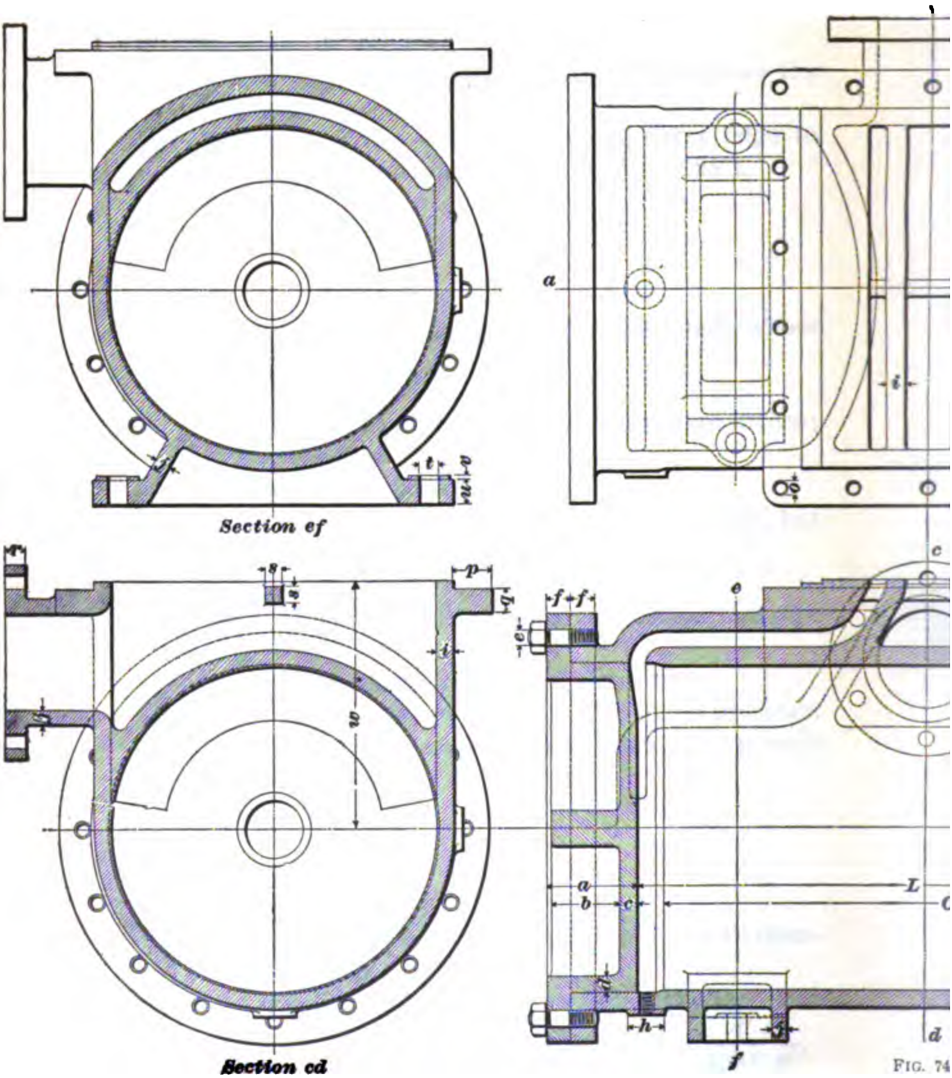
where A = the area of cylinder head in square inches;
P = the steam pressure;
n = the number of bolts.

The pitch of the bolts may be from 4.5 to 5.5 inches, but should never be more than $5\,f$.

$f = 1.5\,i.$
$g = .04\,D + .125''$. Take the nearest nominal size pipe tap.
h = twice the outside diameter of drain pipe.
$i = .0003\,P\,D + .375''$, where P is the steam pressure. If the steam pressure is less than 100 pounds make $P = 100$.
$j = .85\,i.$
$k = 4\,i.$
$l = .75\,i.$
$m = 1.01\,D + .125''.$
$n = m + 6\,e$, never less. Here e is the nominal diameter of the bolt.
o = the nominal diameter of steam-chest bolts. The net area of a single steam-chest bolt $= \frac{A'\,P}{4{,}000\,n'}$,

where A' = area of steam chest,
n' = number of bolts in steam chest.

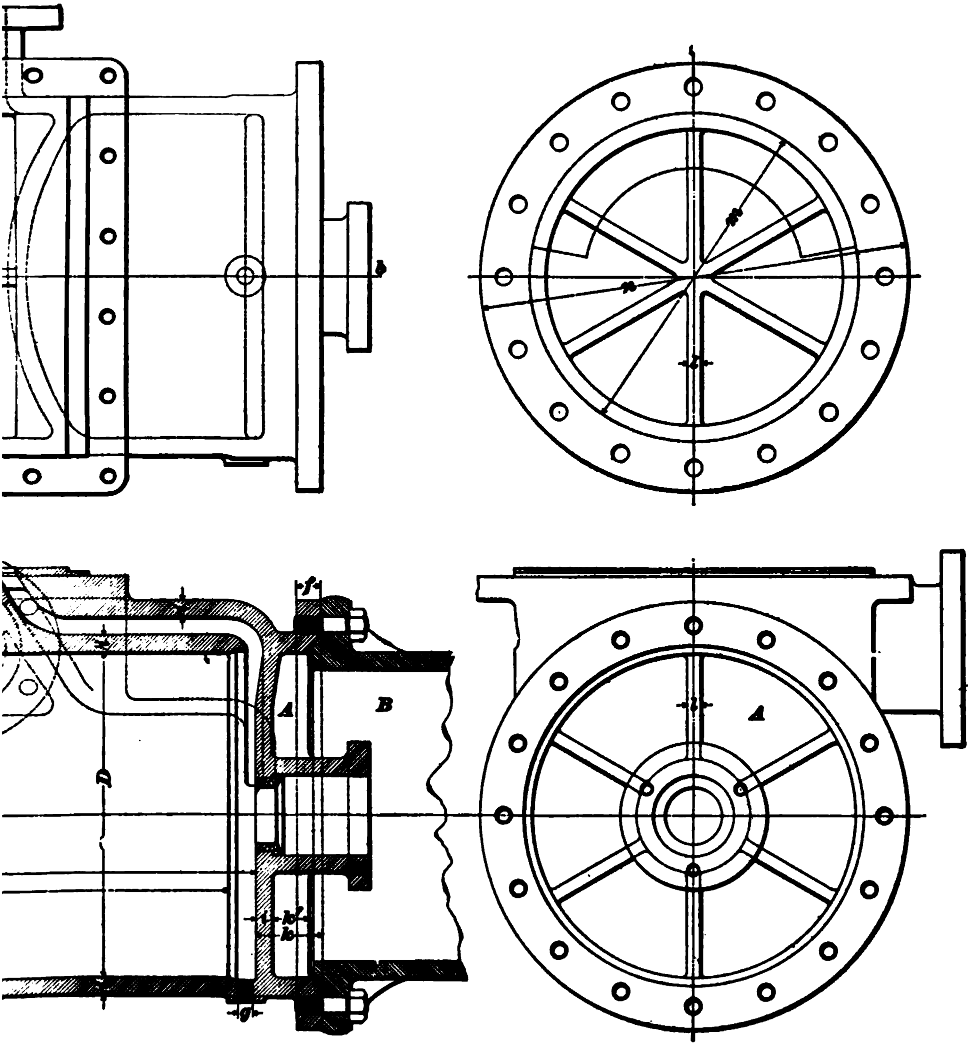
$p = 2.75\,o.$



Section ef

Section cd

FIG. 74


A
B
D
f
g
A

$q = 1.5\,r$.
$r = 1.25\,i$.
$s = i$. This is required only when the length of the port is greater than 12″.
$t = 1.25\,i$. When D is greater than 24″ use 4 bolts in the standard and make $t = 1.1\,i$.
$u = 1.5\,i$.
$v = .25''$, constant.

2083. Steam Ports and Passages.—The dimensions of the *steam ports*, *exhaust ports*, and other steam passages, depend upon the velocity of the flow of steam. The ports and passages must be large enough to allow the steam to follow up the advancing piston without loss of pressure. The maximum allowable velocity of the steam in the passages, when they are short, is about 160 feet per second. But, with the ordinary ratio between the length of connecting-rod and length of crank, the average velocity is about $\frac{5}{8}$ of the maximum. Hence, the allowable average velocities are 100 to 125 feet per second for long and short passages, respectively.

Let l = length of port in inches;
b = breadth of port in inches;
A = area of cylinder;
S = average piston speed in feet per second;
v = average velocity of steam in feet per second.

Then, area of port × velocity of steam = area of piston × velocity of piston,

$$\text{or } l\,b\,v = A\,S,$$

$$\text{whence,} \quad l\,b = \frac{A\,S}{v}. \qquad \textbf{(294.)}$$

Take $v = 100$ for long indirect passages, and 125 for short direct passages.

The constant 100 may be used for v when designing plain slide-valve engines of the ordinary type which cut off late in the stroke, and 125 may be used for high-speed engines with early cut-off, and for the Corliss type.

The area of the exhaust port or ports may be from $1\frac{3}{8}$ to $2\frac{1}{2}$ times the area of a steam port.

The area of the cross-section of the steam pipe is approximately equal to the area of the steam port; likewise, the area of the exhaust pipe should be equal to that of the exhaust port.

The length l of the port may be $.6\ D$ to $.9\ D$ for slide-valve engines, and about $.9\ D$ to D for the Corliss type.

The height w, Fig. 741, of the valve seat must be such that the area of the most contracted part of the exhaust port is not less than 75% of the area of the steam port.

2084. Fig. 742 shows a design for a cylinder having the steam chest cast solid with it. The front head in this

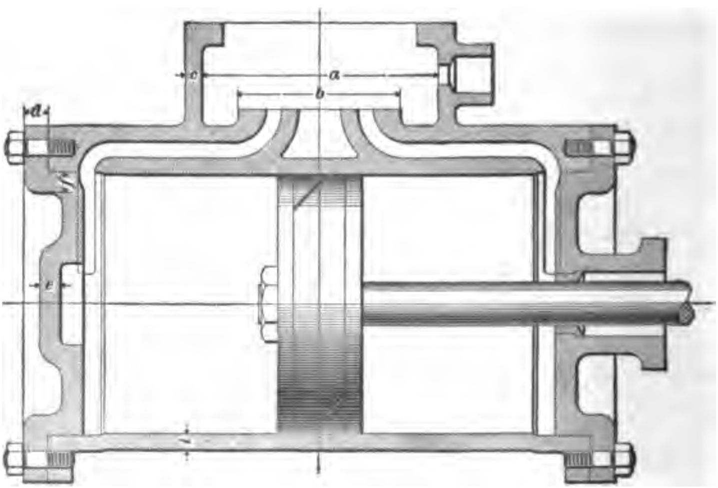


FIG. 742.

case is a separate casting fitted to the cylinder in the same manner as the back head. The heads, which are cast without ribs, are well suited for cylinders of small diameters. For larger diameters, the ribbed heads shown in Fig. 741 are better.

The following proportions apply to Fig. 742:

$i = .0003\,PD + .375''$.

a = length of valve + travel of valve + twice the clearance between valve and steam chest at ends of valve travel.

b = the valve travel + length of valve − $\frac{1}{4}''$ to $\frac{1}{2}''$.

$c = i$.

$d = 1.5\,i$.

$e = 1.25\,i$.

$f = 1.25\,i$.

All other dimensions are to be determined by the proportions given for Fig. 741.

2085. Fig. 743 is a steam chest for the cylinder shown

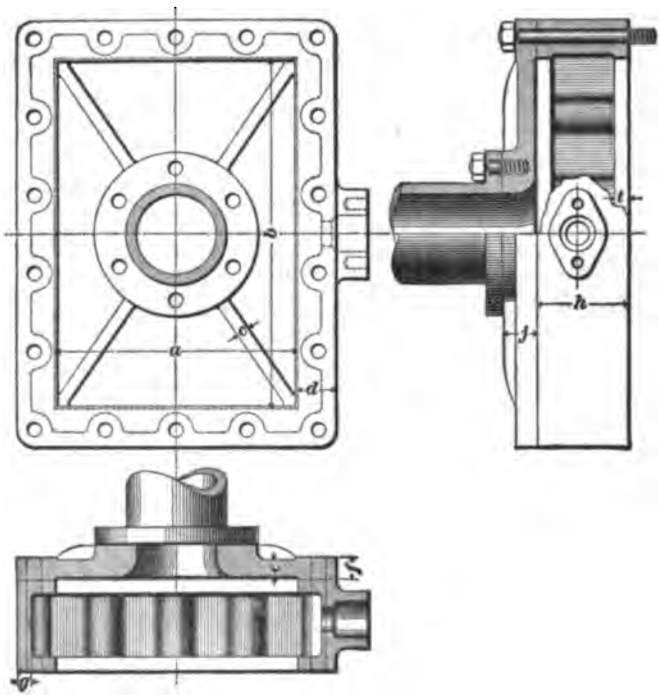


FIG. 742.

in Fig. 741. The principal dimensions are to be determined

by the following proportions, which are based upon the thickness i of the cylinder walls, and upon the travel and dimensions of the valve:

a = length of valve + travel of valve + twice the clearance between the valve and the steam chest at ends of valve travel.
b = breadth of valve + twice the clearance between one end of valve and steam chest.
$c = .75\,i$.
$d = 2.75\,o$, where o is the nominal diameter of the steam-chest bolts, as in Fig. 741.
$e = .04\sqrt{A'} + .125''$ for all areas above 100 sq. in. A' = area of steam chest, outside measurement, in square inches.

$f = 1.3\,e$.
$g = .85\,i$.
h = height of valve + necessary clearance.
$j = 2.5\,i$.
$t = .85\,i$.

NOTE.—When the area of the steam-chest cover is less than 100 square inches, its thickness e may be equal to i. When the area of the steam-chest cover exceeds 600 square inches, the height of the ribs should be 3.5 i, and their number should be increased.

2086. Fig. 744 shows a design for a steam-chest cover when the steam-pipe flange is on one side of the steam chest. Determine the thickness e by the same formula and rules as for the cover in Fig. 743. The other dimensions are found as follows:

$c = .75\,e$.
$f = 1.3\,e$.
$j = 2.6\,c$.
$r = 6\,e$.

p should never exceed the distance in inches given by the formula $\sqrt{\dfrac{40\,e_1^2}{p_g}}$, where e_1 is the numerator of the fraction expressing the thickness of the cover in sixteenths of an inch, and p_g = gauge boiler pressure.

EXAMPLE.—Find the maximum pitch of the ribs for a cover $\frac{15}{16}''$ thick, subjected to a steam pressure of 160 pounds per square inch.

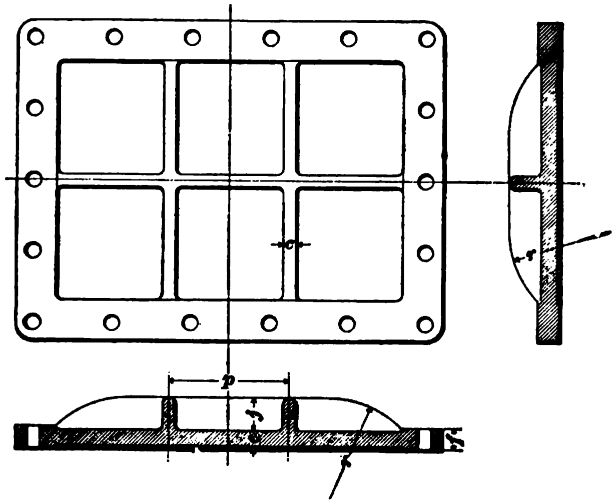


FIG. 744.

SOLUTION.—Substituting in the formula for p, we have

$$p = \sqrt{\frac{40 \times e_1^2}{p_g}} = \sqrt{\frac{40 \times 15^2}{160}} = 7.5 \text{ in.} \quad \text{Ans.}$$

2087. Fig. 745 shows a Corliss engine cylinder which may be designed according to the following proportions:

$D =$ diameter of cylinder.
$a = 1.21\,D + 2\,e + 1.25''$.
$b = .2\,D + 1.125''$.
$c = .048\,D$.
$c' = .079\,D$.
$d = .17\,D$.
$e = .0003PD + .375''$, if boiler pressure is above 100 lb.; otherwise, $e = .03\,D + .375''$.
$f = .82\,e$.
$g = .9\,e$.
$h = b + 2\,(c + g)$.
$h' = h$.
$i = 1.8\,e$.
$j = e$.
$k = 1.2\,e$.
$l = 1.7\,x + 2'' - 1.2\,e$, where $x =$ diameter of piston rod.
$l' = .32\,D$, about.
$m = .25\,D$.
$n = .32\,D$.
$o = 1.25\,e$.
$p = 1.3\,e$.
$q = .25\,D$.
$q' = .32\,D$.
$r = 1.2\,e$.

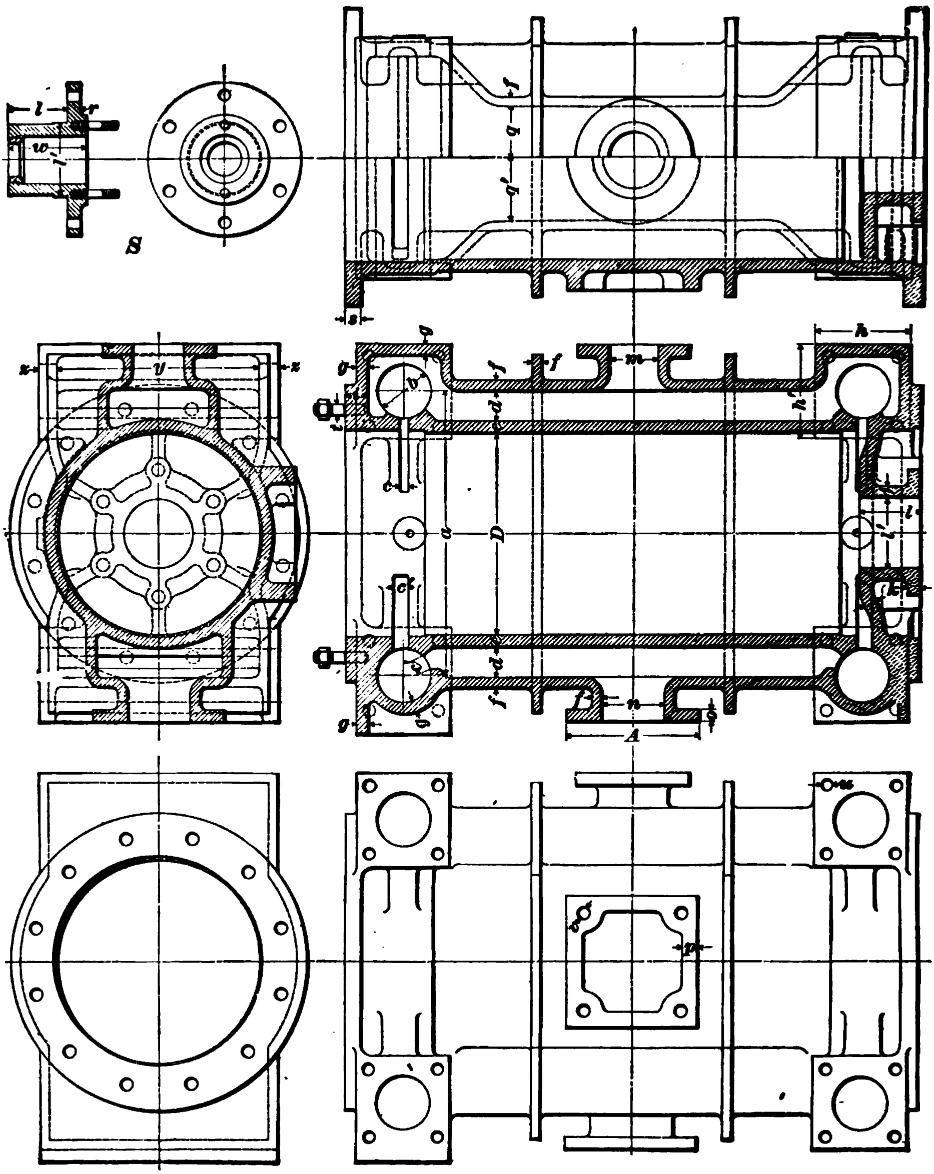


FIG. 745.

$s = 1.5\,e$.

$t =$ (see note).

$u = e$, take nearest standard size bolt.

$v = 1.2\,e$, take nearest standard size bolt.

$w = 1.7\,x + 2.25''$, where x = diameter of piston rod.

$y = D$.

$z = 1.5\,e$.

A is to be made according to Table 54, Art. **2072**. Bolts to be made according to the same table.

NOTE.—The bolts for cylinder heads are to be calculated from the formula given for cylinder-head bolts in connection with Fig. 741.

In this cylinder the stuffing-box S is a separate piece that is to be bolted to the cylinder head.

Fig. 746 shows a cylinder head that is suitable for cylin-

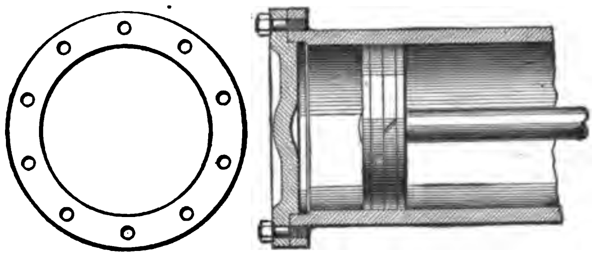


FIG. 746.

ders of small diameter. Its thickness may be the same as the thickness of the cylinder.

CRANK-SHAFTS.

2088. The general methods of computing the dimensions of crank-shafts, given in Arts. **1990** to **1992**, should be used for all important cases, since account is there taken of all the principal stresses.

Several instances have occurred where the shafts of large engines proved failures, because they were calculated from some simple approximate formula, and made too small. In one case a shaft 15 inches in diameter broke, causing great damage; it was found that had it been calculated by the methods given in the above-named articles, the shaft would have been 19½ inches in diameter.

When a series of different sizes of engines of the same type are to be built, however, it may be assumed that they will run under about the same conditions. In such a case it is unnecessary to use the above general method of calculating the crank-shaft, and short empirical formulas may be deduced from the practice of the best makers.

For high-speed automatic short-stroke engines, the following formula corresponds with good practice:

$$d = .44\,D + \tfrac{1}{2}'', \qquad \textbf{(295.)}$$

where d is the diameter of shaft and D = the diameter of cylinders.

For the Corliss type, in which the stroke is equal to or greater than twice the diameter,

$$d = .34\,D + 2\tfrac{1}{2}'', \qquad \textbf{(296.)}$$

when D is equal to or greater than 16 inches. When D is less than 16 inches,

$$d = \tfrac{1}{2}\,D. \qquad \textbf{(297.)}$$

A solid forged double crank is shown in Fig. 747. The crank, or cranks, are forged in the main shaft, as shown.

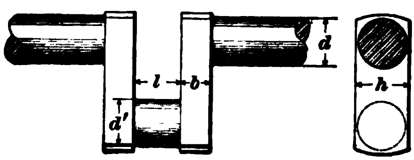


FIG. 747.

The following formula, given by Unwin, may be used to find the diameter of the shaft at journals:

$$d = 4.55\sqrt[3]{\frac{H}{N}}, \qquad \textbf{(298.)}$$

where H is the indicated horsepower, and N the revolutions per minute of shaft.

The proper cross-section $b\,h$ of the crank web may be obtained from the formula:

$$b\,h^2 = .9\,d^3 \text{ to } d^3. \qquad \textbf{(299.)}$$

Usually, $b = .6\,d$ to $.8\,d$; whence,

$$h = 1.05\,d \text{ to } 1.3\,d.$$

2089. Crank-Pins.—These are really end journals, and may be calculated by formulas **241** and **242,** Art. **1975.** There is, however, a growing tendency towards larger diameters and smaller ratios between length and diameter. Hence, in modern engine design, the pins are made much larger than mere strength would require.

The diameter of the crank-pin in marine engine practice (see Fig. 747) is made equal to or slightly greater than the diameter of the shaft journal. That is, in Fig. 747,

$$d' = d.$$

In this case the length of the pin subjected to pressure may be,

$$l = 1.1\,d.$$

2090. Fig. 748 shows a style of crank much used on

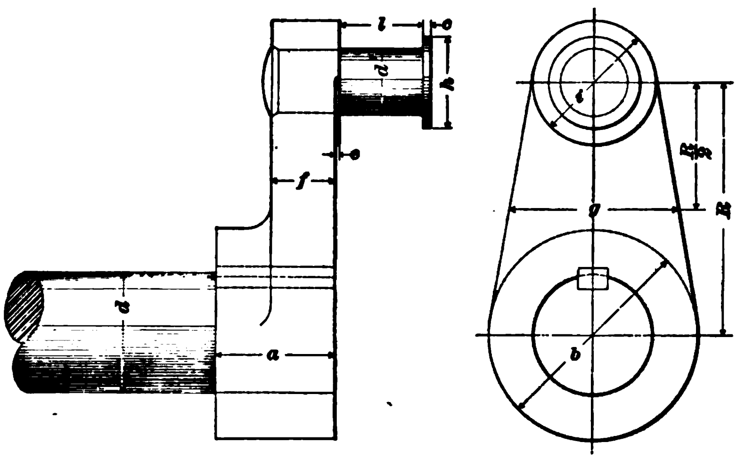


FIG. 748.

slow-speed engines, such as the Corliss type. The dimensions are to be computed by the following proportions:

D = diameter of engine cylinder.	$b = 1.75\,d.$
d = diameter of crank-shaft $= .5\,D.$	$c = .045\,d + .0625''.$
$a = d.$	$d' = .28\,D.$
	$e = .25''$, constant.
	$f = .375\,g.$

g is found by drawing the lines tangent to i and b.

$h = 1.35\,d'$.

$i = 1.125\,d$.

$l = .26\,D + .5''$ for cylinder diameters of $26''$ or less, and $l = d'$ for cylinder diameters greater than $26''$.

2091. Modern high-speed engines require counterweighting, and the crank usually takes the form of a disk, as shown in Fig. 749.

The disk is hollowed out, as shown, but a portion of the material is left in the side opposite the crank-pin, to form the counterweight which, by its centrifugal force, counteracts the centrifugal force resulting from the rotation of the crank-pin and rotating end of connecting-rod. The counterweight is a separate part of the disk proper, it being only cast to the hub a. It is thus made to allow for expansion and contraction of the casting in the larger crank disks. The width of the split is $\frac{3}{4}$ inch for engines of 48-inch stroke or less, and 1 inch for all larger sizes.

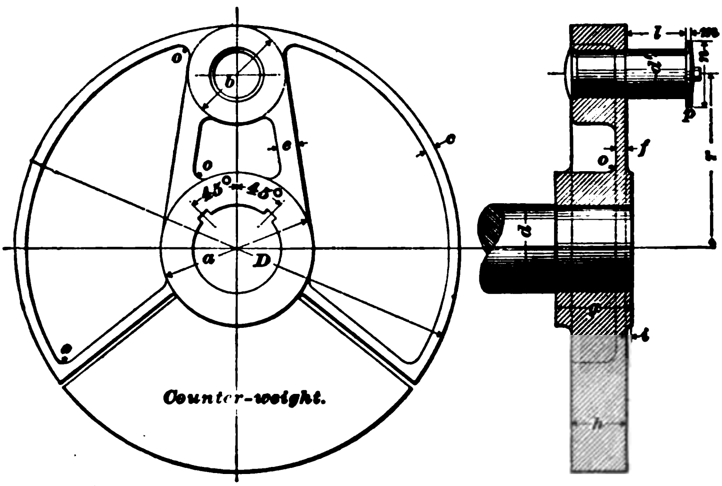


FIG. 749.

The following proportions represent the practice of good engine builders:

The unit is d, the diameter of the crank-shaft.

$a = 1.75\,d.$

$b = 1.125\,d.$

$c = \frac{r}{24} + .5''.$

$d' = .56\,d.$

$e = 2\,c.$

$f = c.$

i = should be given such a value that the connecting-rod will have about $\frac{1}{8}''$ clearance.

$g = .875\,d.$

$h = .625\,d.$

$l = .52\,d + 1''$ for cylinder diameters up to $26''$, above that size make $l = d'$.

$m = .045\,d + .0625''.$

$n = 1.35\,d'.$

$o = c.$

$D = 2\,r + b.$

NOTE.—p is a plate held to end of pin by tap bolt; o is the radius of all fillets except that of the boss i.

THE PISTON.

2092. Engine pistons are made in a great variety of forms. For small engines, that is, for cylinder diameters less than $8''$ or $10''$, the piston is often a solid disk of cast iron.

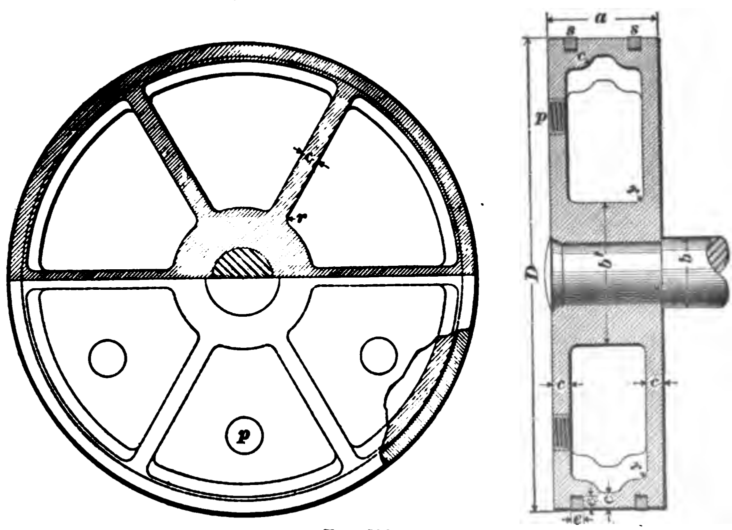


FIG. 750.

The Hollow Piston.—A form of piston that is much used for small engines is shown in Fig. 750. It consists

simply of a hollow circular disk of cast iron. The packing rings *s*, *s* are made of cast iron, and are split and sprung into place. Their elasticity causes them to press against the cylinder walls and thus prevent the leakage of steam.

The following proportions will give dimensions suitable for this piston:

D = diameter of cylinder in inches.
$a = .2D + 1.5''$.
b = diameter of piston rod.
$b' = 2b$.
$c = .18\sqrt{2D} - .1875''$.
$e = .75c$.
$r = .5c$.
p = core plug.
Number of ribs $= .08\,(D + 34)$.

2093. The Built-Up Piston.—The piston shown in Fig. 751 is made in two parts; the main part A is called the **spider**; the **follower plate** B is bolted to it.

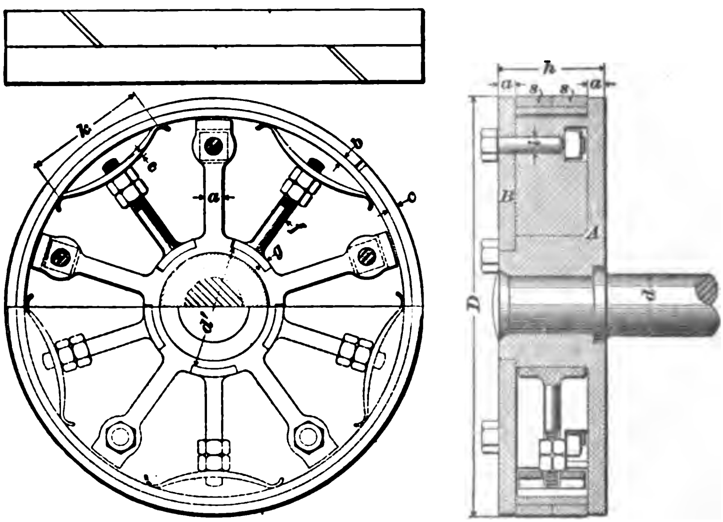


FIG. 751.

The spider is cast hollow, with radiating arms and lugs for the follower-plate bolts i. The split cast-iron **bull ring**

b is placed around the spider and is supported by the steel springs *e*, which are in turn supported by the brass studs *f*. The bull ring forms a support for the packing rings *s*, *s*.

The dimensions of this piston are given by the following proportions:

$D =$ diameter of cylinder in inches.

$a = .18\sqrt{2D} - .1875''$.	$g = .5f$.
$b = .45a$.	$h = .2D + 1.5''$.
$c = .65a$.	$i = a - .125''$.
$d =$ diameter of piston rod.	$k = \dfrac{1.4D}{n}$.
$d' = 2d$.	
$e = \dfrac{.06D}{n}$.	$n =$ number of ribs $= .08(D + 34)$.
$f = a - .125''$.	

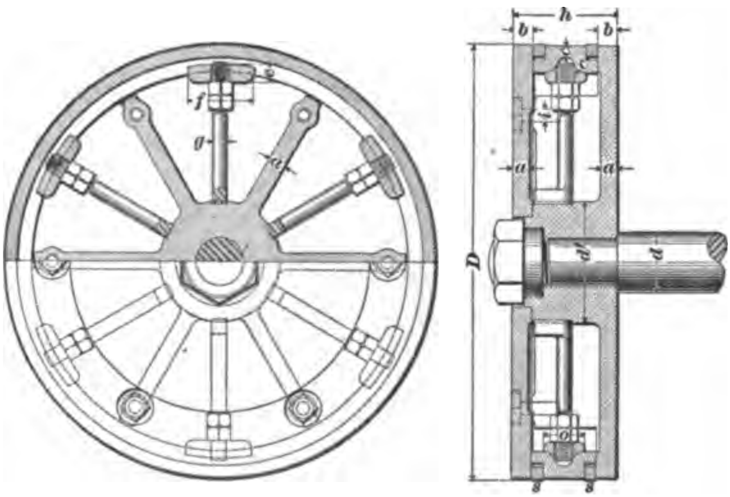


FIG. 752.

2094. Another form of built-up piston is shown in Fig. 752.

The proportions to be used for this piston are:

$D =$ diameter of cylinder in inches.

$a = .18\sqrt{2D} - .1875''$.	$d =$ diameter of piston rod.
$b = 1.25a$.	$d' = 2d$.
$c = .75a$.	$e = 1.25a$.

$f = 3.75\,a.$ $\quad$ $o = 2.5\,a.$

$g = a - .125''.$ $\quad$ $n = \text{number of ribs} = .08\,(D + 34).$

$h = .2\,D + 1.5''.$

$i = a - .125''.$

2095. The Solid Piston.—A form of piston much used in locomotive practice is shown in Fig. 753. It may be made of cast iron, but is usually a steel casting.

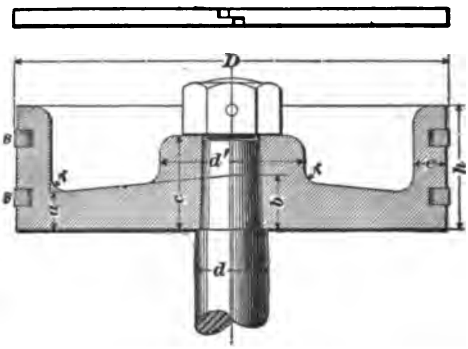


FIG. 753.

Suitable proportions for a cast-iron piston are:

D = diameter of cylinder in inches.

$a = .08\,D.$ $\quad$ $d' = 2\,d.$

$b = .12\,D.$ $\quad$ $e = 1.5\,(.18\sqrt{2\,D} - .1875'').$

$c = .15\,D + 1.125''.$ $\quad$ $h = .2\,D + 1.5''.$

d = diameter of piston rod. $\quad$ $r = .5\,e.$

If it is a steel casting, use the following proportions:

$a = .05\,D.$ $\quad$ $d' = 1.75\,d.$

$b = .075\,D.$ $\quad$ $e = 1.3\,(.18\sqrt{2\,D} - .1875'').$

$c = .15\,D + 1.125''.$ $\quad$ $h = .2\,D + 1.5''.$

d = diameter of piston rod. $\quad$ $r = .5\,e.$

Since the piston is solid, the rings must be cut and sprung to place. The form of the cylinder head is made to conform to the piston, so as to make the clearance small.

2096. Fig. 754 shows a steel-casting piston designed for the high-pressure cylinder of a marine engine. The dotted lines show how the cylinder head is made in order to fit closely around the piston. The conical form is given the piston in order to increase its strength.

The following proportions will give suitable dimensions for this piston for cylinders from 7" to 50" in diameter:

$D =$ diameter of high-pressure cylinder in inches.
$a = .1\,D + 1.5''$.
$b = \sqrt{20\,D} - 7.5''$.
$c = .1\sqrt{D}$.
$d =$ diameter of piston rod.
$d' = 1.63\,d$.
$e = .225\sqrt{D} + .625''$.
$f = .1\sqrt{D} + 25''$.

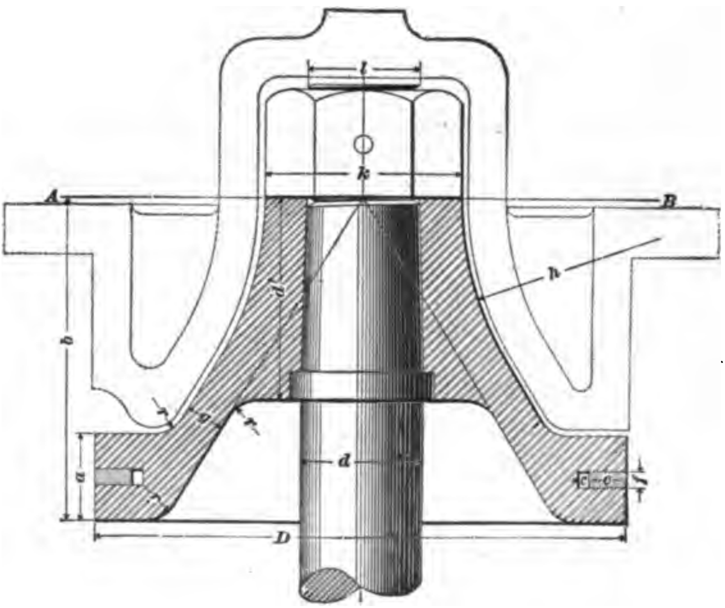


FIG. 754.

$g = .07\,D$.
h is to be found by trial with the center on the line $A\,B$.
$k = 1.73\,l + .1875''$.
$l =$ diameter of threaded end of piston rod.
$r = \dfrac{a - f}{2}$.

A piston suitable for the intermediate and low-pressure cylinders of the engine for which the piston in Fig. 754 is designed is shown in Fig. 755. To compute the dimensions for this piston, take the dimensions a, b, c, d, d', e, f, k, l, and r, the same as for the high-pressure piston in Fig. 754. If the engine is compound, let D' represent the diameter of the low-pressure cylinder in inches, and make $s = .02\ D'$.

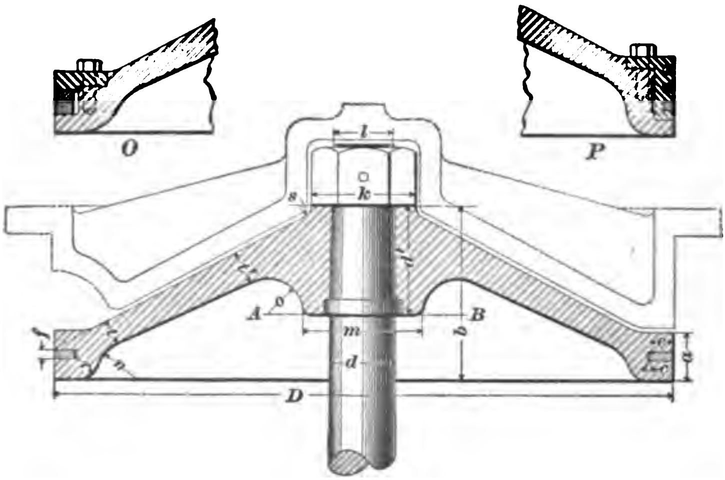


FIG. 755.

If a triple-expansion engine, let D' represent the diameter of the intermediate cylinder, and compute s as before, using the value found for both intermediate and low-pressure pistons. Make the remaining dimensions as follows:

$m = 1.8\,d$.

n is to be found by trial with the center on the line representing the bottom edge of the piston.

o is to be found by trial with the center on the line $A\,B$.

$$\left.\begin{aligned} t &= .2\sqrt{2\,D_1} \\ t' &= .275\sqrt{2\,D_1} \end{aligned}\right\}$$ where D_1 is the diameter of the cylinder for which the piston is made.

At O and P, Fig. 755, are shown methods of attaching the packing rings to large pistons in such a way that the

rings may be removed for inspection or repairs without taking the piston out of the cylinder. This is a very important advantage in many cases, especially for marine work, where the pistons are often very heavy, and facilities for handling them poor.

PISTON PACKING.

2097. It is, of course, impossible to turn the piston to exactly fit the cylinder at all temperatures; therefore, the piston is made slightly smaller than the cylinder bore, and some form of packing is used to prevent the steam from leaking through between the piston and cylinder walls.

The simplest and about the best form of packing, particularly for small pistons, is a cast-iron ring shown in cross-section at s, s, Figs. 750 and 752. The rings are generally of uniform thickness. Many makers, however, prefer to make the thickness where the rings are cut about half the thickness at the opposite side.

The proportions used for the spring packing rings shown in Figs. 750 and 752 are as follows:

Thickness and depth of rings the same and equal to

$$.135\sqrt{2D} - .14'',$$

where D is the diameter of the cylinder.

At A and C, Fig. 756, the packing for the pistons shown in Figs. 754 and 755 is shown in detail. In addition to the dimensions given in connection with the pistons, the following proportions are to be used:

$b = 3''$. $\qquad l = 9''$. $\qquad t = .0625''$.

Details of the packing shown in the section O of Fig. 755 are shown in Fig. 756 at B and D. The proportions applying here are:

$b = 3''$. $\qquad t = .09375''$.

The length of the segments should be about 15", and two springs are placed behind each segment. The packing rings

shown in Fig. 756 are usually of cast iron, and the springs of steel.

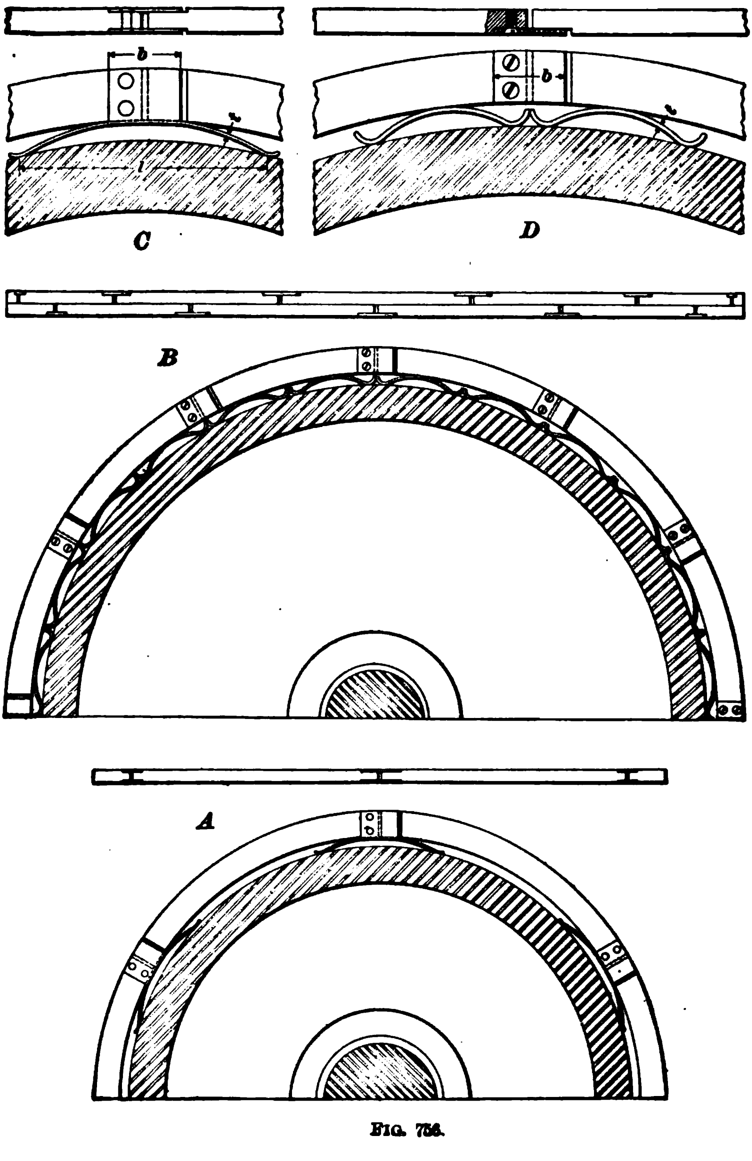


FIG. 756.

2098. Fig. 757 shows Tripp's patent piston packing. The rings S, S are made of cast iron, split so as to spring outwards against the cylinder walls. They are supported by an adjustable ring B, which is made with conical surfaces, and against which the packing rings bear. The pressure of the steam against the packing ring forces it against this conical surface, thus tending to open the ring out and make it press against the cylinder. The spiral springs t, t are for the purpose of holding the packing rings in place when they are not acted on by the steam pressure.

2099. Fig. 758 shows a style of ring packing much used for piston valves. A is a split or sectional cast-iron ring, which is forced out against the walls of the cylindrical valve

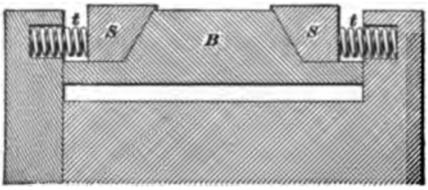


FIG. 757.

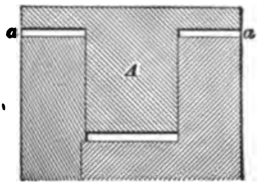


FIG. 758.

seat by the pressure of the steam in the spaces a, a between the overhanging parts of the ring and the main part of the piston or valve.

PISTON RODS.

2100. The piston rod is subjected alternately to tension and compression. If the rod is short in comparison to its diameter, it may be calculated as though simply subjected to tension or compression.

Let d = diameter of rod;
p = greatest pressure per square inch on piston;
D = diameter of cylinder;
l = length of rod;
S = safe crushing strength or tensile strength, whichever may be smaller.

Then, $$\tfrac{1}{4}\pi d^2 S = \tfrac{1}{4}\pi D^2 p,$$

or $$d = D\sqrt{\frac{p}{S}}.$$

For wrought iron the strength in compression is least, and a safe value for S is 3,600 pounds. For steel the safe strength in tension is least, and S may be taken at about 7,200 pounds. These values give a factor of safety of 10 to 12.

Substituting the value of S,

$$\left.\begin{aligned} d &= .0167\,D\sqrt{p} \quad \text{(wrought iron);} \\ d &= .0118\,D\sqrt{p} \quad \text{(steel).} \end{aligned}\right\} \qquad \textbf{(300.)}$$

2101. When the length of the piston rod is great in comparison with its diameter, it is liable to buckle, and must be treated as a long column. Therefore, formula **119**, Art. **1411**, may be used in such cases.

A simpler formula may, however, be obtained as follows:

Let P = maximum load on piston rod;

P_0 = load which will just cause the rod to begin to buckle;

k = factor of safety = 10.

The symbols l, d, D, etc., are the same as given above.

E = coefficient of elasticity of material of rod.

I = moment of inertia of rod $= \dfrac{\pi d^4}{64}$.

According to Weisbach,

$$P_0 = \left(\frac{\pi}{2\,l}\right)^2 E\,I.$$

Then, $$P_0 = 10\,P = \left(\frac{\pi}{2\,l}\right)^2 E\,I = E\left(\frac{\pi}{2\,l}\right)^2 \frac{\pi d^4}{64}.$$

But $$P = \tfrac{1}{4}\pi D^2 p;$$

hence, $$P_0 = \frac{10\,\pi D^2 p}{4} = E\left(\frac{\pi}{2\,l}\right)^2 \times \left(\frac{\pi d^4}{64}\right),$$

or $$d = \sqrt[4]{\frac{640\,D^2 l^2 p}{\pi^2 E}}.$$

For wrought iron, take $E = 25{,}000{,}000$, and for steel $= 30{,}000{,}000$.

$$\left.\begin{aligned} &\text{Then, } d = .04\sqrt[4]{D^2 l^2 p}, \text{ for wrought iron;} \\ &\text{similarly, } d = .038\sqrt[4]{D^2 l^2 p}, \text{ for steel.} \end{aligned}\right\} \qquad \textbf{(301.)}$$

2102. For some values of the ratio $\frac{l}{d}$, formulas **300** and **301** should give the same diameter of rod. To determine this ratio, we proceed as follows:

The first of the formulas **301** may be written as follows by squaring both members of the equation:

$$d^2 = .0016\, D\, l \sqrt{p},$$

or

$$d = .0016\, D \frac{l}{d} \sqrt{p}.$$

But, from formula **300** for wrought iron,

$$d = .0167\, D \sqrt{p}.$$

Hence,

$$.0016\, D \frac{l}{d} \sqrt{p} = .0167\, D \sqrt{p},$$

and $\frac{l}{d} = \frac{.0167}{.0016} = 10\frac{1}{2}$, nearly, for wrought-iron rod.

Similarly, $\frac{l}{d} = \frac{.0118}{.038^2} = 8.2$, for steel.

The length l of the rod is known approximately as soon as the cylinder is designed. For calculation, the length should be taken between the points where the rod enters the piston and where it enters the cross-head.

To calculate the diameter of the rod, proceed as follows: First find the diameter by formula **300.** If the rod is of wrought iron and the ratio $\frac{l}{d}$ (using the value of d just found) is less than $10\frac{1}{2}$, the value of d is correct. If the ratio $\frac{l}{d}$ exceeds $10\frac{1}{2}$, calculate d by formula **301.** The same process is applicable to a steel rod, except that the discriminating ratio is $\frac{l}{d} = 8.2$.

EXAMPLE.—Calculate the diameter of a wrought-iron piston rod 21 inches long, the diameter of the cylinder being 13 inches, and the steam pressure 90 lb.

SOLUTION.—By formula **300,**

$$d = .0167\, D \sqrt{p} = .0167 \times 13 \times \sqrt{90} = 2\tfrac{1}{16}''.$$

Since $\frac{l}{d} = \frac{21}{2\frac{1}{16}}$ is less than $10\frac{1}{2}$, the value found is correct. Ans.

Supposing the rod to be of steel, the diameter would be from formula **300**,

$$d = .0118\,D\sqrt{p} = .0118 \times 13\sqrt{90} = 1.46 \text{ inches.}$$

But $\dfrac{l}{d} = \dfrac{21}{1.46}$ is greater than 8.2; hence, formula **301** must be used.

$$d = .038\sqrt[4]{D^2\,l^2\,p}$$

$$= .038\sqrt[4]{13^2 \times 21^2 \times 90} = 1.934 = 1\tfrac{15}{16}''. \quad \text{Ans.}$$

APPROXIMATE FORMULAS FOR PISTON RODS.

2103. In many cases the length of the piston rod is about twice the cylinder diameter; that is, $l = 2\ D$. Substitute this value of l in formula **301**, and take $p = 80$ lb.

Then, for wrought iron,

$$d = .04\sqrt[4]{4\,D^4 \times 80} = .169\,D = \frac{D}{6}, \text{ nearly.}$$

For steel,

$$d = .038\sqrt[4]{4\,D^4 \times 80} = .161\,D = \tfrac{4}{25}\,D.$$

Many engine builders thus make the diameter of the piston rod a certain fixed fraction of the cylinder diameter. This fraction $\left(\dfrac{d}{D}\right)$ may be as small as $\frac{1}{10}$ in exceptional cases, and in a few instances it may be as large as $\frac{1}{3}$ or $\frac{3}{8}$. In the best modern practice, however, $\dfrac{d}{D}$ is about $\frac{1}{6}$ or $\frac{1}{7}$.

2104. Connection of Rod to Piston.—Modes of fastening the piston rod to the piston are shown in Figs. 750 to 755. The end of the rod is tapered and threaded to receive a nut, or it is riveted. The taper may vary in different cases from $\frac{1}{24}$ to $\frac{1}{4}$. The cross-section of the rod at root of threads should be such as to give a tensile strength of 5,000 lb. per sq. in. for wrought iron and 7,000 lb. for steel. Letting d_1 = diameter of rod at root of thread,

$$\frac{\pi}{4} d_1^2 \times 5,000 = \frac{\pi}{4} D^2 p \text{ for wrought iron;}$$

$$\frac{\pi}{4} d_1^2 \times 7,000 = \frac{\pi}{4} D^2 p \text{ for steel;}$$

Or,

$$\left.\begin{array}{l} d_1 = .014\, D \sqrt{p} \text{ for wrought iron;} \\ d_1 = .012\, D \sqrt{p} \text{ for steel.} \end{array}\right\} \quad (302.)$$

The rods with a collar forged on to bear against the piston are much to be preferred for heavy pressures. When the section of a rod is to be reduced it is important, especially for a steel rod, that a liberal fillet be provided, as shown in Fig. 751, so as to leave no sharp corners.

CONNECTING-RODS.

2105. The connecting-rod is subjected to various and severe stresses, the calculation of which is difficult and complicated, and the formulas derived from theoretical considerations are, therefore, somewhat unreliable.

The proportions for connecting-rods given below are based on the results of practical experience of standard engine builders. Fig. 759 shows a style of rod that gives excellent results in stationary work. The cross-head end is forged solid, and cut out for the brasses, which are made without top and bottom flanges on one side, so that they can be slipped into the rod. The brasses are held in place and adjusted by the steel wedge w_1 and the adjusting screws S_1. The brasses on the cross-head end of connecting-rods are seldom babbitted, as experience shows that it gives unsatisfactory results.

The crank end of the rod is made fork-shaped, so that the brasses can be put on to the crank-pin and then slipped into the rod from the end. If the wrist-pin can not be removed from the cross-head, such a construction must also be used for the cross-head end of the rod. The bolt B, which is turned and fitted in a reamed hole, holds the brasses, which are adjusted by a steel wedge and screws, in the same manner as for the cross-head end. The crank-pin brasses are babbitted, as shown.

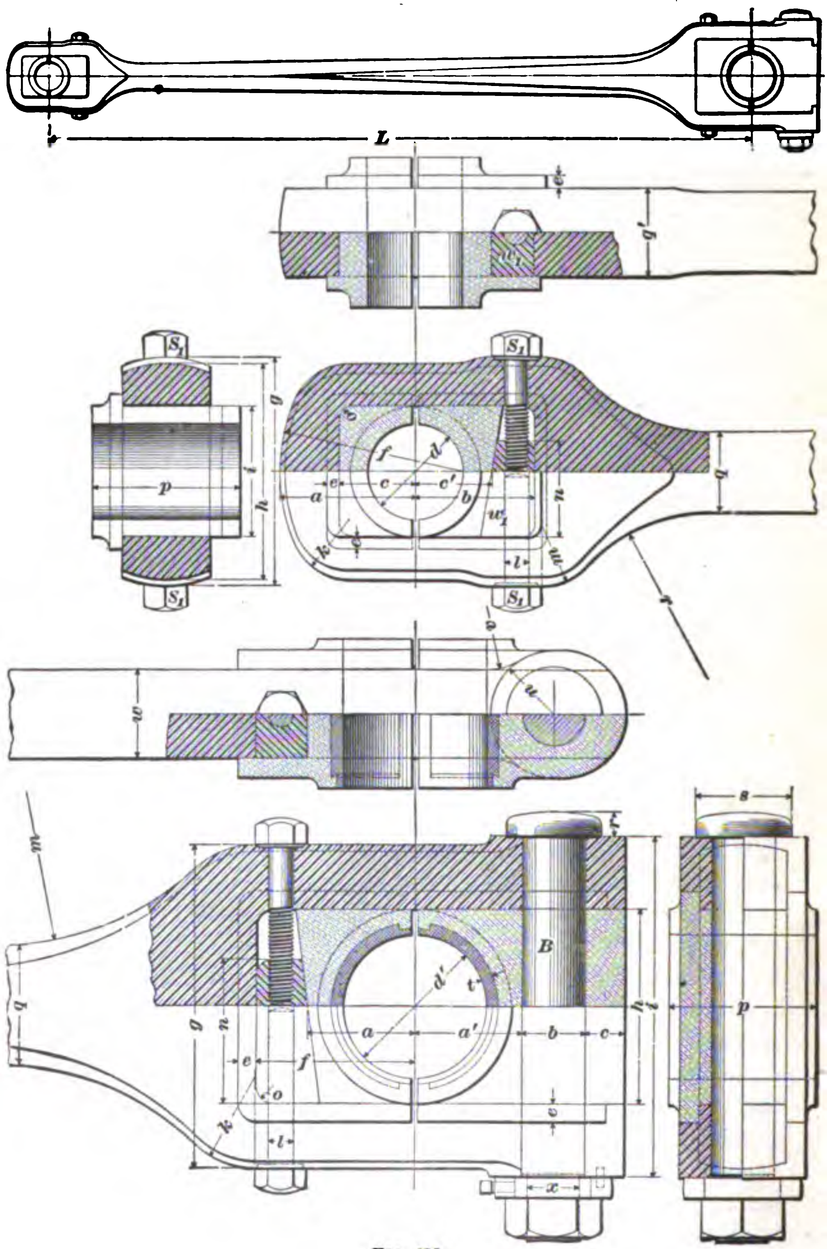


FIG. 759.

The dimensions of this rod are to be made according to the following proportions, which are suitable for a rod of either steel or wrought iron:

For the wrist-pin (cross-head pin) end:

$D =$ diameter of cylinder in inches.

$d = .2\,D =$ diameter of wrist-pin.

$a = 1.42\,d.$

$b = 1.125\,d + .375''.$

$c = .75\,d + .125''.$

$c' = .75\,d + .125''.$

$e = .125\,d.$

$f = 1.92\,d.$

$g = 2.375\,d.$

$h = 2.25\,d.$

$i = 1.35\,d.$

$k = .625\,d.$

$l = .035\,D + .25''.$

$m = .625\,d.$

$n = d.$

$o = .125\,d.$

$p = .26\,D + .5''$ for cylinders up to $26''$ in diameter, and $p = .28\,D$ for cylinders above $26''$ in diameter.

$q = .155\,D + .0625''.$

$q' = .17\,D + .0625''.$

$r = 1.75\,d.$

Taper of adjusting wedges $= 1\frac{1}{2}''$ per foot.

For the crank-pin end:

$D =$ diameter of cylinder in inches.

$d' = .28\,D =$ diameter of crank-pin.

$a = .75\,d'.$

$a' = .75\,d'.$

$b = .1\,D + .4375''.$

$c = .625\,b.$

$e = .125\,d'.$

$f = d' + .5''.$

$g = 2.25\,d'.$

$h = 1.35\,d'.$

$i = 2.375\,d'.$

$k = .625\,d'.$

$l = .035\,D + .25''.$

$m = 2\,d'.$

$n = d'.$

$o = .125\,d'.$

$p = .26\,D + .5''$ for cylinders up to $26''$ in diameter, and $p = .28\,D$ for cylinders above $26''$ in diameter.

$q = (.155\,D + .0625)\sqrt{\frac{L}{S}}$, in which $L =$ length of connecting-rod and $S =$ length of stroke.

$r = .375\,b.$

$s = 1.5\,b.$

$t = .02\,D + .0625''.$

$u = b.$

$v = b.$

$w = .17\,D + .0625''.$

$x = .8\,b.$

For dimensions of the nut and locking collar, see Art. **1946.**

Taper of adjusting wedge, $1\frac{1}{8}''$ per foot.

2106. In Fig. 760 is shown a **strap end** connecting-rod, The straps c_1 and c_2 are fastened to the ends of the rods by means of the gibs a_1 and a_2 and the cotters b_1 and b_2. The cotters are held in place by the set-screws s_1 and s_2. Small steel blocks, shown between the ends of the set-screws and the cotters, are used to prevent injury of the cotter by the set-screws.

The rod, cotters, gibs, and straps may be made of either wrought iron or steel. The crank-pin brasses are shown babbitted, and wrist-pin brasses without babbitt. The brasses are adjusted by means of the cotters, which draw the straps farther on to the rod when they are driven in.

The dimensions for the rod shown in Fig. 760 are given by the following proportions:

For wrist-pin end,

$D =$ diameter of cylinder.

$d = .2D =$ diameter of wrist-pin.

$n = .155\,D + .0625''$.

$x = \frac{\pi}{4}n^2 =$ a factor for use in finding proportions below.

$a = .75\,d + .125''$.

$a' = .75\,d + .125''$.

$b = \sqrt{2.5\,x}$.

$c = .25\,b$.

$e = .125\,d$.

$f = .26\,D + .5''$ for cylinders up to $26''$ in diameter, and $f = .28\,D$ for cylinders above $26''$ in diameter.

$g = 1.3\,n$.

$h = \frac{.5\,x}{g - c}$.

$i = \frac{.32\,x}{h}$.

$k = \frac{x}{1.8\,d}$.

$l = .375\,b$.

$m = 1.35\,d$ for wrist-pins up to $3.5''$ in diameter, and $m = 1.48\,n$ for pins above $3.5''$ in diameter.

$o = .25\,b$.

$p = .33\,b$.

$q = 1.125\,d$ for wrist-pins up to $3.5''$ in diameter, and $q = 4''$, constant, for pins above $3.5''$ in diameter.

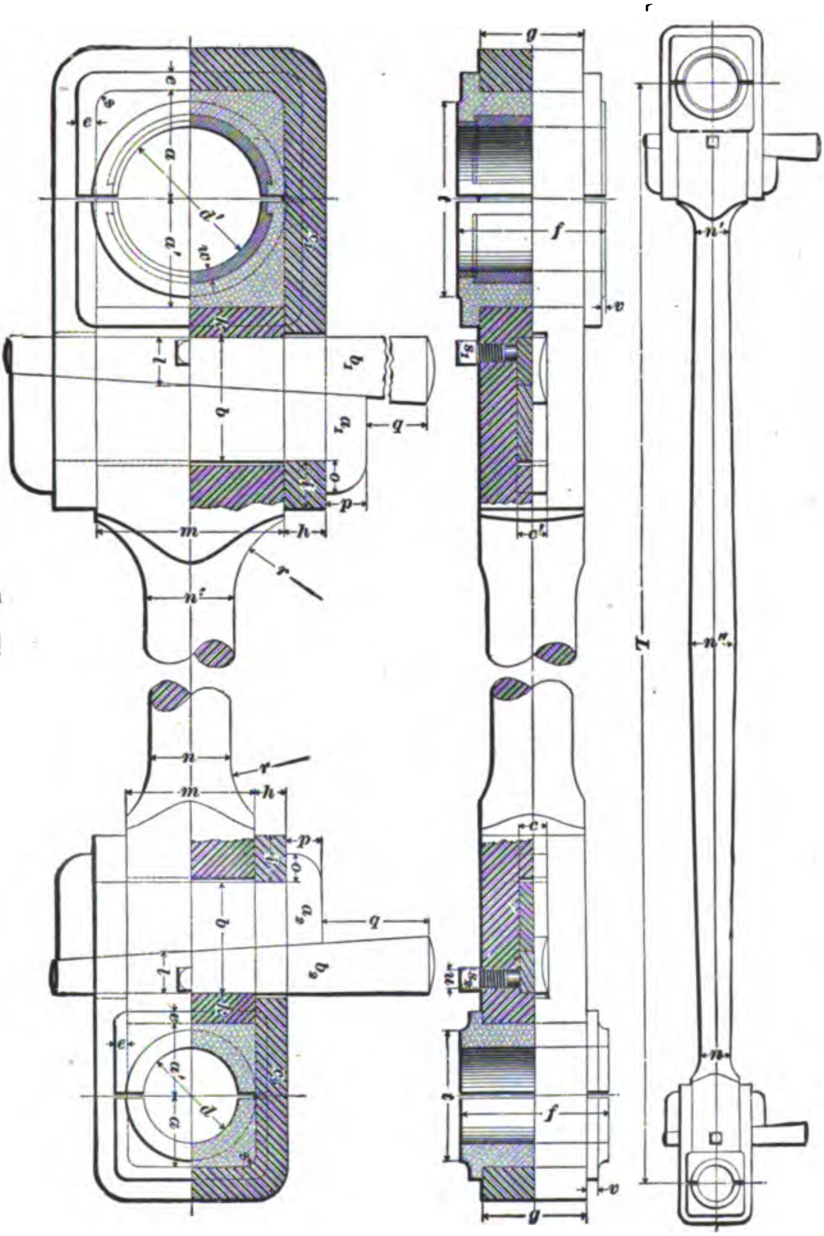


FIG. 700.

$r = n$.

$s = .125\,d$.

$t = 1.35\,d$.

$u = .02\,D + .25''$.

$v = .125\,d$.

The taper of the cotter is $\frac{3}{4}''$ per foot.

Proportions for the crank-pin end:

$D =$ diameter of cylinder in inches.

$d' = .28\,D =$ diameter of crank-pin.

$n' = 1.1\,n$. ($n = .155\,D + .0625''$.)

$x' = \frac{\pi}{4}n'^2 =$ a factor used below.

$a = .75\,d'$.

$a' = .75\,d'$.

$b = \sqrt{2.5\,x'}$.

$c' = .25\,b$.

$e = .125\,d'$.

$f = .26\,D$ for cylinder diameters up to $26''$, and $f = .28\,D$ for cylinders above $26''$ in diameter.

$g = 1.3\,n =$ same as wrist-pin end.

$h = \frac{.5\,x'}{g - c'}$.

$i = \frac{.32\,x'}{h}$.

$k = \frac{x}{1.8\,d} =$ same as wrist-pin end.

$l = .375\,b$.

$m = 1.3\,d'$.

$o = .25\,b$.

$p = .33\,b$.

$q =$ same values as for wrist-pin end.

$r = 1.1\,n$.

$s = .125\,d'$.

$t = 1.35\,d'$.

$v = .125''$ (constant).

$w = .02\,D + .0625''$.

$n'' = n\left(\sqrt{\frac{L}{S}} - .22''\right)$, where $L =$ length of rod and $S =$ stroke, both in inches.

Taper of cotter, $\frac{3}{4}''$ per foot.

2107. Fig. 761 shows a connecting-rod with **marine ends.** For marine engines, and in some cases for stationary engines, the cross-head end is forked as shown. For most stationary work, however, the cross-head end is not forked, and a solid, or strap, end is often used with a marine crank-pin end. The brasses are held to the ends of the rod, which are forged T shaped, by turned bolts. These bolts pass through steel or wrought-iron caps and the brasses. Liners are fitted between the two parts of each brass, and when

the brasses become worn the liners are taken out and either filed or planed down, thus allowing the brasses to be tightened. The crank-pin brasses are babbitted, and the brass is chipped out between the babbitt blocks, so that there will be bearing only on the babbitt. This is common practice in marine work, but is seldom done in stationary work.

The liners in the crank-pin end are secured by small pins p_1, which pass through the liner and project into the holes h_1, shown in the view of the brass given at A. B is a view of one of the wrist-pin brasses. The rod and caps are usually made of steel, although wrought iron may be used.

The dimensions for the rod shown in Fig. 761 are to be determined by the following proportions :

$a = \sqrt{.0004\,A\,P}$, where A = area of piston and P = the boiler pressure.

$a' = a\left(\sqrt{\frac{L}{S}} - .05''\right)$, where L = length of rod and S = stroke.

$b = 1.3\,d.$

$b' = .6\,a.$

$b'' = .5\,a'.$

$c = .7\,a.$

$c' = .7\,a.$

d = diameter of cross-pin.

$d' = 1.1\,d.$

$e = .25\,d.$

$f = .35\,c.$

$g = 1.4\,c.$

h = area of bolts at bottom of thread = .00008 $A\,P$ for steel and .0001 $A\,P$ for wrought iron, where A = area of cylinder and P = boiler pressure.

$i = 1.5\,h.$

$j = h.$

$k' = .875\,h.$

$k'' = h.$

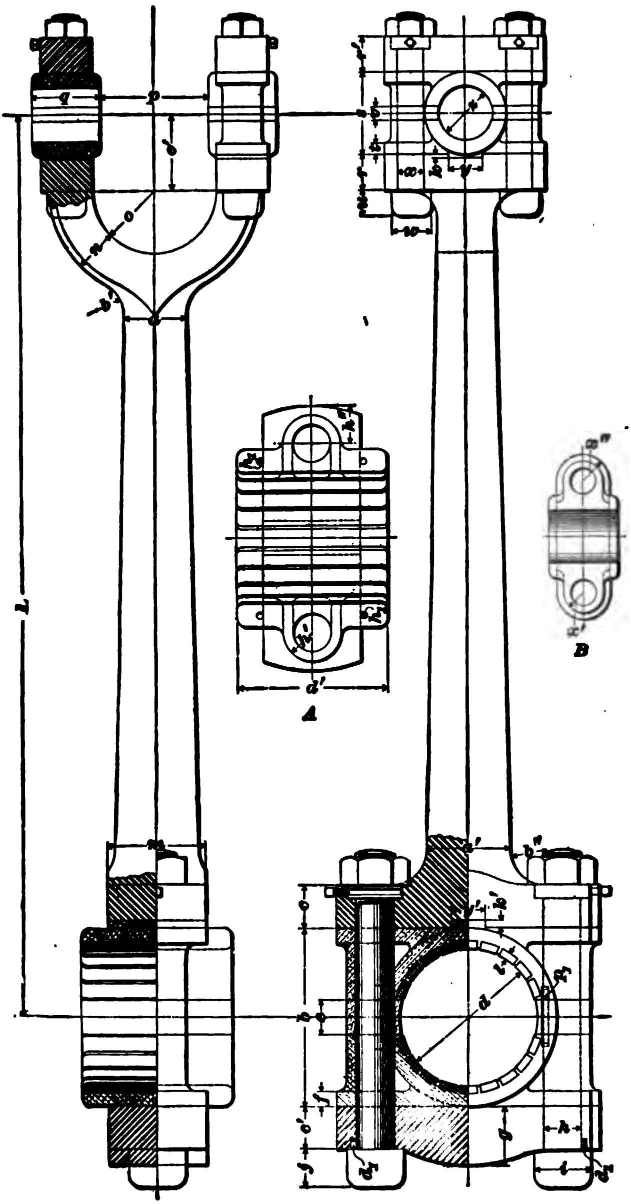


FIG. 761.

$k = .05z + .0625''$.
$l = .05d + .0625''$.
$k' = l$.
$m = 1.1a'$.
$n = .7a$.
$o = .5p + .5(q - 2x)$.
$o' = .6a + .675z$.
p to fit cross-head, i. e., $2B + .25''$, where $B =$ diameter of piston rod.
$q =$ length of wrist-pin as made for cross-head.
$r = .6a$.
$r' = .6a$.
$s = 1.35z$.
$t = .35r$.
$u = x$.
$v = .25z$.
$w = 1.5x$.
$x = .00004\,AP$ for steel, and $.00005\,AP$ for wrought iron.
$y = .25d$.
$y' = .25d$.
$z =$ diameter of wrist-pin as made for cross-head, Fig. 763.
$x' = .875x$.
$x'' = x$.

NOTE.—x and h should be taken as the diameter of the nearest larger size of bolt, and the diameter of that bolt should be used as the unit where proportions are given in terms of x or h.

This rod may be used with the cross-head shown in Fig. 763. If the rod is used for a compound or triple-expansion engine, design it for the cylinder that does the most work, and make the rods alike for all cylinders. The same rule applies to the cross-head.

Proportions for locknuts to be made as shown in Art. **1946.** The dowel-pins d_1 are used to keep the bolt from turning with the nut.

The distance between the bolts should be such that the brass will be about $\frac{1}{4}$ inch thick at the thinnest part between the bolt and the pin.

CROSS-HEADS.

2108. The **cross-head pin** is a simple neck journal supported at the ends and loaded at the middle. It may, therefore, be designed by the rules given under the head of journals.

Cross-heads are made in a great variety of forms, some of the most important of which, together with proportions for their design, are given below. Fig. 762 shows a cross-head much used on Corliss and similar engines. It is composed of a cast-iron box with a boss for the piston rod, and bosses for the wrist-pin. The wrist-pin is steel or wrought iron, with the ends turned tapering so as to fit snugly into the cross-head. It is held by a nut and washer, and has a projecting part y which is drilled for an oil cup. Holes are drilled into the pin, as shown by the dotted lines, for the purpose of leading oil to the connecting-rod brasses.

Boss gibs g_1 are fitted to the cross-head on tapered ways, and these gibs can be adjusted by means of the bolts t_1. The gibs shown on the cross-head have cylindrical bearing surfaces, but a gib is shown at A with the bearing surface V shaped. The guides are made either cylindrical or V shaped, to correspond to the gibs.

The following proportions give dimensions for designing the cross-head shown in Fig. 762:

D = diameter of cylinder.
d = diameter of piston rod.
a must be found by making a scale drawing of the connecting-rod in its extreme position. Care must be taken that the rod will clear the gibs, and also the guides, for all positions.
$c = 2.125\,d$.
$d' = 2\,d$.
$d'' = 2.125\,d$.
$e = .01\,D + .5''$.
$e' = e$.

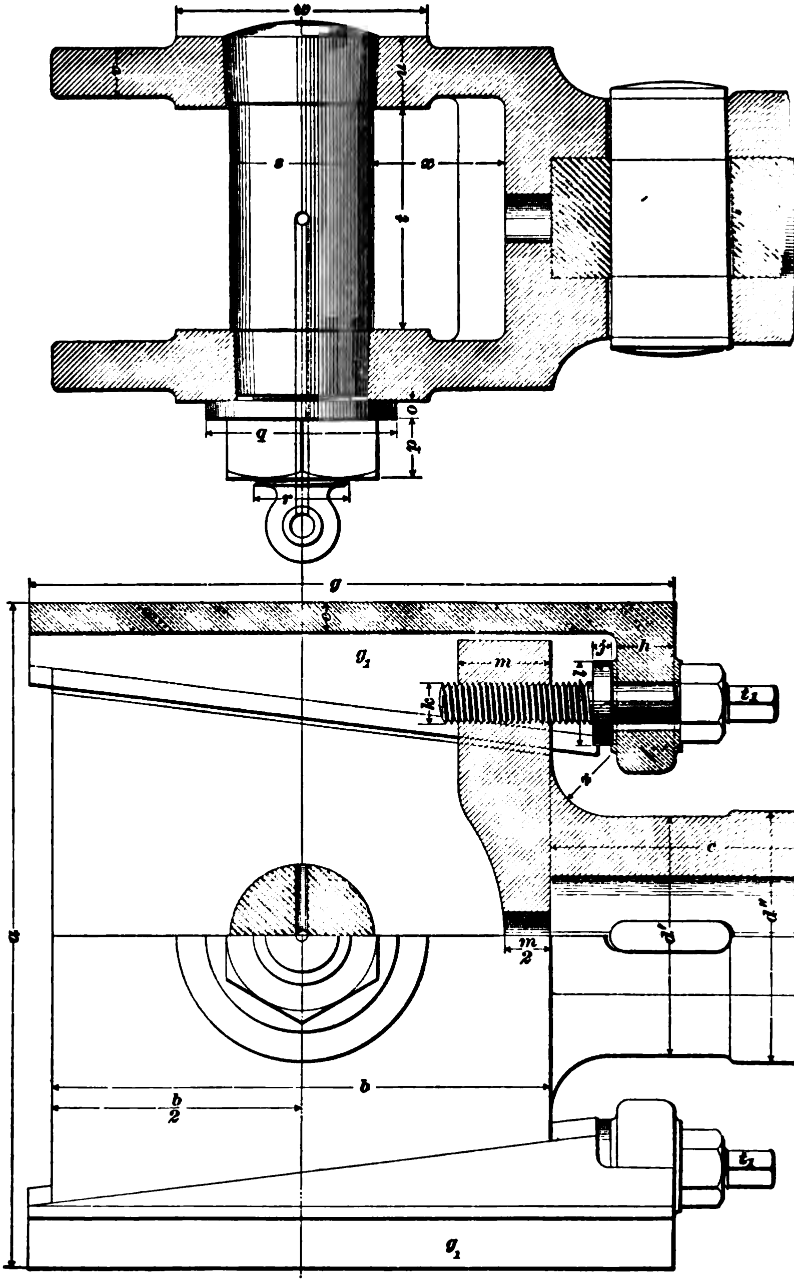
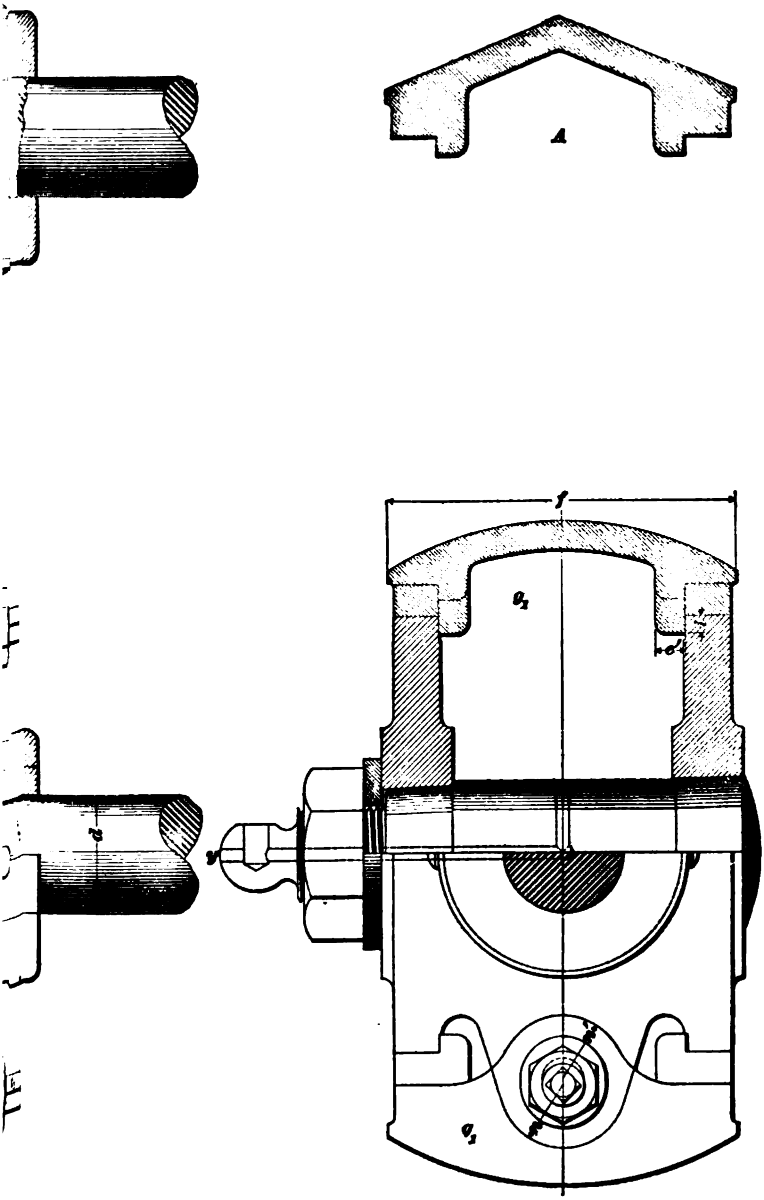


FIG. 7



32.

$f = .357\,D + 1.625''$ for cylinders up to $26''$ in diameter, and $f = .376\,D + 1''$ for cylinders larger than $26''$ in diameter.
$g = 3\,s + .225\,D + 1.375''$.
$h = .04\,D + .5625''$.
$i = .6\,e$.
$j = .015\,D + .125''$.
$k = .04\,D + .3125''$.
$l = .08\,D + .625''$.
$m = .12\,D + .125''$.
$n = .06\,D + 5''$.
$n' = n$.
$o = .19\,r$.
$p = .625\,r$.
$q = 2.125\,r$.
$r = .66\,s$.
$s = .2\,D$.
$t = .26\,D + .5''$ for cylinders up to $26''$ in diameter, and $t = 28\,D$ for cylinders above $26''$ in diameter.
$u = .05\,D + .625''$.
$v = .043\,D + .3125''$.
$w = 1.75\,s$.
$x = s$.
$y =$ plug for oil hole.
$z = .5\,d$.

Taper of gibs $1.5''$ per foot.

For cylinders above $20''$ in diameter the gibs should be ribbed.

This cross-head is designed to be used with a solid-end connecting-rod; see Fig. 759.

2109. Fig. 763 shows a style of cross-head used mostly for marine work. The wrist-pin is in two parts a_1, a_1, forged solid with the block b_1, which is of either wrought iron or steel. This cross-head requires a forked-end connecting-rod similar to that shown in Fig. 761. The bearing surfaces are composed of two cast-iron shoes fastened to the block by

bolts. These shoes are babbitted, the babbitt being dove-tailed into and raised a little above the surface of the iron.

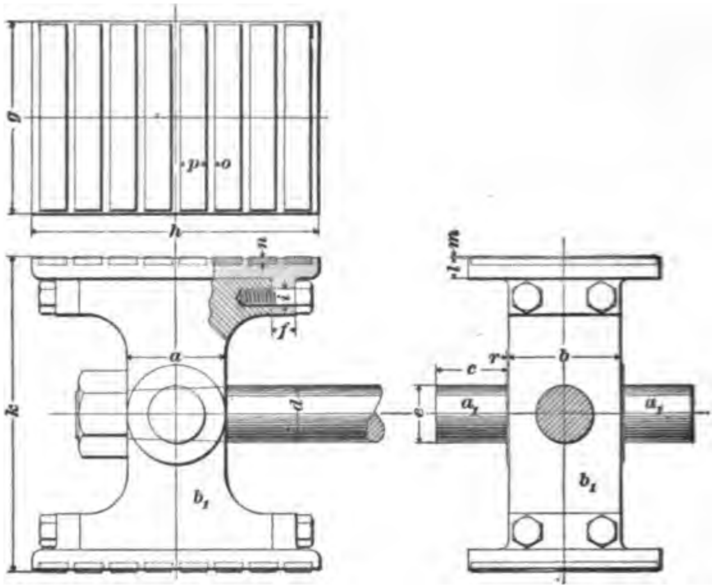


FIG. 763.

so that no wear will come directly on the iron. The piston rod is tapered in the block and fastened with a nut.

The dimensions of this cross-head are based on the following proportions:

$D =$ diameter of cylinder.
$d =$ diameter of piston rod.
$a = 1.75\ d.$
$b = 2\ d.$
$c = \sqrt{.0005\ A\ P.}$
$e = .8 \sqrt{.0005\ A\ P.}$
$A =$ area of piston in square inches.
$P =$ boiler pressure in pounds per sq. in.
$f = .05 D + .5''.$

$$g = .66 \sqrt{\frac{.035\ A\ P\ C}{L}}.$$

$h = \sqrt{\frac{.035\,A\,P\,C}{L}}$.

C = length of crank in inches.

L = length of connecting-rod in inches.

$i = \frac{3}{4}''$ for $D = 15''$ or less; $\frac{7}{8}''$ for D = from $15''$ to $20''$; $1''$ for D = from $20''$ to $25''$, and $1\frac{1}{4}''$ for D above $25''$.

k must be such that the connecting-rod will clear cross-head and guides.

$l = .05\,D + .25''$.

$m = .125''$, constant.

$n = .016\,D = .1875''$.

$o = .5''$, about.

$p = 1.75''$, about.

$r = .125''$, constant.

2110. Fig. 764 shows a modification of the marine

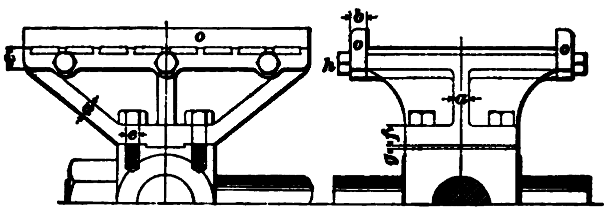


FIG. 764.

cross-head, in which the cast-iron shoe forms a much larger proportion, and is provided with guiding strips o, o.

The proportions that apply to this form are:

$a = .03\,D + .5''$.

$b = .03\,D + .5''$.

$c = .05\,D + .5''$.

$e = \frac{7}{8}''$ up to $D = 20''$; $1''$ up to $D = 25''$, and $1\frac{1}{4}''$ for D above $25''$.

$f = .05\,D + .5''$.

$g = .02\,D$.

$h = \frac{3}{4}''$ for $D = 20''$ or less; $\frac{7}{8}''$ for $D = 20''$ to $25''$, and $1''$ for D above $25''$ in diameter. Space bolts h not more than $7''$ between centers.

The other dimensions of this style of cross-head may be obtained from the proportions given for Fig. 763.

The shoes for the cross-heads, Figs. 763 and 764, are adjusted by placing liners behind them.

2111. Fig. 765 is an example of a cross-head that is much used on box-bed engines. The main part *A* is of cast

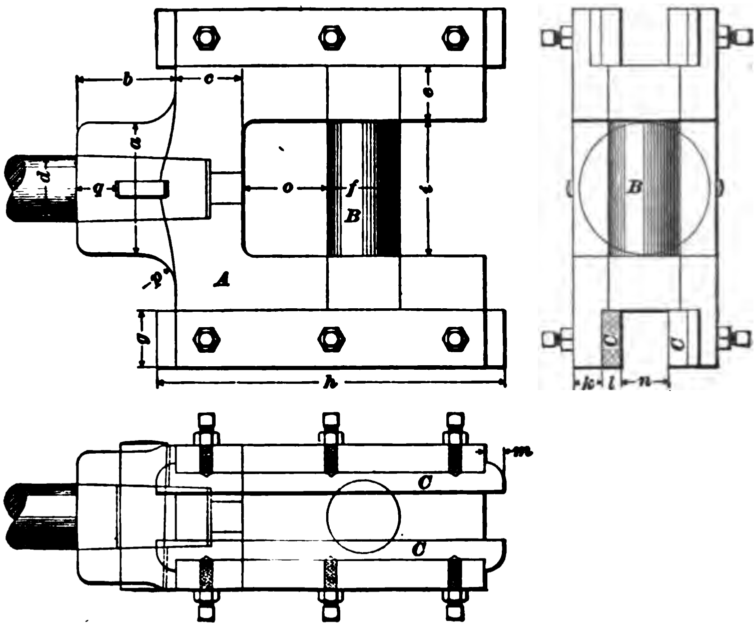


FIG. 765.

iron, with a boss for the end of the piston rod, which is secured by means of a cotter. The wrist-pin *B* is either wrought iron or steel, forced into the cross-head. Brass or bronze gibs *C* that are adjustable by means of the set-screws furnish the surfaces that bear on the guides.

Proportions for the cotter are to be taken from Art. **1968,** and the other proportions are as follows:

$D =$ diameter of cylinder.
$d =$ diameter of piston rod.
$a = 2\,d.$
$b = 1.5\,d.$
$c = d.$
$e = .75\,f.$
$f = .2\,D.$
$g = \frac{h}{6}.$
$h = \sqrt{\frac{PAC}{16.5\,L}}.$
$P =$ boiler pressure.
$A =$ area of piston in square inches.
$C =$ length of crank in inches.
$L =$ length of connecting-rod in inches.
$i = .26\,D + .5''.$
$k = .075\,D.$
$l = \frac{h}{18}.$
$m = l.$
$n =$ thickness of guides.
$o =$ space required to clear connecting-rod.
$p = .5\,d.$
$q = .75\,d.$

Set-screws may be $\frac{3}{8}''$ for cylinders up to 8″ in diameter; $\frac{7}{16}''$ for cylinders from 8″ to 12″ diameter, and $\frac{1}{2}''$ for all sizes above. Use two set-screws for cylinders up to 8″ in diameter, and three for larger cylinders. The above proportions are suitable for cylinders up to 16″ in diameter.

VALVES, VALVE STEMS, ECCENTRIC-RODS.

2112. The motion of the slide valve has been fully described under the subjects of Steam and Steam Engines, and Applied Mechanics. Hence, it is only necessary here to give the proportions of the various parts. Two sections of a slide valve are shown in Fig. 766.

First, take the design of the valve seat. The width b and length l of steam port are obtained from formula **294,** Art. **2083.** The width of the bridges c, c between the ports is usually made equal to the thickness of the cylinder walls. The width of the exhaust port is, as before stated, from $1\frac{3}{8}$ to $2\frac{1}{2}$ times the width of the steam port. In any given case the exhaust port must be wide enough so that when the valve is at the end of its travel (see Fig. 249, Art. **1238,**) the width of the portion of exhaust port remaining open is at least equal to the width of the steam port.

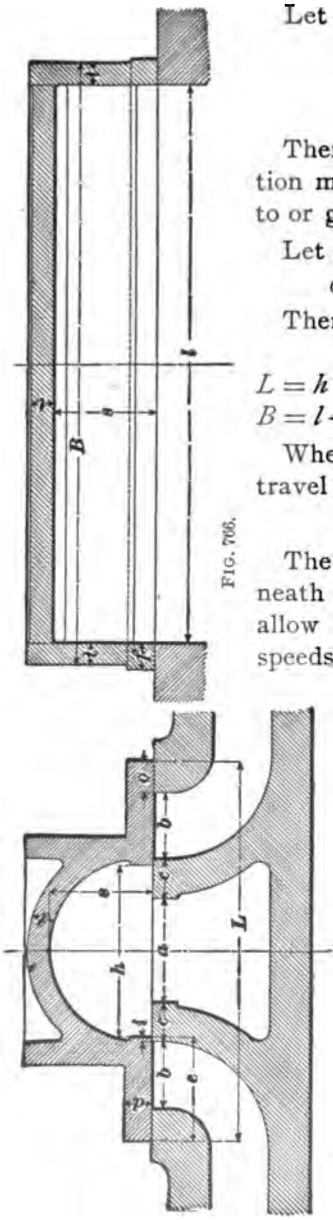


FIG. 766.

Let a = width of exhaust port;
b = width of steam port;
c = width of bridge;
k = half of travel of valve.

Then, in order that the above condition may be fulfilled, a should be equal to or greater than $b + k + i - c$.

Let i = inside lap;
o = outside lap.

Then, in Fig. 766, $h = a + 2(c - i)$.
$e = b + i + o$.
$L = h + 2e = a + 2c + 2b + 2o$.
$B = l + 2t$.

When the valve at the end of its travel just opens the steam port fully,

$$k = b + o.$$

The height s of the hollow underneath the valve must be sufficient to allow a free exhaust. For low piston speeds, s may be made equal to or slightly greater than the width b of the steam port; that is, $s = b$. More often, $s = \frac{3}{4}a$ to a, where a is the width of the exhaust port.

Also, d may be $1.2t$,

and f may be $1.1t$.

The thickness t of the metal of the valve equals

$t = .03D + .25''$ (cast iron),

where D is the cylinder diameter in inches.

The lead, lap, valve travel, etc., are readily determined from the valve diagram; see Arts. **1621**, etc.

These quantities, in addition to the proportions given above, furnish sufficient data to design a plain slide valve. The above proportions and formulas may be readily applied to the design of more complicated forms of slide valves, such as the Allen valve, double-ported valve, piston valve, etc.

2113. The **valve stem** must be designed to move the valve under the most unfavorable conditions that may occur in practice; hence, we may assume that the valve is unbalanced, for, even if balanced, the joint may leak.

Further, the valve may run dry upon the seat, thus increasing the coefficient of friction.

Let L = length of valve; (See Fig. 766.)
B = breadth of valve;
f = coefficient of friction;
p = pressure on back of valve which may be taken as the boiler pressure;
d = diameter of valve stem.

Then, the load which the valve stem must move equals

$$f p B L \text{ pounds.}$$

Under the most unfavorable circumstances f should not exceed $\frac{1}{5}$, or .20 (when the valve is properly lubricated, $f = \frac{1}{10}$ to $\frac{1}{16}$). For designing purposes, assume $f = \frac{1}{4}$, or .25. Then, the load on the valve stem is

$$\frac{pBL}{4} \text{ pounds.}$$

The valve stem is really a long column alternately in tension and compression; but, since under all ordinary circumstances, the load is very much less than given above, it will be safe to treat it simply as a rod in tension, and use a low value for the safe stress, S.

We then have

$$\frac{pLB}{4} = \frac{\pi d^2}{4} S,$$

or

$$d = \sqrt{\frac{pLB}{\pi S}}.$$

Take $S = 3,200$ for wrought iron and 3,850 for steel.

Then,
$$\left.\begin{array}{l} d = .01\ \sqrt{p\,L\,B} \text{ for wrought iron.} \\ d = .0091\sqrt{p\,L\,B} \text{ for steel.} \end{array}\right\} \quad \textbf{(303.)}$$

EXAMPLE.—A locomotive valve is $18\frac{1}{4}'' \times 10\frac{1}{4}''$ on the face; the boiler pressure is 150 lb. per sq. inch. Find the diameter of a wrought-iron valve stem for the same.

SOLUTION.—Using formula **303**,

$d = .01\ \sqrt{p\,L\,B} = .01\ \sqrt{150 \times 10\frac{1}{4} \times 18\frac{1}{4}} = 1.7'' = 1\frac{3}{4}''$, nearly. Ans.

NOTE.—The above example is taken from a locomotive in actual operation.

2114. The valve stem may be fastened to the valve in

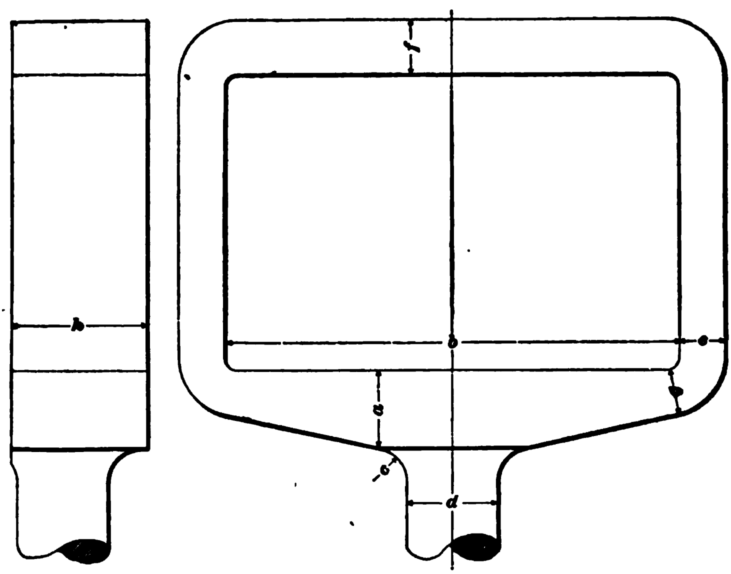


FIG. 767.

different ways. Examples are shown in Fig. 242, Art. **1226,** and Fig. 300, Art. **1308.**

Fig. 767 shows a wrought-iron yoke suitable for a simple slide valve like the one shown in Fig. 766.

The following proportions will give the proper dimensions:

$a = \sqrt{\frac{.21\,b\,d^2}{h}}$.

$b =$ dimension to fit valve.

$c = .375\,d$.

$d =$ diameter of valve stem in inches.

$e = .5\,d$.

$f = .75\,a$.

$g = .5\,d$.

$h =$ dimension to fit valve.

2115. Two other methods are shown in Fig. 768. At

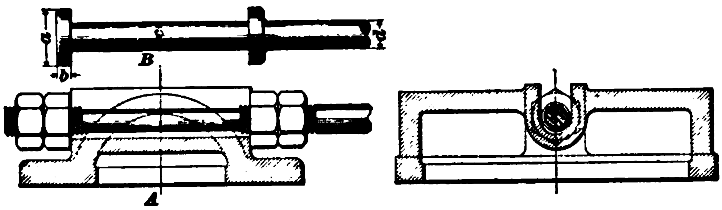


FIG. 768.

B the valve stem is forged with two collars, and at A the end of the stem is threaded for two sets of nuts.

This allows the valve to adjust itself more or less, and thus prevents the stuffing-box from wearing. The arrangement at B also furnishes a means for setting the valve so as to get the proper location over the ports.

The proportions for the valve-stem fastening shown in Fig. 768 are:

$c = d$. $b = .5\,d$.

$a = 2\,d$. $d =$ diameter of valve stem.

The outer end of the valve stem may terminate in some form of cross-head running in guides (see Fig. 299, Art. **1308**), or it may be jointed to a rocker-shaft, as shown in Fig. 242, Art. **1226.**

2116. Eccentric-rods may be rectangular or circular in cross-section. Quite often the rod is tapered, being largest where it joins the eccentric strap. It is a common practice to make the area of the smallest section of the rod equal to

about .8 the area of cross-section of valve stem, the latter being calculated from formula **303**.

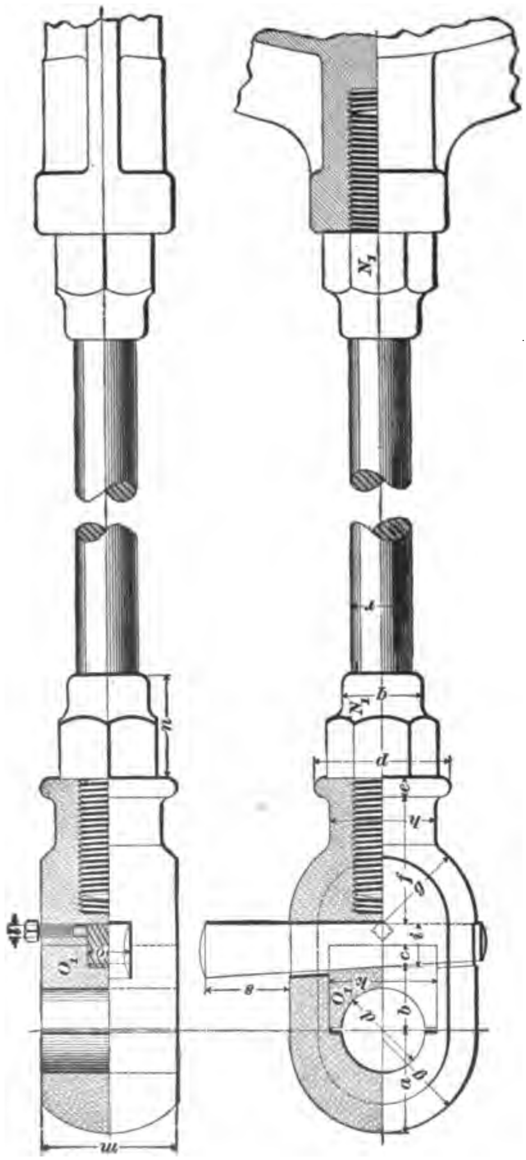


FIG. 768

The area at the large end may then be made about $\frac{1}{3}$ larger than the area at the small end.

The end of the eccentric-rod that joins the valve stem is often forked, as shown in Fig. 642, Art. **1955.** In such cases the proportions given on the figure may be used.

Fig. 769 shows an eccentric-rod with right and left threaded ends, one of which is attached to the eccentric strap, and the other to a brass bearing for connecting to the valve-rod pin or rocker-arm pin. The threaded ends of the rod furnish means for adjusting the valve, and the locknuts N_1, N_1 prevent the rod from turning after the valve is properly set.

The brass has a loose piece O_1 which is adjustable by means of the cotter, thus furnishing means of taking up wear.

The following proportions will give the dimensions for this rod and its brass:

$D =$ Diameter of valve stem.
$d = 1.77\,D$.
$a = 1.2\,d$.
$b = .75\,d$.
$c = .25\,d$.
$e = .3\,r$.
$f = 1.75\,d$.
$g = 1.1\,d$.
$h = 1.75\,r$.
$i = .25\,d + .25\,D + .1875''$, but never less than $.25\,d + .4375''$.
$k = 1.3\,d$.
$l = .25\,D$, but never less than $.25''$.
$m = 1.6\,d$.
$n = 1.75\,r$.
$q = 1.3\,r$.
r to be designed as a long column.
$s = d$.

Taper of cotter $= \frac{3}{4}''$ per foot.

2117. Fig. 770 shows an eccentric-rod with a modification of the marine end for the valve-stem pin bearing. The rod passes through a boss on the eccentric strap, and is fastened by the two nuts. This construction admits of adjusting the valve when necessary. The bearing for the valve-stem pin is composed of two brass seats that are held in place by a wrought-iron cap, and the stud bolts; liners

are placed between the end of the rod and the cap; the brasses may be adjusted by filing.

The proportions for this rod, and of the boss for fastening it to the eccentric strap, are as follows:

D = diameter of valve stem.

$d = 1.77\,D.$	$j = d.$
$a = d + f + .375''.$	$k = 1.6\,d.$
$b = 1.25\,d.$	l = diameter of eccentric-rod at eccentric end.
$c = 1.5\,d.$	
$e = 1.5\,d + .1875''.$	$m = 2.1\,l.$
f = area at root of thread $= .38\,D^2.$	$n = .75\,D.$
	$o = 3\,l.$
$g = .5\,i.$	$p = 1.5\,D.$
$h = 1.5\,d.$	$s = .125\,d.$
$i = 3.5 f + d + .375''.$	

Determine the diameter of the rod by treating it as a long column.

ECCENTRIC SHEAVES AND STRAPS.

2118. Fig. 771 shows a design for an eccentric, with a cast-iron sheave and a steel strap, that is especially adapted for vertical engines. The sheave is made in two parts, so that it can be readily put on or removed from the shaft without disturbing fly-wheels or bearings. In some cases this is necessary, owing to the construction of the shaft, which will not permit an eccentric to be slipped into place over the end. The end of the eccentric-rod is forged **T** shaped, and fastened to the strap by tap bolts.

The two halves of the strap are held apart by liners, which permit of adjustment for wear. Split pins are put through the holes in the ends of the bolts n to keep the nuts from turning off.

The following proportions give the necessary dimensions for this eccentric:

D = diameter of valve stem.

d = diameter of shaft.	$b = 2.5\,n$ at least, but never less than w.
$a = d + 2\,k + 2\,f$, never less.	

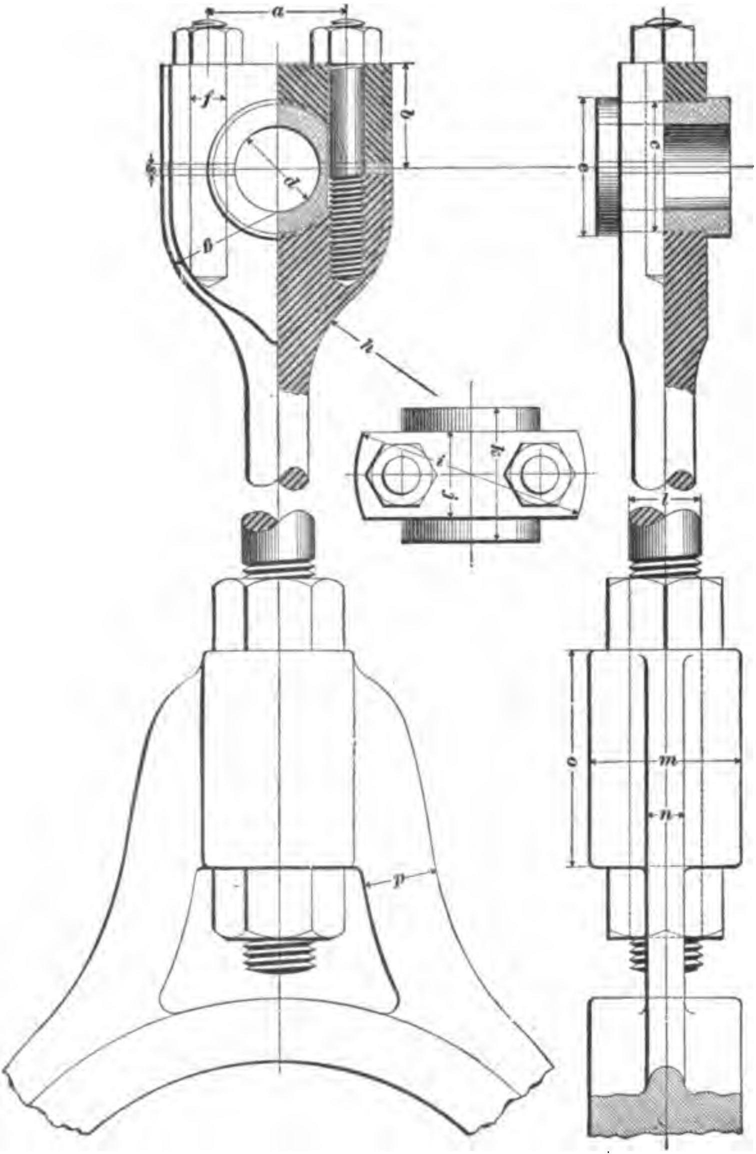


FIG. 770.

$b' = b + .5\,c.$

$c = .6\,D + .375''.$

$e = .25\,c.$

$f = .7\,D + .5''$, unless more is required to allow nuts to be placed on stud m.

$g = .6\,D + .375''.$

k = eccentricity.

l = .75 of diameter of bolt m.

Area of bolt m at root of thread $= .38\ D^2$; use nearest standard size of bolt.

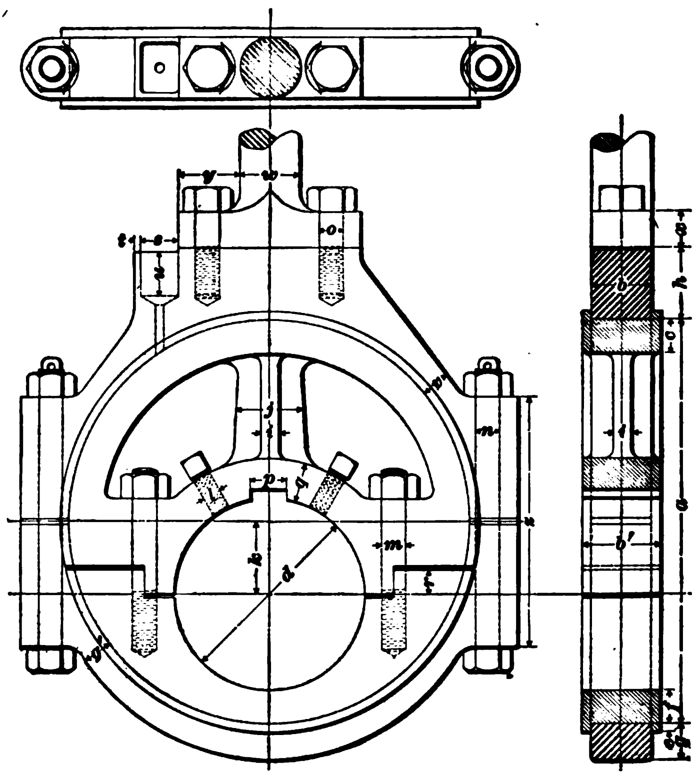


FIG. 771.

$g' = .5\,D + .25''.$

$h = 3\,o.$

$i = .5\,c.$

$j = 2\,c.$

n = diameter of bolt m.

o = diameter of bolt m.

$p = .7\,D + .5''.$

$q = .7\,D + .5''.$

$r = .125\,d.$
$s = D.$
$t = .25''$, constant.
$u = 1.25\,D.$
$v = .5\,D + .25''.$
$w =$ diameter of eccentric-rod.
$x = .6\,w.$
$y = 2.5\,o.$

z is to be found by laying out. The bolt n should clear the eccentric sheave by $\frac{1}{4}''$ on all sizes up to $D = 1\frac{1}{2}''$; $\frac{3}{8}''$ for sizes of D from $1\frac{1}{2}''$ to $2''$, and $\frac{1}{2}''$ for all sizes above $D = 2''$.

2119. In Fig. 772 the eccentric sheave and strap are both made of cast iron. The eccentric sheave is cast solid,

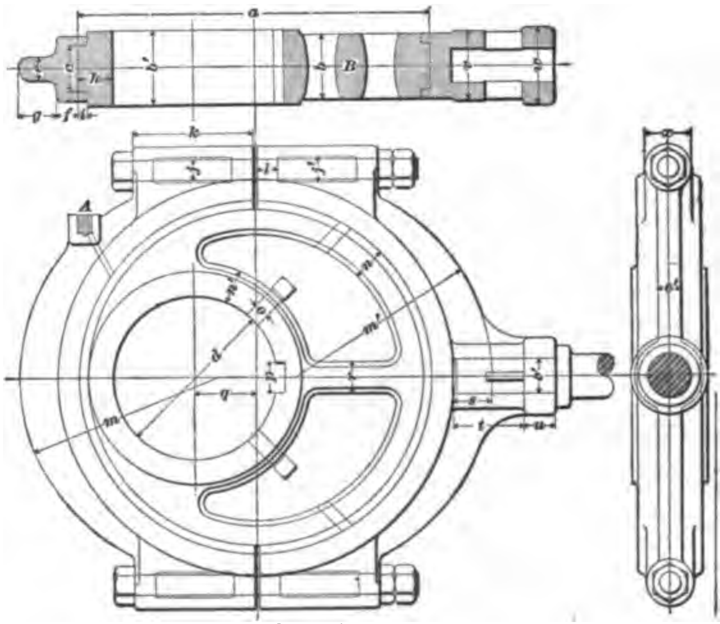


FIG. 772.

and must, therefore, be slipped over the end of the shaft. The eccentric-rod is held in a boss on the strap by means of a cotter.

It will be seen that in this case the strap is grooved for the sheave, while in Fig. 771 the groove is in the sheave. The construction which places the groove in the strap has the advantage of retaining the oil better.

For eccentrics used with valve stems $\frac{1}{2}''$ in diameter or less the holes for bolts j are not to be cored.

A = boss for oil cup.

B = cross-section of rib r.

The proportions are as follows:

D = diameter of valve stem.
d = diameter of shaft.
$a = d + 2q + 2h$.
$b = 2D + .125''$.
$b' = 2.25D + .125''$.
$c = 1.5D$.
$e = .75D$.
$e' = .75D$.
$f = .7D$.
$g = 1.25D$.
$h = D + .125''$.
$i = .25D + .0625''$.
j = area of bolt at root of thread = $.38\ D^2$; use the nearest standard size bolt.
$j' = j + .1875''$.
$k = 4D$.

$l = j$.
$m = \dfrac{d + 2q + 2h + 2f}{2}$.
$m' = m$.
$n = D + .125''$.
$n' = D + .125''$.
$o = .75j$.
$p = D$.
q = eccentricity.
$r = D$.
$s = 1.25D$.
$t = 2.25D + 1.25''$.
$u = D$.
$v = 2.25D$.
$v' = 1.125D$.
$w = 2.5D$.
$x = 2.25j$.

STUFFING-BOXES.

2120. A stuffing-box of the ordinary form is shown in

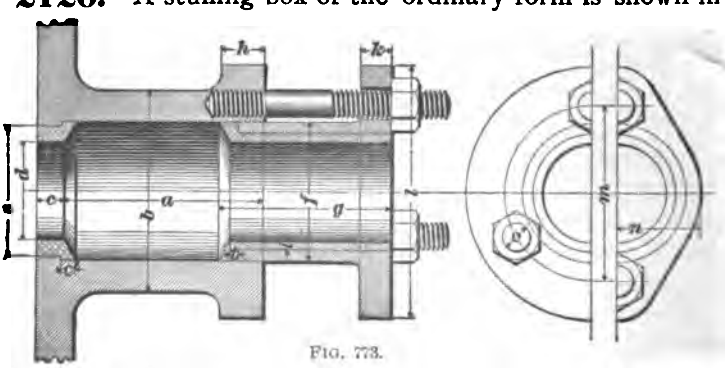


FIG. 773.

Fig. 773. The gland may be made of brass, of cast iron lined with brass, or simply of cast iron.

The brass lining, however, injures the rod less than the harder iron. The gland is usually held in place by two stud bolts, but for large rods the gland is sometimes made circular instead of oval, and fastened by three or more studs.

The proportions used for a gland of the form shown are:

d = diameter of rod.
$a = 1.6\,d + 1.5''$.
$b = 1.75\,d + 1.125''$.
$c = .1\,d + .75''$.
$c' = .5\,c$.
$e = 1.25\,d + .375''$.
$f = 1.25\,d + .625''$.
$g = 1.5\,d + 1''$.
$h = .3\,d + .5''$.
$i = .04\,d + .1875''$.
$k = .25\,d + .25''$.
$l = 2.25\,d + 1.75''$.
$m = 1.6\,d + 1.25''$.
$n = .75\,d + .375''$.
$o = .25\,d + .25''$ for two bolts.
$= .2\,d + .25''$ for three bolts.
$= .05\,d + 1.0625''$ for four bolts.
$t = i$.

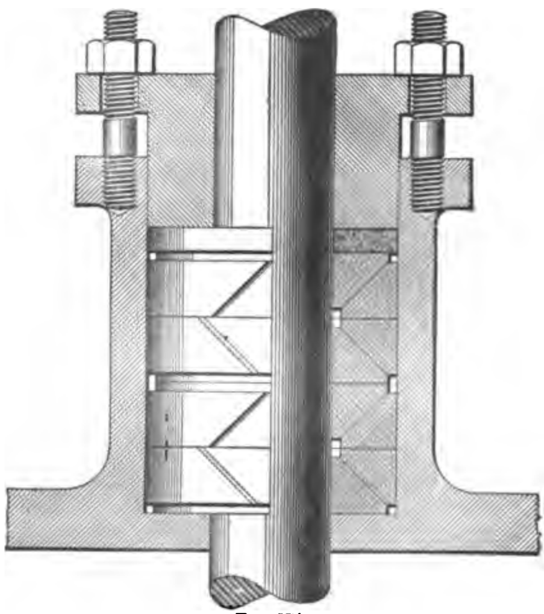


FIG. 774.

Use two bolts for glands on rods up to 3.5 inches in diameter; above that size make gland round, and use three bolts

for rods up to 5.5″ in diameter, and four bolts for all larger sizes.

2121. For very high steam pressures various styles of metallic piston and packing are used. One of the best of these is Katzenstine's, shown in Fig. 774.

The construction of the stuffing-box and gland for this packing is very similar to the form shown in Fig. 773, but the packing is made up of rings of brass or similar anti-friction metal. These rings are made in two or more segments, depending on the size of the rod; by reason of their conical shape, the pressure of the gland forces the inner ones against the rod, while the outer ones are pressed against the sides of the stuffing-box. A rubber or fibrous ring placed between the gland and the first ring serves as a cushion to make the packing slightly elastic.

2122. A stuffing-box of the form shown in Fig. 775 is

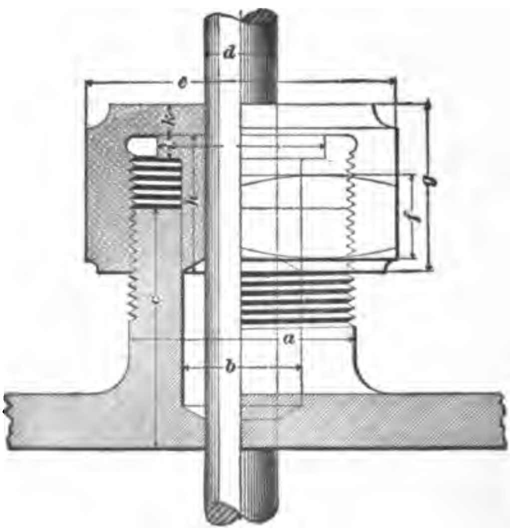


Fig. 775.

generally used for small work, such as the spindles of valves, etc. The outside of the stuffing-box is threaded to receive a hexagonal nut which fits over the gland. As the nut is

screwed down, the gland is pressed downwards and compresses the packing.

The proportions used are:

d = diameter of rod.

$a = 2.5\,d + .5''$.

$b = 1.5\,d + .125''$.

$c = 3\,d + .25''$.

$e = 3.5\,d + .625''$.

$f = d + .125''$.

$g = 2\,d + .25''$.

$h = 1.5\,d + .25''$.

$i = .25\,d + .0625''$.

$k = .5\,d$.

This design may be used for rods up to $1\frac{1}{4}''$ diameter.

Make the number of threads per inch the same as for a bolt whose diameter is equal to the diameter of the rod.

ENGINE FLY-WHEELS.

2123. Fly-wheels are subjected to a variety of complicated stresses. It is therefore difficult to base the design upon theory alone, and reliance must be placed upon empirical rules representing successful practice.

First, consider a fly-wheel which is intended merely to equalize the turning moment, and which does not carry a driving belt. The weight of the rim for any given case may be found from formula **107**, Art. **1328.** Then, knowing the diameter of the wheel, it is easy to find the required cross-section.

Let W = required weight of rim in pounds;
D = mean diameter of rim in inches;
w = weight of material per cubic inch;
A = area of cross-section in square inches.

Then,
$$W = \pi D A w,$$
or
$$A = \frac{W}{\pi D w}.$$

For example, assume in the problem given in Art. **1328** that the fly-wheel is 12 feet in diameter; since the rim is required to weigh 5,288 pounds, the cross-section of the rim must be

$$A = \frac{W}{\pi D w} = \frac{5{,}288}{3.1416 \times 144 \times .261} = 44.7 \text{ sq. in.}$$

Fly-wheels of this type usually have rims of rectangular section, the depth being from 1.1 to 1.4 times the breadth.

The usual form of the rims is shown in Fig. 776. The only precaution necessary in designing such a rim is to limit the

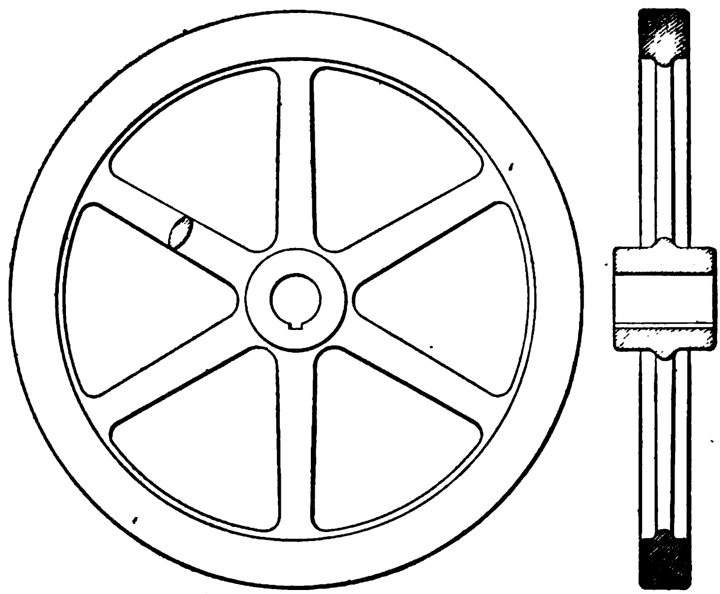


FIG. 776.

velocity to a safe value. The maximum allowable rim speed is usually taken at a mile per minute = 88 feet per second. Occasionally this may run as high as 100 feet per second.

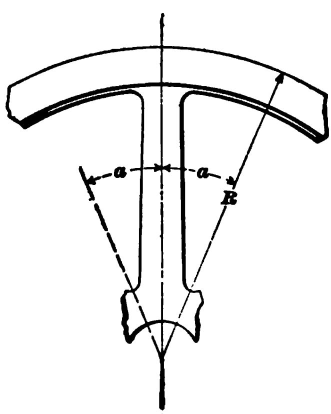


FIG. 777.

The arm of a fly-wheel is subjected to the following straining forces:

1. The centrifugal force due to the rim.

2. A transverse pull at certain points of the stroke; for example, when the crank effort is less than the resistance, the rim is retarded (see Art. **1327**) and gives up energy to help turn the shaft. This energy must be transmitted through the arms.

3. If the fly-wheel carries a belt, the belt pull is, of course, transmitted through the arms.

Suppose the wheel to be made in sections, each section containing one arm.

Let W_1 = weight of one section of rim;
W_2 = weight of one arm;
R = outer radius of wheel in feet;
r = outer radius of wheel in inches;
v = velocity of rim in feet per second;
w = angular velocity;
h = distance in feet from center of wheel to center of gravity of section of rim;
n = number of arms;
a = half the angle subtended by section of rim (see Fig. 777).

The centrifugal force due to one section of the rim is

$$F = \frac{W_1 h w^2}{g}.$$

According to Weisbach,

$$h = \frac{R n \sin a}{\pi}, \text{ very nearly.}$$

Also,
$$w = \frac{v}{R}.$$

Substituting these values of h and W, the centrifugal force of the section of the rim is

$$\frac{W_1 h w^2}{g} = \frac{W_1 n v^2 \sin a}{\pi g R} \text{ pounds.}$$

The center of gravity of the arm may be taken as $\frac{1}{2} R$ from the center, and the velocity of this center of gravity is, therefore, $\frac{1}{2} v$. Consequently, the centrifugal force due to the arm is

$$\frac{W_2 (\frac{1}{2} v)^2}{g \times \frac{1}{2} R} = \frac{W_2 v^2}{2 g R}.$$

To these stresses must be added the weights of the section and arm, as they exert a pull when hanging vertically downwards. The direct tension on each arm is, therefore,

$$T = \frac{W_1 n v^2 \sin a}{\pi g R} + \frac{W_2 v^2}{2 g R} + W_1 + W_2. \qquad (a)$$

NOTE.—The general formula for centrifugal force is $F = \frac{m v^2}{R}$, when m = the mass of the revolving body, v = the velocity of the center of gravity in feet per second, and R = radius to center of gravity in feet. This may be easily changed to correspond with formula **19**, Art. **903**, as follows:

$$m = \frac{W}{g};\ N = \frac{60\,v}{2\pi R},\ \text{or}\ v = \frac{2\pi R N}{60}.$$

Substituting in the above formula,

$$\frac{W}{g R} \times \left(\frac{2\pi R N}{60}\right)^2 = .00034\ W R N^2.$$

The term **angular velocity** is applied to the result obtained by dividing the circumferential velocity by the radius; hence, $w = \frac{v}{R}$.

Therefore, if h = radius to center of gravity in feet,

$$F = \frac{m v^2}{h} = \frac{m v^2}{h} \times \frac{h}{h} = m h \frac{v^2}{h^2} = m h \left(\frac{v}{h}\right)^2 = m h w^2 = \frac{W h w^2}{g}.$$

To take account of the second of the straining actions above mentioned, we may proceed as follows:

From a crank-effort diagram, such as is shown in Fig. 315, Art. **1327,** the greatest difference between the crank effort and the constant resistance may be found. For example, in Fig. 315, this difference is represented by the ordinate $A\,M$ or $B\,N$. *Multiply this pressure, expressed in pounds, by the area of the cylinder and by the diameter of the fly-wheel (in feet), and divide by the stroke in feet. This will give the equivalent pull at the ends of the arms.* Call it P_1. Then, the bending moment on one arm is

$$\frac{P_1 r}{n}\ \text{inch-pounds.}$$

In a similar manner, if the fly-wheel is belted, the pull of the belt is found from the formula

$$P_2 = \frac{63{,}025\,H}{r\,n} \text{ (see formula } \mathbf{231}\text{, Art. } \mathbf{1963}\text{)},$$

and the moment on each arm is

$$\frac{P_2 r}{n} \text{ inch-pounds. Let } P_1 + P_2 = P_0.$$

Then, in accordance with formula **116**, Art. **1398**,

$$\frac{P_2 r}{n} + \frac{P_1 r}{n} = \frac{(P_2 + P_1)\,r}{n} = \frac{P_0 r}{n} = \frac{S_4 I}{c}, \qquad (b)$$

where $S_4 =$ safe transverse strength, and $I =$ moment of inertia, etc.

Let $A =$ area of cross-section of arm.

Then, the stress on the arm due to direct tension is, from equation (a),

$$S_1 = \frac{T}{A},$$

from equation (b), $\qquad S_4 = \frac{P_0 r c}{n I}.$

$S_1 + S_4$ must not exceed a safe value, k.

Adding, $\qquad S_1 + S_4 = \frac{T}{A} + \frac{P_0 r c}{n I} = k. \qquad$ **(304.)**

This formula may be used for finding the arm dimensions of a wheel cast in sections. If, however, the rim is cast solid, the centrifugal force developed is chiefly borne by the rim itself, and the expression $\frac{T}{A}$ may be dropped. In this case,

$$k = \frac{P_0 r c}{n I},$$

or $\qquad \frac{I}{c} = \frac{P_0 r}{n}. \qquad$ **(305.)**

The value of $\frac{I}{c}$ for the desired section may be found from the Table of Moments of Inertia.

An example will best show the application of formula **304.** Take a fly-wheel 18 feet in diameter, with 8 arms. The wheel is in sections. The cross-section of the rim is 60 square inches. There is no belt, but the stress induced by acceleration of the rim is 9,000 pounds at the rim; that is, $P_1 = 9{,}000$ pounds. The rim runs at a speed of 70 feet per second. It is required to design the arms, which are to be rectangular in cross-section.

Each section contains $\frac{1}{8}$ of the rim, and, therefore, it weighs $\frac{3.1416 \times 18 \times 12 \times 60 \times .261}{8} = 1{,}328$ lb. (approximately). For the present leave the arm out of consideration.

Then, the direct tension on the arm due to centrifugal force is, from equation (a),

$$T = \frac{W_1 n v^2 \sin a}{\pi g R} + W_1 =$$

$$\frac{1{,}328 \times 8 \times 70^2 \times \sin 22\tfrac{1}{2}^\circ}{\pi \times 32.16 \times 9} + 1{,}328 = 23{,}236 \text{ lb.}$$

Using a safe stress of 3,000 lb. per sq. in., the cross-section of the arm must be, to withstand this tension,

$$\frac{23{,}236}{3{,}000} = 7\tfrac{3}{4} \text{ sq. in.}$$

In reality, this cross-section must be largely increased to provide for the bending force at the ends of the arms. Assume that the final section may be double that given above, or $2 \times 7\frac{3}{4} = 15\frac{1}{2}$ sq. in. Then, the weight of the arm is

$$15\tfrac{1}{2} \times 108 \times .261 = 437 \text{ lb.} = W_2,$$

and $$\frac{W_2 v^2}{2 g R} + W_2 = \frac{437 \times 70^2}{64.32 \times 9} + 437 = 4{,}136 \text{ lb.}$$

Then, $T = 23{,}236 + 4{,}136 = 27{,}372$ lb.

For the stress k, take 3,000 lb. per sq. in.

From formula **304,**

$$\frac{T}{A} + \frac{P_0 r c}{n I} = k, \text{ or } \frac{27{,}372}{A} + \frac{9{,}000 \times 108}{8} \frac{c}{I} = 3{,}000.$$

From the Table of Moments of Inertia,

$$A = b\,d, \text{ and } \frac{c}{I} = \frac{6}{b\,d^2}.$$

Let $b = .4\,d$; then, $A = .4\,d^2$, and $\frac{c}{I} = \frac{15}{d^3}$.

Hence, $$\frac{27,372}{.4\,d^2} + \frac{9,000 \times 108 \times 15}{8\,d^3} = 3,000;$$

or $$\frac{68,430}{d^2} + \frac{1,822,500}{d^3} = 3,000.$$

It will be found that a value of $d = 9\frac{1}{2}$ inches, nearly, satisfies the equation. Hence, take $d = 9\frac{1}{2}''$ and $b = 9.5 \times .4 = 3.8 = 3\frac{3}{4}''$.

The area of the cross-section from these dimensions might now be used to give more accurately the centrifugal force of the arm. It would make little difference in the final result, however.

2124. Construction of Fly-Wheels.—Fly-wheels of small diameter are usually cast solid; the arms are of oval cross-section, and the wheel has the general appearance of a belt pulley with a heavy rectangular rim (see Fig. 776).

A heavy fly-wheel with oval arms is shown in Fig. 778. This wheel is built in four sections, with two arms to each section, and in addition the hub is formed of two separate rings that are bolted and keyed to the segments forming the inner ends of each pair of arms. The segments of the rim are joined by means of steel or wrought-iron rings R which are shrunk on to bosses formed by recesses cast in the rim. Besides these rings, bolts B pass through lugs on the inner surface of the rim. In order that the wheel may be amply strong, the net section of the bolt and two rings must be sufficient to withstand the action of centrifugal force tending to separate the rim through the section which they join. The bolts and keys joining the inner ends of

the arms to the hub are in double shear, and must be calculated to withstand the centrifugal force due to the arms and rim plus the force necessary to retard or accelerate the rim.

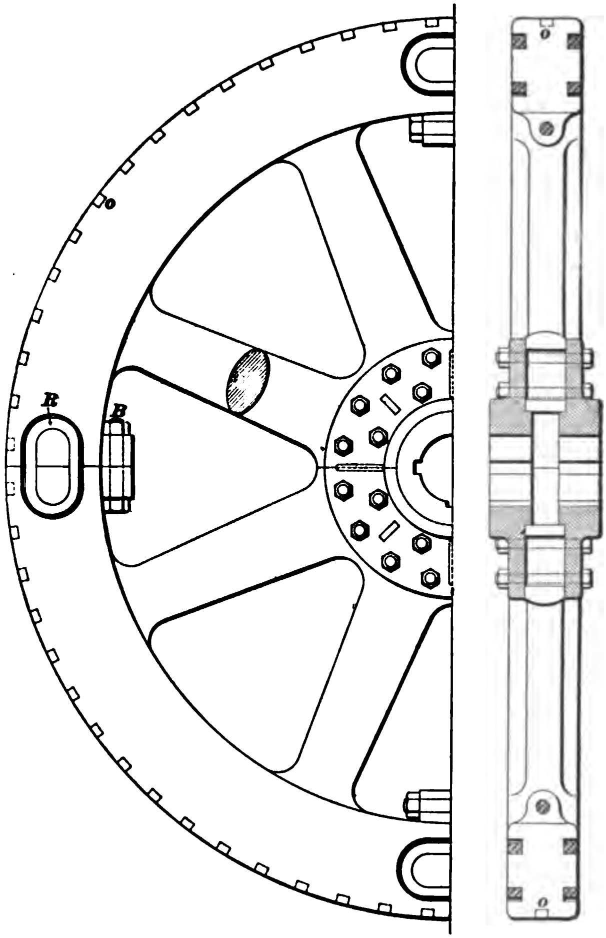


FIG. 778.

2125. Fig. 779 shows a fly-wheel with the face of the rim turned to serve as a belt pulley. The arms are oval in

section and cast hollow, thus giving them increased stiffness for a given weight. The rim is also given a channel-shaped section, which increases its ability to withstand the bending

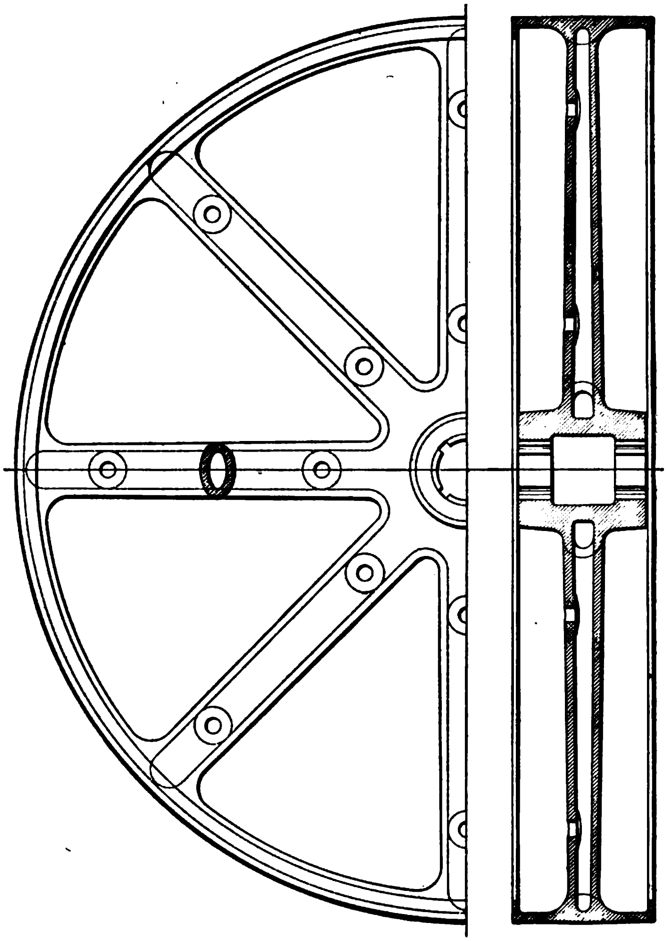


FIG. 779.

stresses produced by centrifugal force in the sections between the arms.

2126. Fig. 780 is a fly-wheel of large diameter, with the face of the rim turned for a belt. This wheel has ten

arms, and the rim is cast in ten segments that are bolted together by means of flanges on the inner surface. The

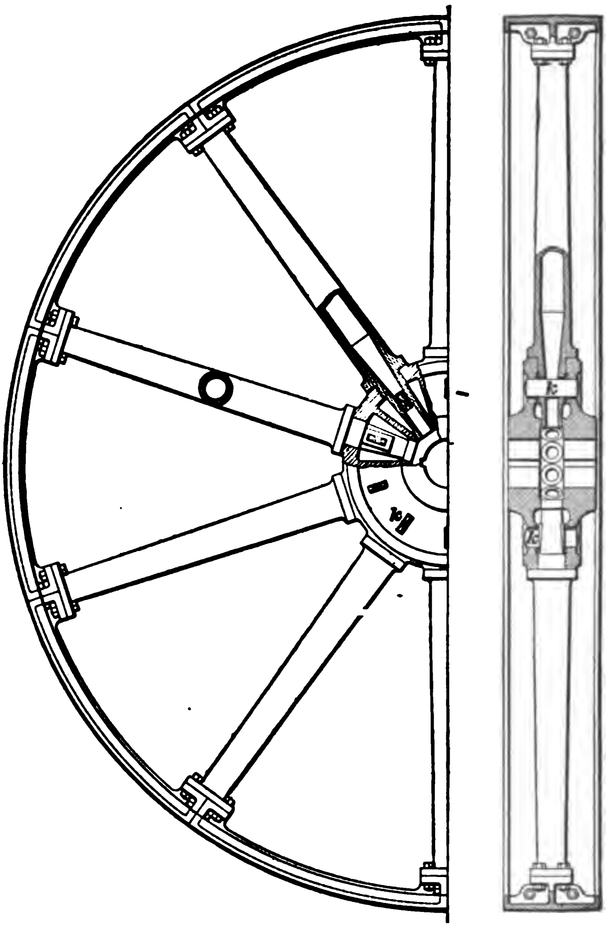


FIG. 780.

segments of the rim are also bolted to the ends of the arms by means of flanges cast on the rim and arms.

The arms are circular in section, and cast hollow. Their inner ends are turned to fit recesses bored in the hub, and held in the hub by steel or wrought-iron keys *k*.

2127. Another method of fastening the arms to the hub is shown in Fig. 781. Here the ends of the arms are cast with flanges, and they also have a circular boss turned on them which fits into a recess bored in the hub. Bolts

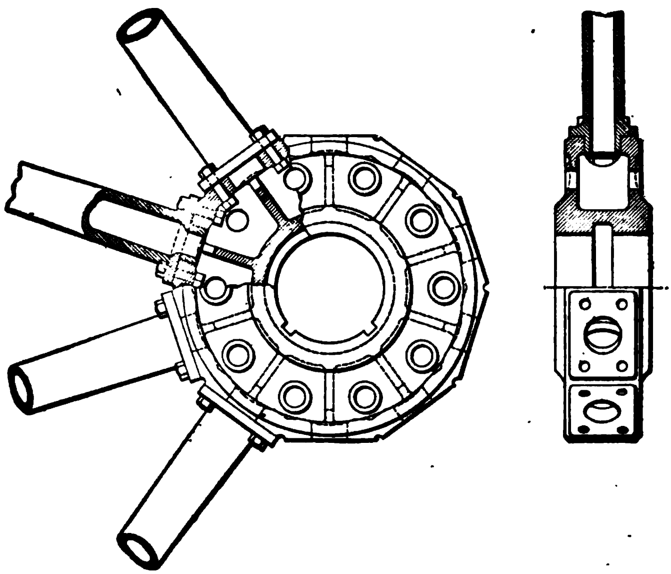


FIG. 781.

pass through the flanges in the ends of the arms and in the face of the hub, thus holding the arms securely in place.

2128. Fig. 782 is an example of a fly-wheel cast in halves. The sections of the rim are joined by means of steel or wrought-iron bars *B*, inserted in holes cast in the ends of the rims. These bars are fastened to the rim by keys *K* that pass through holes fitted for that purpose. The arms are oval in section, and cast solid. The hub is

provided with bosses through which four bolts are passed, thus joining the two parts of the hub securely. The holes *o* cored in the rim of the fly-wheels, shown in Figs. 778 and

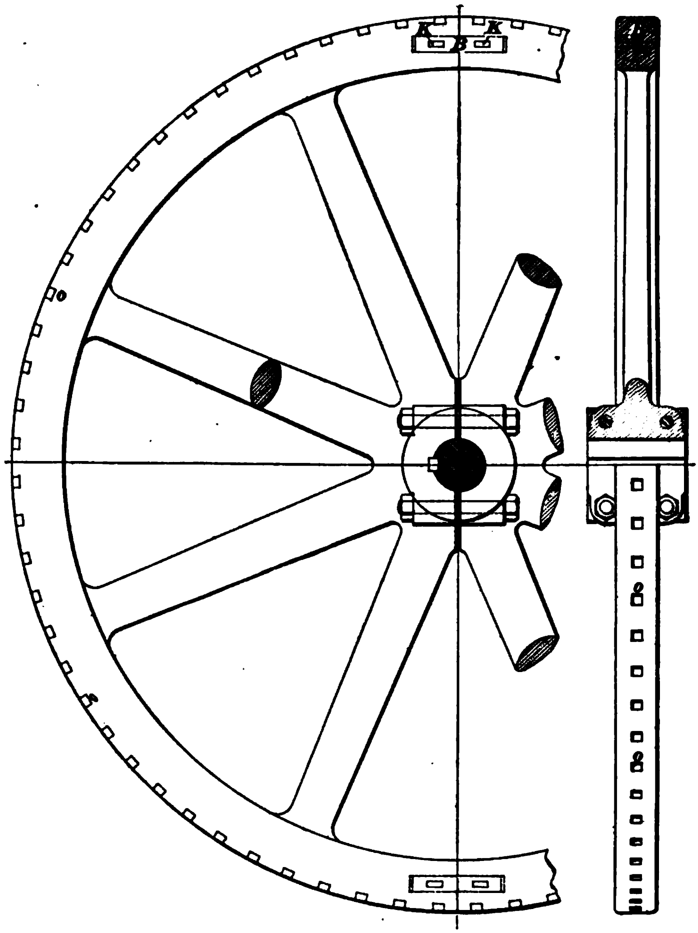
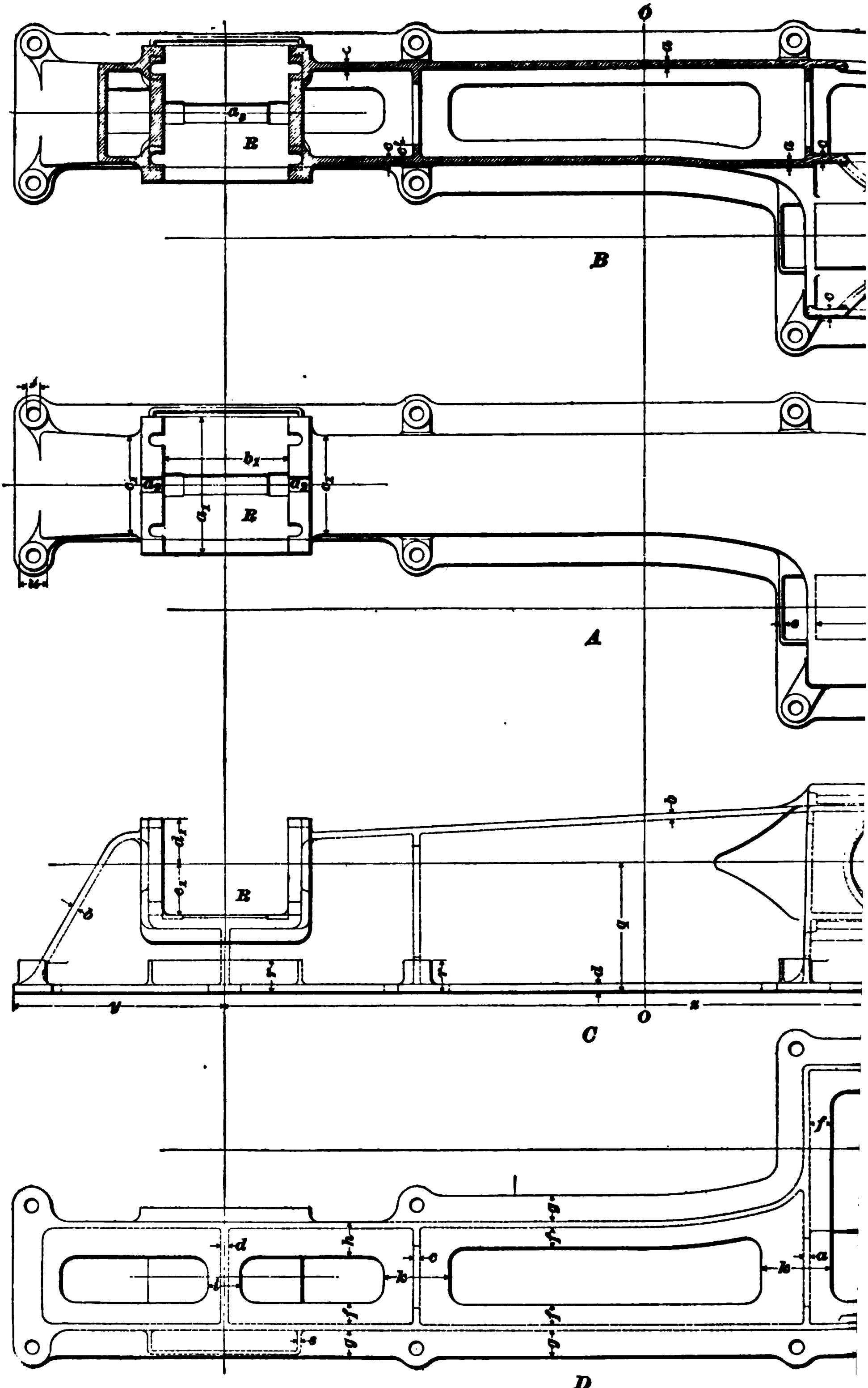


FIG. 782.

782, are for the purpose of inserting the end of a bar when it is required to turn the engine, either to get the crank off from the center in starting, or for any other purpose.



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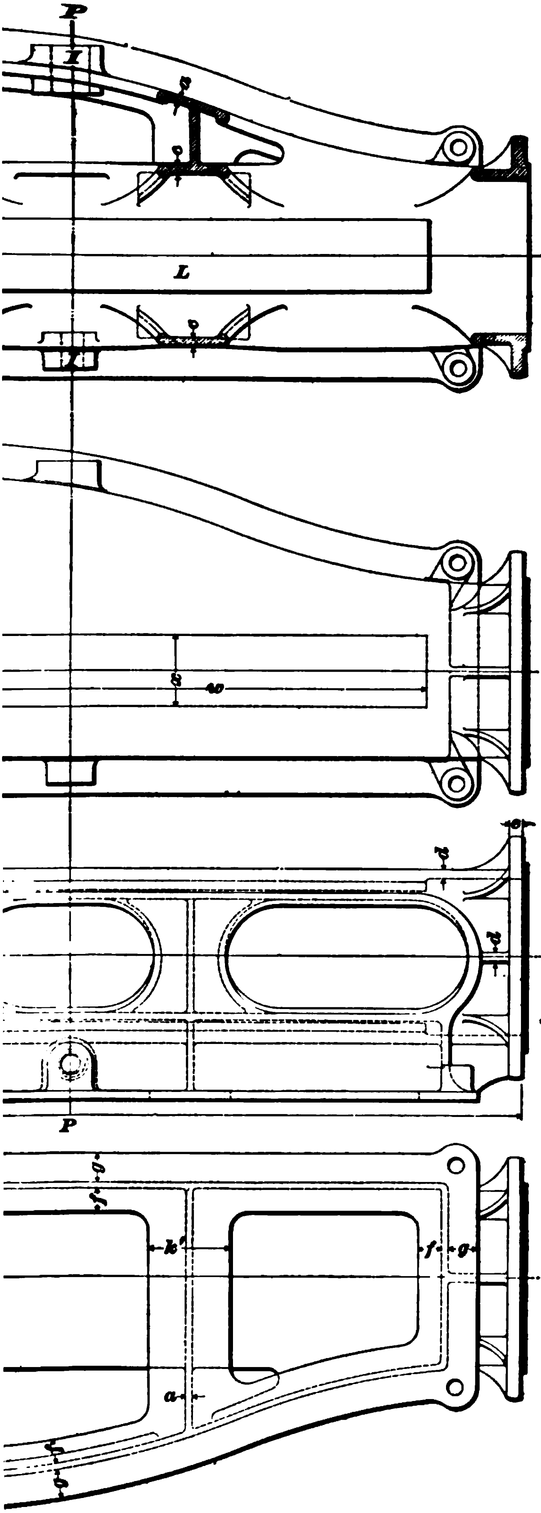

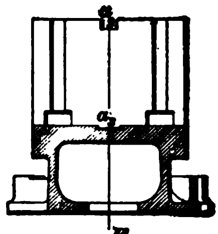

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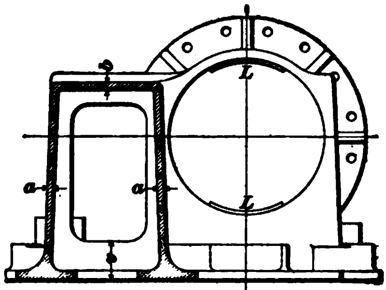

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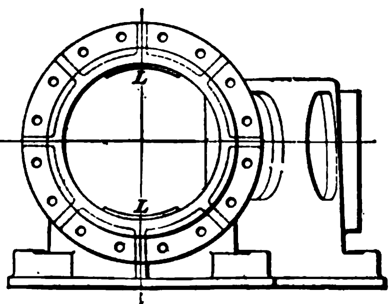

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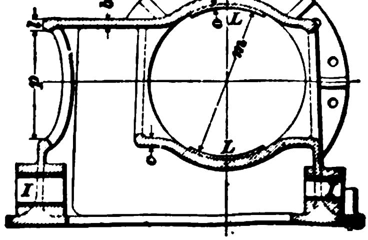

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88.

ENGINE FRAMES, OR BEDS.

2129. The **frame,** or **bed,** of an engine is the main structure to which the other parts are attached. It provides the necessary stiffness and rigidity, and, on account of its mass, absorbs more or less of the vibration due to the movement of the reciprocating parts.

Engine beds are made in a great variety of forms, each type of engine having its peculiar type of bed.

Fig. 783 shows a substantial style of engine bed for horizontal engines. *A* is a top view; *B* a horizontal section on the center line looking towards the bottom; *C* a front side view, and *D* a view of the bottom. *E* is a cross-section through the center line of the main bearing; *F* a cross-section on the line *O O*; *G* an end view, and *H* a cross-section through the guides on the line *P P*. The guides *L*, *L* are cast solid with the bed, and bored out to form the bearing surface for the cross-head, which is of the form shown in Fig. 762. *I*, *I* are bosses which form bearings for the rocker-arm shaft. These bearings are provided with brass or babbitt bushings. The main bearing, which is separate from the frame, rests in the opening *R*.

Proportions for designing this bed are based on the diameter of the cylinder, the length of stroke, dimensions of cross-head, and length of connecting-rod, as follows:

$D =$ diameter of cylinder.

$a = .027\,D + .1875''$.	$l = 2\,a$.
$b = 1.1\,a$.	$m =$ the distance required to clear the connecting-rod.
$c = 1.25\,a$.	
$c' = 2.5\,a$.	
$d = 1.5\,a$.	$n = 2\,a$.
$e = 2.25\,a$.	$o = .5\,a$.
$f = 4\,a$.	$p = .75\,m$.
$g = 4.75\,a$, but never less than u.	$q = .8\,D$ to D.
	$r = 6\,a$.
$h = 6.5\,a$.	$s = .5\,a$.
$i = 6\,a$.	$t = .06\,D + .5''$ (use the nearest standard size of bolt).
$k = 12\,a$.	
$k' = 13.5\,a$.	$u = 2.1\,t$.

$v = 6.5\,a$.

w = length of stroke + length of rubbing surface of cross-head − (.01 D + .1875″).

x = same width as cross-head.

y = about 1.3 D. In all cases the crank must clear the bosses and nuts for the foundation bolts.

The length z must be such that the hub of the cross-head will clear the stuffing-box bolts when at the end of the stroke. Its value approximately = length of crank + length of connecting-rod + distance from center of cross-head pin to end of cross-head hub + clearance between cross-head hub and stuffing-box bolts + the distance which the stuffing-box bolts project into the frame. This distance z is best determined by laying out the various parts to scale.

The dimensions for the seat for the main bearing are:

d' = diameter of crank-shaft journal.

$a_1 = 1.75\,d'$.

$b_1 = 1.65\,d' - .5''$.

$c_1 = .62\,D$.

$d_1 = .5\,d' + 1.25''$.

$e_1 = .66\,d'$.

The bearing for the frame shown in Fig. 783 is shown in detail in Fig. 784.

Proportions for designing this bearing are:

d' = diameter of journal.
D = diameter of cylinder.

$a = d' + 1''$.

$a' = .2\,d' + 2''$.

$b = .5\,d' + 1''$.

$b' = .12\,d' + 1.25''$.

$c = .66\,d'$.

$c' = .06\,d' + .625''$.

$c_1 = .62\,D$.

$e = 1.65\,d' - .5''$.

$e' = 1.22\,d'$.

$f = .25\,d' + .375''$.

$f' = 1.35\,d'$.

$g = .1\,d' + .5625''$.

$g' = .1\,d' + 1''$.

$h = .85\,d'$.

$i = .1\,d' + .25''$.

$i' = .2\,d' + .5''$.

$j = .1\,d' + .25''$.

$k = .5\,d' + 1.25''$.

$l = .375''$, constant.

$l' = .1\,d' + .375''$.

$m = .175\,d' + .3125''$.

$n = .25\,d' + .25''$.

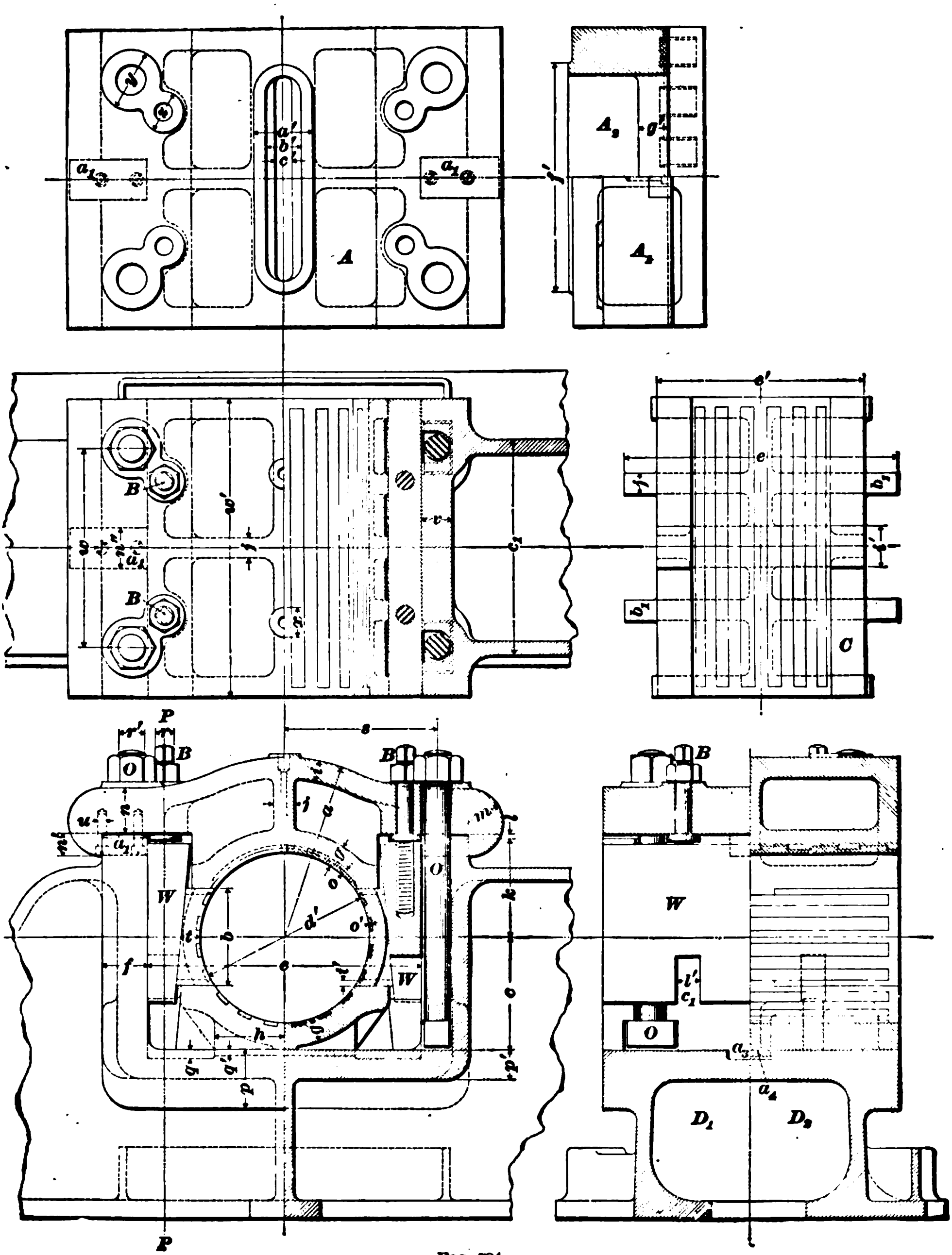
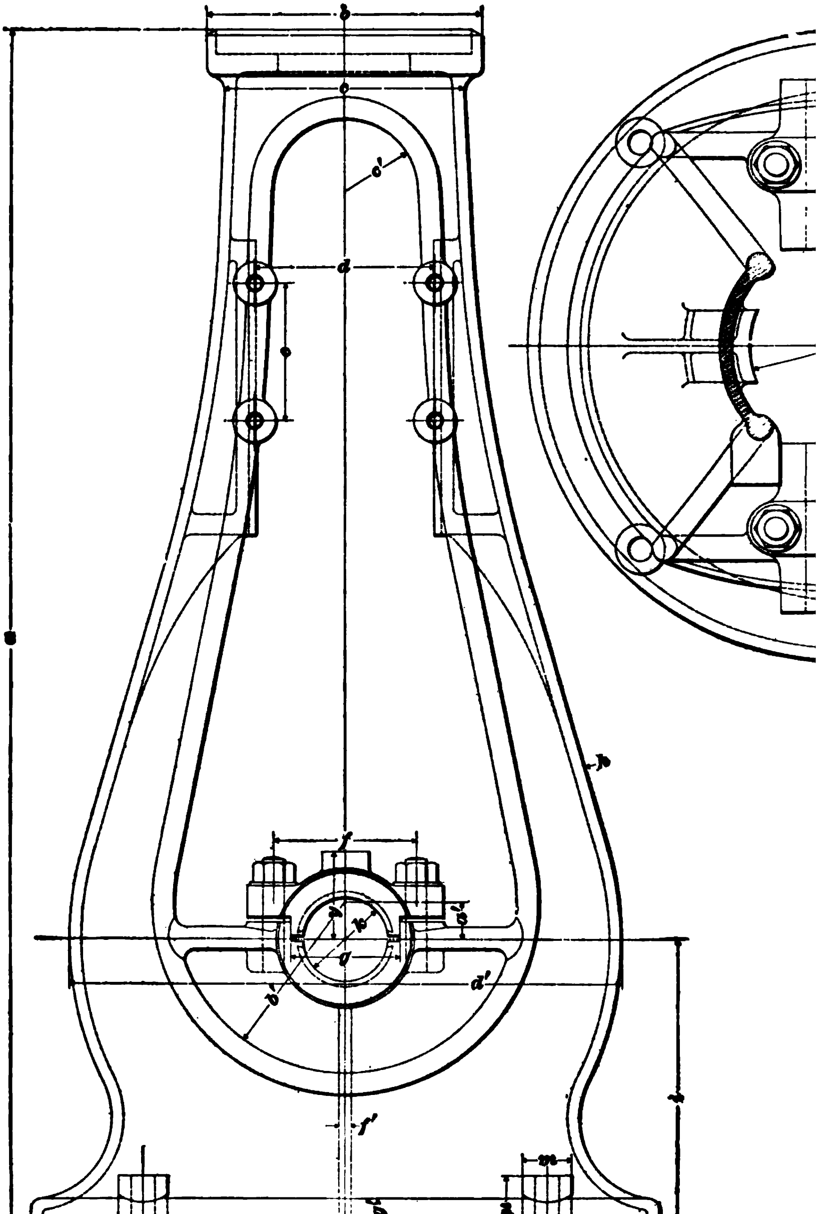


FIG. 784.

$n' = .1\,d' + .375''$.
$n'' = .2\,d' + .5''$.
$o = .625''$, constant.
$o' = .375''$, constant.
$p = .3\,d' + .5''$.
$p' = .15\,d' + .375''$.
$q = .02\,d' + .5''$.
$q' = .02\,d' + .25''$.
$r = .1\,d'$.
$r' = .15\,d'$.
$s = .9\,d'$.
$t = .11\,d'$.
$t' = .02\,d + .25''$.
$u = .04\,d' + .125''$; use nearest standard size bolt.
$v = .15\,d' + .375''$.
$w = 1.2\,d'$.
$w' = 1.75\,d'$.
$x = 2.5''$, constant.
$y = .3\,d' + .75''$.
$z = .2\,d' + .5''$.

It will be seen that this bearing has four seats, including the cap, which is lined with babbitt so as to form the top seat. The two side seats are adjustable by means of the wedges *W*, which are moved by the bolts *B*. The bearing is held to the engine bed by the T-head bolts *O*, *O*, which fit into slots cast in the bed for this purpose. The side and bottom seats are of brass, with babbitt linings. *A* is a top view of the cap, A_1 a side view of the cap, and A_2 a section of the cap on the center line. Lugs a_1, usually of wrought iron, fit in slots a_2, Fig. 783, to prevent end motion of the cap. *C* is an inside, or top, view of the bottom seat. It has lugs b_1 which fit into the slots c_1 of the wedges *W* (see view D_1, which is a half-section through the bearing on the line *P P*, with the wedge in place). D_2 is a half-section of the bearing on the center line. The bottom seat has a lug a_4 (see section D_2) that fits in a corresponding slot in the bed. This slot is shown at a_3 in section D_1; also at a_3, Fig. 783.

2130. Fig. 785 is an example of a frame for a vertical engine, as made by a well-known builder. The dimensions for this frame are given in Table 55, for various sizes of cylinders:



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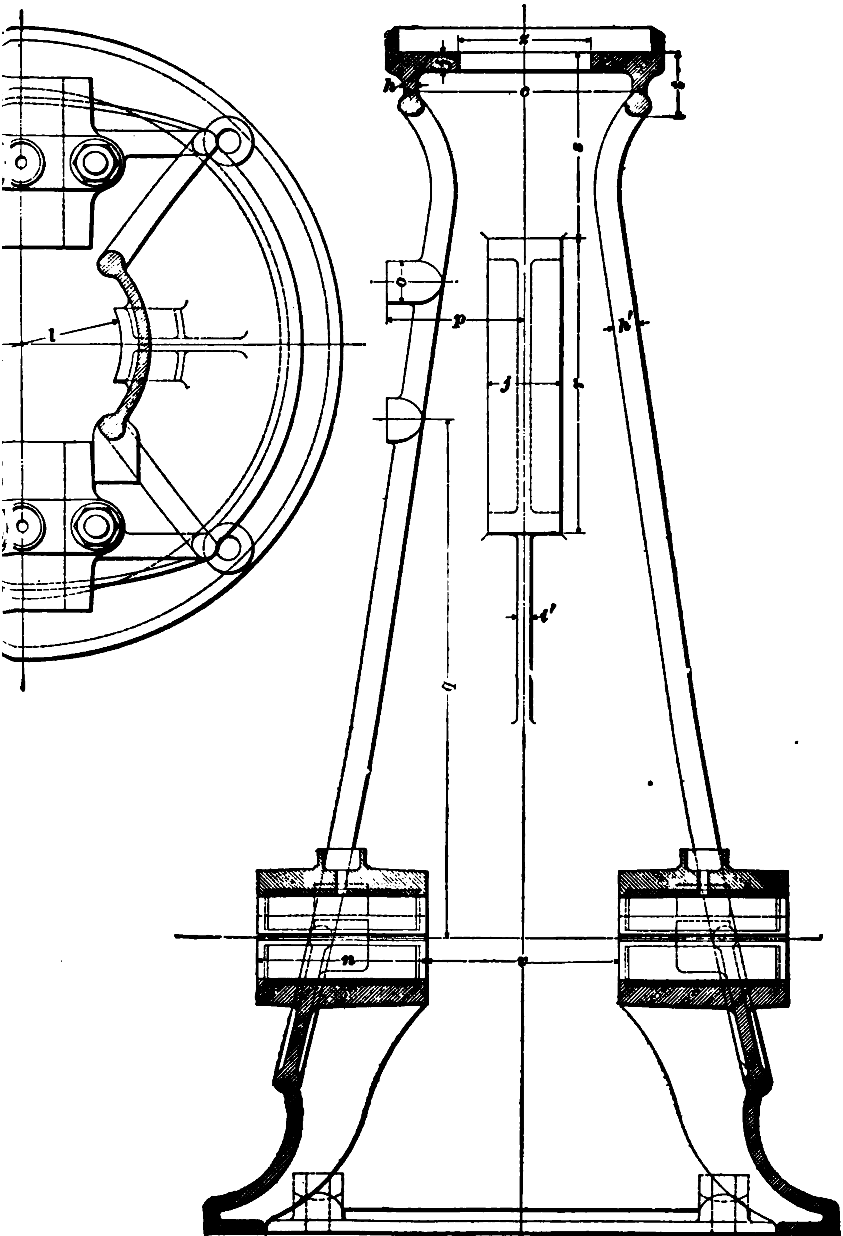

786.

TABLE 55.

Size.	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z
3″ × 5″	28 1/4	6 7/8	6	3 3/4	3 1/4	3 5/8	2 1/4	1/4	6 3/4	1 1/4	1 3/4	4 1/8	1 1/8	3 3/8	1	3 1/16	12 3/8	7 3/4	2 15/16	1 3/4	1 5/8	3 3/8	1/2	12 3/4	2	3 5/8
4″ × 6″	34 3/4	7 7/8	7 1/4	5 1/4	4 1/2	4 1/2	3	3/8	7 3/4	1 3/4	2	5	1 1/8	4 7/16	1 1/8	3 7/8	13 3/4	9 3/8	4 3/4	2 5/8	1 5/8	4 1/8	5/8	15	2 1/4	4 1/8
5 1/4″ × 7″	40 1/4	9 1/8	8	6	5 1/2	5	3 5/8	3/8	10	2 1/2	2 1/2	6	1 3/4	5 13/16	1 1/8	4 3/8	15 1/2	10 1/2	4 3/4	2 1/4	2 5/8	4 7/8	1	18	3 1/2	4 3/4
7″ × 9″	50 1/4	11 1/2	10	7 1/2	5 3/4	6 1/4	4	1/2	12 3/8	2 1/2	2 15/16	7 7/8	2	5 5/8	1 3/4	5	21 5/8	12 1/4	7 3/4	3 3/4	3	6 3/4	2 1/2	26 1/4	4 1/4	5 1/8
9″ × 12″	61 1/4	14 1/4	12 1/2	9	7 1/4	9	6 3/4	5/8	13 3/4	3 1/2	3 15/16	9 1/2	2	5 1/2	2	5 3/4	30 1/8	17 1/2	7 1/2	4 3/4	3 1/2	11	3 1/4	26 1/2	6	7 1/2

Size.	a′	b′	c′	d′	e′	f′	g′	h′	i′	j′
3″ × 5″	11/16	4	2 3/16	11 1/2	5/8	3/8	1	1/2	3/8	1/2
4″ × 6″	15/16	4 5/8	2 1/2	13 1/2	5/8	1/2	1 1/8	3/4	1/2	3/4
5 1/4″ × 7″	1	5 1/4	2 5/8	15 3/4	3/4	1/2	1 1/4	3/4	1/2	3/4
7″ × 9″	2 3/16	7 7/8	3 11/16	23	7/8	9/16	1 1/2	1	9/16	1
9″ × 12″	3/4	7 5/8	4 7/16	23 1/2	7/8	5/8	1 3/4	1 1/4	5/8	1 1/4

e′ = size of foundation bolt.
For 9″ × 12″ engine a disk-center crank is used.

EXAMPLES OF ENGINE PROPORTIONS.

2131. It has been previously stated that when a standard line of engines is to be manufactured, the rules and formulas for the design of the various parts need not be applied to each individual engine. It is found that under the conditions in which engines are to work a certain ratio may be assumed to exist between the sizes of the parts. For example, a certain line of engines work uniformly at a steam pressure of 75 lb., and the length of the piston rod bears a fixed relation to the length of stroke. Under these circumstances, the diameter of the piston rod may be a fixed fraction of the diameter of the cylinder for all sizes, and it is only necessary, therefore, to multiply the cylinder diameter by this fraction to find the diameter of the piston rod.

In the following tables are given the proportions of a standard line of Corliss engines made by a leading manufacturer. They will serve to illustrate the use of fixed proportions in designing, and will also furnish valuable examples of good, modern practice.

TABLE 56.

Diameter of Cylinder, Inches.	Stroke, Inches.	Revolutions per Minute.	Piston Speed in Feet per Minute.	Diameter of Shaft Journal.
10	24	90	360	5
12	24	90	360	$5\frac{1}{2}$
14	30	80	400	7
16	30	80	400	$7\frac{1}{2}$
16	36	70	420	8
18	36	70	420	$8\frac{1}{2}$
20	36	70	420	9
16	42	65	455	$8\frac{1}{4}$
18	42	65	455	$8\frac{3}{4}$
20	42	65	455	$9\frac{1}{4}$
20	48	60	480	$9\frac{1}{2}$
22	48	60	480	10
24	48	60	480	10

TABLE 56—(Continued).

Diameter of Cylinder, Inches.	Stroke, Inches.	Revolutions per Minute.	Piston Speed in Feet per Minute.	Diameter of Shaft Journal.
22	54	60	540	$10\frac{1}{4}$
24	54	60	540	$10\frac{3}{4}$
26	54	60	540	$11\frac{1}{4}$
24	60	60	600	11
26	60	60	600	$11\frac{1}{2}$
28	60	60	600	12
30	60	60	600	$12\frac{1}{2}$
32	60	60	600	13
34	60	60	600	$13\frac{1}{2}$
36	60	60	600	14

TABLE 57.

Diameter of Cylinder.	Diam. and Length of Crank-Pin.	Diameter and Length of Cross-Head Pin.	Diameter of Valve Stem.	Depth of Piston.	Diameter of Piston Rod.	Width of Crank Disk.	Clearance of Piston.
10	$2\frac{1}{2}$	$1\frac{5}{8} \times 2\frac{1}{2}$	$\frac{7}{8}$	4	$1\frac{5}{8}$	$2\frac{1}{2}$	$\frac{1}{4}$
12	3	2×3	1	$4\frac{1}{2}$	2	3	$\frac{1}{4}$
14	$3\frac{1}{2}$	$2\frac{3}{8} \times 3\frac{1}{2}$	$1\frac{1}{8}$	5	$2\frac{1}{4}$	$3\frac{1}{2}$	$\frac{1}{4}$
16	4	$2\frac{5}{8} \times 4$	$1\frac{1}{4}$	$5\frac{1}{2}$	$2\frac{1}{2}$	4	$\frac{1}{4}$
18	$4\frac{1}{2}$	$3 \times 4\frac{1}{2}$	$1\frac{3}{8}$	6	$2\frac{7}{8}$	$4\frac{1}{2}$	$\frac{1}{4}$
20	5	$3\frac{3}{8} \times 5$	$1\frac{1}{2}$	$6\frac{1}{2}$	$3\frac{1}{4}$	5	$\frac{1}{4}$
22	$5\frac{1}{2}$	$3\frac{5}{8} \times 5\frac{1}{2}$	$1\frac{1}{2}$	7	$3\frac{1}{2}$	$5\frac{1}{2}$	$\frac{5}{16}$
24	6	4×6	$1\frac{5}{8}$	$7\frac{1}{2}$	$3\frac{7}{8}$	6	$\frac{5}{16}$
26	$6\frac{1}{2}$	$4\frac{1}{4} \times 6\frac{1}{2}$	$1\frac{3}{4}$	$7\frac{3}{4}$	$4\frac{1}{8}$	$6\frac{1}{2}$	$\frac{5}{16}$
28	7	$4\frac{5}{8} \times 7$	$1\frac{3}{4}$	$8\frac{1}{4}$	$4\frac{1}{2}$	7	$\frac{5}{16}$
30	$7\frac{1}{2}$	$5 \times 7\frac{1}{2}$	$1\frac{7}{8}$	$8\frac{1}{2}$	$4\frac{7}{8}$	$7\frac{1}{2}$	$\frac{5}{16}$
32	8	$5\frac{1}{4} \times 8$	$1\frac{7}{8}$	$8\frac{1}{2}$	$5\frac{1}{8}$	8	$\frac{5}{16}$
34	$8\frac{1}{2}$	$5\frac{5}{8} \times 8\frac{1}{2}$	2	$8\frac{3}{4}$	$5\frac{1}{2}$	$8\frac{1}{2}$	$\frac{3}{8}$
36	9	$5\frac{7}{8} \times 9$	2	9	$5\frac{3}{4}$	9	$\frac{3}{8}$

TABLE 58.

Diameter of Cylinder.	Steam Inlet Area, sq. in.	Exhaust Area, sq. in.	Diameter of Valve.	Inlet Ports.	Exhaust Ports.	Width of Steam Chest.	Width of Exhaust Chest.	Depth of Steam and Exhaust Chests.	Diameter of Steam Pipe.	Diameter of Exhaust Pipe.
10	4.71	7.85	$3\frac{1}{8}$	$\frac{1}{2} \times 10$	$\frac{13}{16} \times 10$	5	$6\frac{1}{2}$	$1\frac{3}{4}$	$2\frac{1}{2}$	$3\frac{1}{2}$
12	6.79	11.31	$3\frac{1}{2}$	$\frac{9}{16} \times 12$	$\frac{15}{16} \times 12$	6	$7\frac{1}{2}$	2	3	4
14	9.23	15.39	$3\frac{7}{8}$	$\frac{11}{16} \times 14$	$1\frac{1}{8} \times 14$	7	9	$2\frac{1}{4}$	$3\frac{1}{2}$	$4\frac{1}{2}$
16	12.00	20.00	$4\frac{1}{4}$	$\frac{3}{4} \times 16$	$1\frac{1}{4} \times 16$	8	10	$2\frac{1}{2}$	4	5
18	15.27	25.44	$4\frac{5}{8}$	$\frac{7}{8} \times 17$	$1\frac{1}{2} \times 17$	9	$11\frac{1}{2}$	$2\frac{3}{4}$	$4\frac{1}{2}$	6
20	18.85	31.42	5	1×19	$1\frac{11}{16} \times 19$	10	$12\frac{1}{2}$	3	5	$6\frac{1}{2}$
22	22.80	38.00	$5\frac{3}{8}$	$1\frac{1}{8} \times 20\frac{7}{8}$	$1\frac{7}{8} \times 20\frac{7}{8}$	11	$14\frac{1}{2}$	$3\frac{1}{4}$	$5\frac{1}{2}$	7
24	27.14	45.24	$5\frac{3}{4}$	$1\frac{3}{16} \times 22\frac{7}{8}$	$2 \times 22\frac{7}{8}$	12	15	$3\frac{1}{2}$	6	$7\frac{1}{2}$
26	31.85	53.09	$6\frac{1}{8}$	$1\frac{5}{16} \times 24\frac{3}{4}$	$2\frac{1}{8} \times 24\frac{3}{4}$	13	$16\frac{1}{2}$	$3\frac{3}{4}$	$6\frac{1}{2}$	$8\frac{1}{2}$
28	36.95	61.58	$6\frac{1}{2}$	$1\frac{7}{16} \times 25\frac{1}{2}$	$2\frac{1}{2} \times 25\frac{1}{2}$	14	$17\frac{1}{2}$	4	7	9
30	42.41	70.69	7	$1\frac{9}{16} \times 27\frac{1}{2}$	$2\frac{5}{8} \times 27\frac{1}{2}$	15	19	$4\frac{1}{2}$	$7\frac{1}{2}$	$9\frac{1}{2}$
32	48.26	80.43	$7\frac{1}{2}$	$1\frac{5}{8} \times 29\frac{1}{4}$	$2\frac{3}{4} \times 29\frac{1}{4}$	16	20	5	8	$10\frac{1}{2}$
34	54.48	90.79	8	$1\frac{3}{4} \times 31\frac{1}{4}$	$2\frac{7}{8} \times 31\frac{1}{4}$	17	$21\frac{1}{2}$	$5\frac{1}{2}$	$8\frac{1}{2}$	11
36	61.07	101.79	$8\frac{1}{2}$	$1\frac{7}{8} \times 33$	$3\frac{1}{8} \times 33$	18	$22\frac{1}{2}$	6	9	$11\frac{1}{2}$

TABLE 59.

Diameter of Cylinder.	Thickness of Cylinder.	Thickness of Chests.	Thickness of Valve Chamber.	Bearing of Valves.
10	$\frac{3}{4}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$
12	$\frac{13}{16}$	$\frac{5}{8}$	$\frac{3}{4}$	$1\frac{1}{16}$
14	$\frac{7}{8}$	$\frac{11}{16}$	$\frac{13}{16}$	$1\frac{1}{8}$
16	$\frac{15}{16}$	$\frac{3}{4}$	$\frac{7}{8}$	$1\frac{1}{4}$
18	1	$\frac{13}{16}$	1	$1\frac{3}{8}$
20	$1\frac{1}{16}$	$\frac{7}{8}$	$1\frac{1}{16}$	$1\frac{1}{2}$
22	$1\frac{1}{8}$	$\frac{15}{16}$	$1\frac{1}{8}$	$1\frac{5}{8}$
24	$1\frac{3}{16}$	$\frac{15}{16}$	$1\frac{1}{8}$	$1\frac{3}{4}$
26	$1\frac{1}{4}$	1	$1\frac{3}{16}$	$1\frac{7}{8}$
28	$1\frac{1}{4}$	1	$1\frac{1}{4}$	2
30	$1\frac{5}{16}$	$1\frac{1}{16}$	$1\frac{5}{16}$	$2\frac{1}{8}$
32	$1\frac{3}{8}$	$1\frac{1}{8}$	$1\frac{3}{8}$	$2\frac{1}{4}$
34	$1\frac{3}{8}$	$1\frac{1}{8}$	$1\frac{3}{8}$	$2\frac{3}{8}$
36	$1\frac{3}{8}$	$1\frac{1}{8}$	$1\frac{3}{8}$	$2\frac{1}{2}$

An inspection of the above tables shows that the following rules are used in designing the above line of engines:

Let $D =$ diameter of cylinder.

Then, diameter of shaft $= .34\,D + 2\frac{1}{2}''$, nearly. A more exact but less simple formula is

diameter of shaft $= .26\sqrt{DL} + 2\frac{1}{2}''$, where L is the stroke.

Diameter of crank-pin $= \frac{D}{4}$.

Length of crank-pin $= \frac{D}{4}$.

Length of cross-head pin $= \frac{D}{4}$.

Diameter of cross-head pin $=$ diameter of crank-pin $\times$ 0.65.

Diameter of steel valve stem $= .19\sqrt[3]{D^2}$.

Depth of piston $= \frac{D}{4} + 1\frac{1}{2}''$ for D less than 24 inches;

$= \frac{D}{8} + 4\frac{1}{2}''$ for D greater than 24 inches.

Diameter of piston rod $= .16\ D$.

Width of crank disk $= \frac{D}{4}$.

Area of steam port $= .06 \times$ area of cylinder.

Area of exhaust port $= .1 \times$ area of cylinder.

Diameter of valve $= \frac{3\ D}{16} + 1\frac{1}{4}''$.

Width of steam chest $= \frac{D}{2}$.

Width of exhaust chest $= .63\ D$.

Depth of steam and exhaust chests

$= \frac{D}{8} + \frac{1}{2}''$ for D less than 28 in.;

$= \frac{D}{4} - 3''$ for D greater than 28 in.

Diameter of steam pipe $= \frac{D}{4}$.

Diameter of exhaust pipe $= 0.31\ D$.

Thickness of cylinder $= .028\ D + \frac{1}{2}''$.

Thickness of chests $=$ thickness of cylinder $\times$ 0.8.

Thickness of valve chamber

$=$ thickness of chest $+ \frac{1}{8}''$ for $D = 10$ to 16 in.;

$=$ thickness of chest $+ \frac{3}{16}''$ for $D = 18$ to 26 in.

$=$ thickness of chest $+ \frac{1}{4}''$ for $D = 28$ to 36 in.

Bearing of valve $=$ diameter of valve $\times$ 0.3.

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STEAM AND STEAM ENGINES

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MECHANICS OF THE STEAM ENGINE

1310. In Fig. 304, let $O\ C$ represent the crank; then, the circle $C'\ C''\ M$ represents the path of the crank-pin.

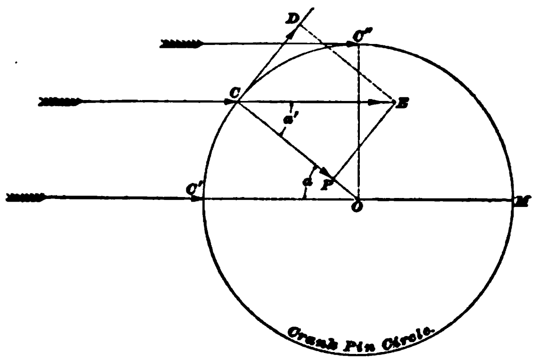


FIG. 304.

The steam exerts, through the piston, piston rod, and connecting-rod, a pressure on the crank-pin C. Let it be assumed that the connecting-rod is very long, so that the direction of the pressure on the pin is always horizontal.

Now, when the crank is in the position $O\ C'$, the horizontal pressure of the steam simply produces a pressure on the bearing of the crank-shaft; there is no tendency whatever to turn the crank around O as a center, and it is, therefore, said to be on the dead center. When the crank is in the

position $O\ C''$ *all* the pressure of the steam on the piston is expended in turning the crank around O as a center, and there is no pressure on the bearing. This is because the direction of the pressure at this point is at right angles to the crank, or, in other words, tangent to the crank circle.

When the crank is in some other position, as $O\ C$, there will be a tendency to turn the crank around O as a center, and also to produce a pressure on the crank-shaft bearing. To find the magnitudes of these forces tending to rotate the crank and tending to produce a pressure on the bearing, proceed as follows: Let $C\ E$ represent to some scale the horizontal pressure of force on the crank-pin at C. The turning force acts in the direction of the tangent $C\ D$, while the force which produces the pressure on the bearing acts along the crank, or in the direction $C\ O$. The force $C\ E$ may be resolved in the two directions $C\ D$ and $C\ O$ by means of the parallelogram of forces. From E, draw $E\ P$ parallel to $C\ D$ and $E\ D$ parallel to $C\ O$. Then, $C\ D$ is the tangential or turning force, and $C\ P$ the force producing pressure on the bearing, both to the same scale as $C\ E$.

Let a be the angle $C\ O\ C'$ which the crank makes with the horizontal. $C\ E$ and $C'\ O$ are parallel, and $C\ P$ and $C\ O$ are parallel (coincide); hence, by geometry, $C\ O\ C'$ and $P\ C\ E$ are equal, or angle $a =$ angle a'.

Tangential force $= C\ D = E\ P = C\ E \sin E\ C\ O = C\ E \sin a$, or *the force tending to turn the crank is equal to the horizontal force on the crank-pin multiplied by the sine of the angle which the crank makes with the horizontal.*

When the crank is at $C'\ O$, a is 0; therefore, $\sin a$ is 0 and the tangential force is 0, as it should be. When the crank is at $O\ C''$, a is 90°, $\sin a$ is 1, and the tangential force is the same as the horizontal force.

The radial force, or the force which produces pressure on the bearing, may be shown in the same manner to be equal to the horizontal force multiplied by the cosine of the angle which the crank makes with the horizontal, thus,

Radial force $= C\ P = C\ E \cos E\ C\ O = C\ E \cos a$.

EXAMPLE.—The horizontal force on the crank-pin is 6,000 pounds. What will be the tangential and radial forces when the crank makes an angle of 60° with horizontal direction of the force?

SOLUTION.—$a = 60°$; $\sin a = .866$; $\cos a = .5$.

Tangential force $= 6{,}000 \times \sin a = 6{,}000 \times .866 = 5{,}196$ lb.

Radial " $= 6{,}000 \times \cos a = 6{,}000 \times .5 = 3{,}000$ lb.

1311. A diagram showing the tangential pressure for every position of the crank may be easily constructed, as

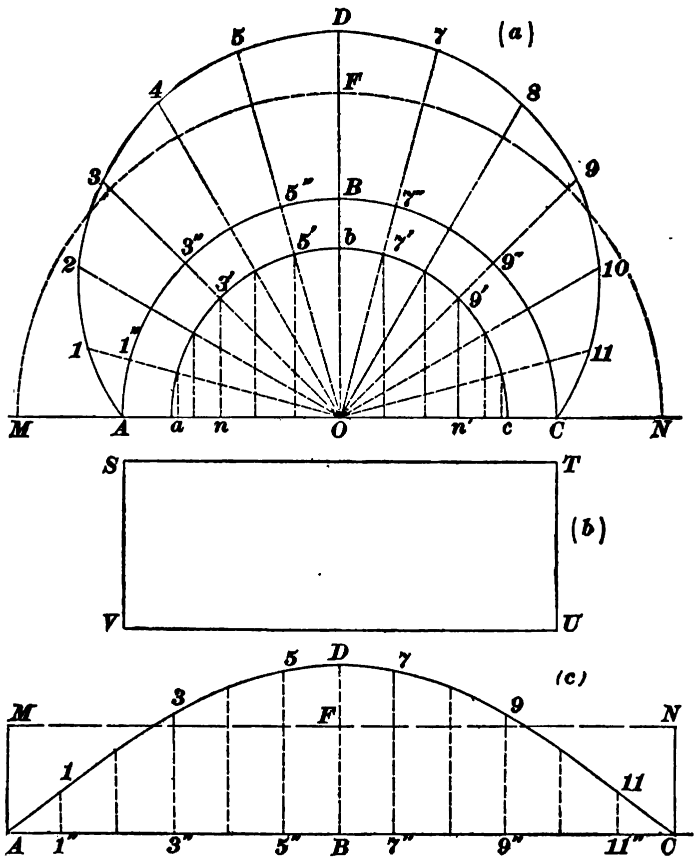


FIG. 305.

follows: For simplicity, assume the piston pressure to be constant throughout the stroke, as shown by the indicator

diagram or card, Fig. 305 (*b*); suppose, also, that the connecting rod is very long, so that the direction of pressure is always horizontal. Let the length of the crank be $O\ A$; then, with O as a center and $O\ A$ as a radius, describe one-half the crank-pin circle $A\ B\ C$. Let $O\ a$ represent the uniform pressure $s\ v$ and describe the semicircle $a\ b\ c$. Divide $a\ b\ c$ or $A\ B\ C$ into a convenient number of equal parts (12 is most convenient), and through each division draw the radial lines $O\ 1$, $O\ 2$, $O\ 3$, etc., prolonging them beyond $A\ B\ C$. From the points where these radial lines intersect the semicircle $a\ b\ c$ drop perpendiculars on line $a\ c$. Then, each perpendicular represents the tangential component of the pressure when the crank is in that position. For example, when the crank is at $O\ 3'$, the length $3'n$ represents the tangential pressure, since

$$3'n = O\ 3' \sin 3'O\ n = O\ a \sin 3'O\ n = \text{horizontal pressure} \times \text{sine of crank angle}.$$

Now, lay off these perpendiculars, each on its own radial line outwards from the crank-pin circle $A\ B\ C$; that is, lay off $3'n$ on the radial line $O\ 3$, the length $3\text{-}3''$ being made equal to the length $3'n$.

We thus obtain the series of points 1, 2, 3, etc.; the curve $A\ D\ C$, drawn through these points, will represent the tangential pressures for all points of the stroke.

It will be noticed that at A and C, the dead points, the tangential pressure is 0, and at D it is equal to the horizontal pressure.

In Fig. 305 (*c*), the tangential pressure diagram is represented with a straight base. The semicircle $A\ B\ C$ has been straightened out, the ordinates $1\text{-}1''$, $2\text{-}2''$, etc., remaining the same as before.

Since the ordinates of (*c*) represent the tangential pressures on the crank to the same scale that the ordinates of (*b*) represent pressures on the piston, and since the length $A\ C$ represents the distance passed through by the crank to the same scale that $V\ U$ represents the distance passed through by the piston, it follows that the area of (*c*)

represents the work done by the crank-pin during a half revolution of the crank, or during one stroke of the piston.

The work done on the piston by the steam must be equal to the work given up by the crank-pin. Therefore, since (*b*) and (*c*) have the same scale of pressures and distances, since (*b*) represents the work done on the piston and (*c*) represents the work done by the crank-pin, their areas must be equal; and so they will be found to be by actual measurement.

The "mean ordinate" of (*c*) may now be found from the above considerations. The length of the semicircle $A\ B\ C$ of (*a*) is $\frac{\pi}{2}$ times the length of the diameter $A\ C$; but the base $A\ B\ C$ of the diagram (*c*) is equal in length to the semicircle $A\ B\ C$, and the length $V\ U$ of (*b*) is equal to the diameter $A\ C$, since both represent the length of stroke. Therefore, the base $A\ B\ C$ of (*c*) is $\frac{\pi}{2}$ times as long as the base $U\ V$ of (*b*). The area of (*b*) and (*c*) are the same; consequently, the mean ordinate of (*b*) must be $\frac{\pi}{2}$ times that of (*c*), or $S\ V = \frac{\pi}{2} A\ M$; therefore, $A\ M = \frac{2\ S\ V}{\pi}$.

The mean ordinate of the diagram of tangential pressures on the crank-pin is always $\frac{2}{\pi}$ times the mean ordinate or M. E. P. of the indicator card.

In both (*a*) and (*c*), $M\ F\ N$ is the line of average tangential pressures, and in both cases is drawn parallel to $A\ B\ C$ at a distance from it equal to the mean ordinate $A\ M$.

The case just described is much simpler than is ever met with in practice. The pressure of the steam is rarely constant throughout the whole stroke, and the connecting rod is never so long that the pressure exerted by it on the crank pin may be regarded as always horizontal.

1312. In order to apply the foregoing principles to an actual case, it is necessary to use the net pressure (forward

pressure, less the back pressure on the other side of the piston) on the piston during one-half a revolution. This is easily done by combining the two curves *A B C D* and *K L C M*, Fig. 267 (which represent the steam pressure on opposite sides of the piston during the forward stroke), as shown in Fig. 306, where *A B a c* represents *A B C D*, and *e f a b* represents *K L C M*. Joining *A* and *e* and *b* and *c* by straight lines, the figure is completed. Any ordinate, as *g h*, measured to the scale of the indicator spring, is the net pressure on the piston urging it ahead when it occupies the position of *h* of its stroke. The net pressure is 0 at *a*,

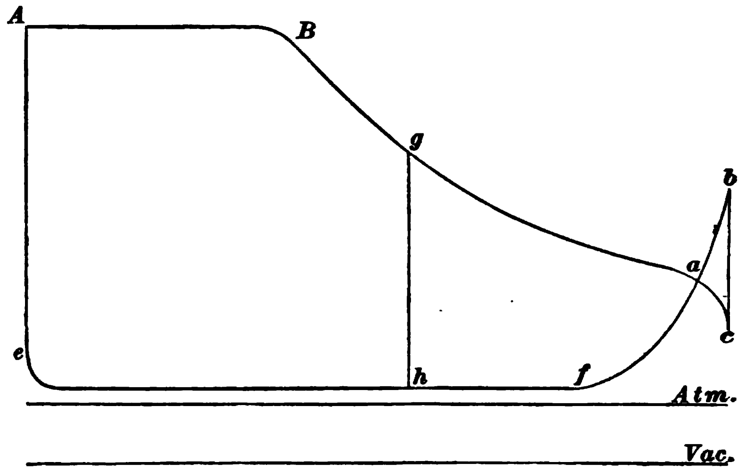


FIG. 306.

i. e., the pressure is the same on both sides of the piston. Between *a* and *b c* the net pressure is negative, or, in other words, the back pressure is greater than the forward pressure, and the piston is carried to the end of its stroke solely by the energy stored in the fly-wheel.

1313. Let us now take a diagram of *net* pressures on the piston like the one in the figure just shown, and, assuming the connecting-rod to be four times the length of the crank, work out the corresponding diagram of tangential pressures.

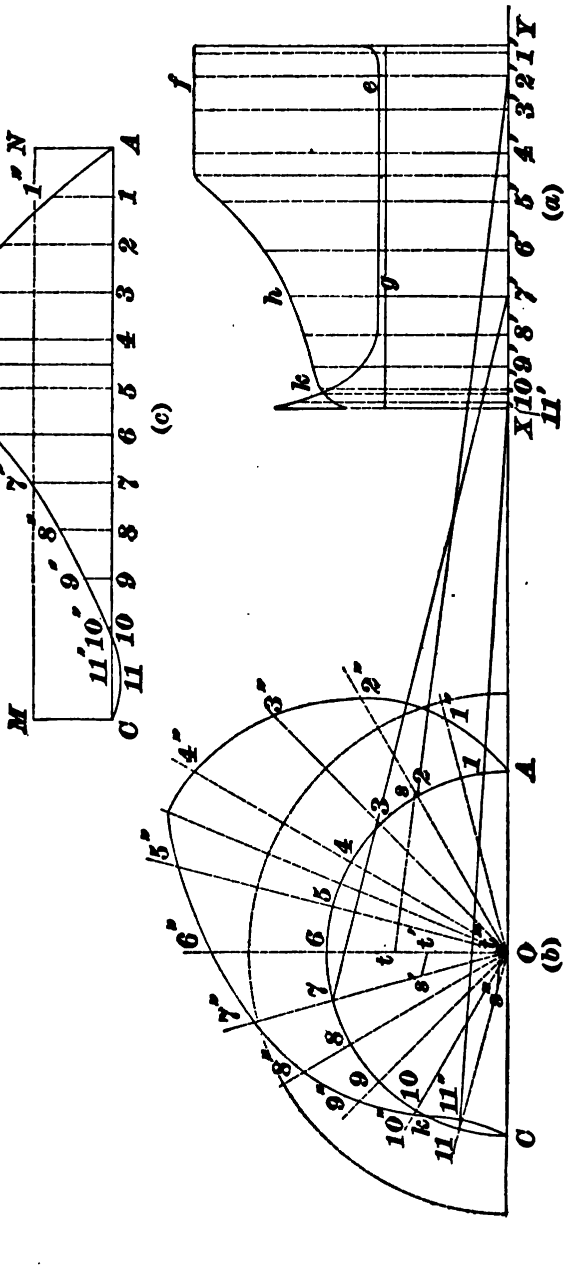


FIG. 307.

The diagram or card is shown at (*a*) Fig. 307. At any convenient distance below the card, draw a line *C Y* parallel to the atmospheric line of the cards. Project the two ends of the card down on *C Y*, locating the points *X* and *Y*. Lay off, on the line *C Y*, $YA = XC = 2\,XY = 4\,OA =$ the length of the connecting-rod. Take the point *O* midway between *A* and *C*, and with *O A* as a radius, strike the semicircle *A 6 C*. *X Y* now represents the length of the stroke to the scale of the diagram, and *A 6 C* the path of the crank-pin. Divide *A 6 C* into a convenient number of equal parts, and through the points of division draw the radial lines *O 1*, *O 2*, *O 3*, etc., prolonging them some distance beyond the semicircle. Now find the positions of the piston corresponding to the crank positions *O 1*, *O 2*, etc. This may be done by taking the length *A Y* of the connecting-rod with a pair of dividers; then, placing one leg of the dividers on *1*, *2*, *3*, etc., in succession, strike off the piston positions *1'*, *2'*, *3'* on the line *X Y*. The tangential pressure may be found in the following manner: Suppose it is desired to find the tangential pressure on the crank when it is in the position *O 2*. The pressure on the piston at this point is shown on the card to be *e f*. Lay off from *O* the length *e f* on the radial line *O 2*, thereby obtaining the length *O s*. Draw in the connecting-rod *22'*, and through *s*, draw *s t* parallel to *22'* and intersecting the vertical line *O 6" t*. Then, *O t* is the tangential pressure to the same scale that $ef = Os$ is the pressure on the piston. When the crank is at *7* the piston is at *7'*, and the pressure on it from the card is *g h*. *O s'* is laid off on *O 7* equal to *g h*, *s' t'* is drawn parallel to *7 7'*; then, *O t'* is the tangential pressure on the crank when it is in the position *O 7*. In this manner, the tangential pressure can be found for all positions of the crank, and, being laid off radially from the crank-pin circle, as in the previous case, they form the curve *A 4" 7" k C*. At *k*, the pressures on both sides of the piston are equal, and, consequently, the tangential pressure is zero. The point *k* in (*b*) is determined by dropping a perpendicular from *k* to *X Y*, and with the point of intersection as a center and a

radius equal to the length of the connecting-rod, describe an arc cutting $C\,6\,A$ in k. Beyond k, the back pressure exceeds the forward pressure, and the tangential pressure must be laid off below the semicircle $C\,k\,6\,A$, as shown. As before, the semicircle $A\,6\,C$ may be laid out straight, and the diagram shown in (*c*) obtained. The area of the latter will again be found to be equal to the area of the diagram (*a*). The line $M\,N$, as before, represents the average tangential pressures on the crank-pin. It is usually a good plan to find the crank and piston positions at cut-off and get the tangential pressure at that point, since a sharp change in the curve will there occur, as shown in the figure.

1314. The load on the engine, or the resistance against which the engine works, is nearly always constant. That is, it requires a practically constant force to drive the shafting or machines. It is, therefore, very desirable that the

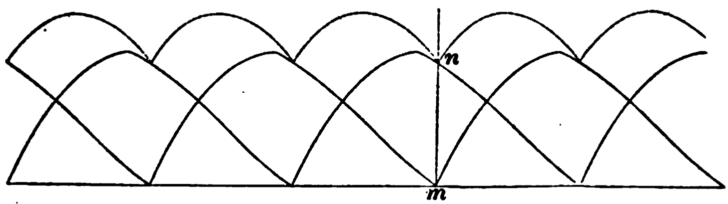


FIG. 308.

force turning the crank-shaft should be as nearly constant as possible. It has been shown by the diagrams of Figs. 305 and 307 that this tangential force is very far from being constant; that at the dead points it is zero and near the middle of the stroke it is greatest. It can now be shown why compound and duplex engines, which have their cranks at right angles, possess advantages over the simple engine. In Fig. 308 is shown a diagram for a cross-compound engine with cranks at right angles. When one crank is beginning its stroke, the other is at the middle of the stroke, and vice versa. Therefore, when the curve of one crank touches the base line, as at m, the other will be at or near its highest position, as at n. Now, adding the corresponding ordinates of the two curves together, we obtain the dotted curve,

which, therefore, represents the total tangential pressure tending to turn the crank-shaft. It is apparent at a glance how much more nearly constant is the tangential pressure of the compound engine diagram as compared with the tangential pressure of the simple engine diagram. Consequently, with the same steam pressure and weight of fly-wheel, the compound or duplex engine with crank at 90° will run more steadily than the simple engine, while a triple-expansion engine with 3 cranks, making angles of 120° with each other, will run steadier than the compound.

1315. A tandem compound engine also has a mechanical advantage over the simple engine, as a little consideration will show. It was proven in Art. **1301** that a compound engine was equivalent to a simple engine whose cylinder was of the same dimensions as the low-pressure cylinder, and which expanded its steam the same number of times as the compound. For convenience of illustration, assume the cut-off in the high-pressure cylinder to be $\frac{1}{2}$, and that the ratio of the volumes of the two cylinders is 1 : 4; that there is no clearance and no receiver; that there is no loss of pressure due to wire-drawing, friction, etc., and, finally, that the steam is carried full stroke in the low-pressure cylinder. The total number of expansions is 8, 2 in the small cylinder and 4 in the large cylinder. Assume the steam pressure to be 120 pounds, absolute, and the absolute back pressure to be 15 pounds. Denote the area of the large piston by A; then, the area of the small piston is $\frac{A}{4}$. The terminal pressure in the small cylinder is, evidently, 60 pounds, and this is the initial forward pressure in the large cylinder and the initial back pressure in the small cylinder.

First consider the simple engine. The initial forward pressure is 120 pounds and the back pressure 15 pounds; hence, the net pressure urging the piston ahead is $120 - 15 = 105$ pounds. The total force acting on the piston is $105\,A$.

Considering the compound engine, the initial forward

pressure in the small cylinder is 120 pounds; the back pressure 60 pounds, and the net pressure $120 - 60 = 60$ pounds. Since the area of the small piston is $\frac{A}{4}$, the total force tending to drive the small piston forward is $60 \times \frac{A}{4} = 15A$.

The initial forward pressure in the low-pressure cylinder is 60 pounds, the back pressure is 15 pounds, and the net pressure urging the piston ahead is $60 - 15 = 45$ pounds. The total force tending to drive the large piston forward is $45A$.

The total initial force acting on both pistons of the compound is $45A + 15A = 60A$; in the simple engine it was $105A$.

Since the greatest strains occur when the forces which produce them are greatest, it is evident, from the above, that the various parts of a compound engine (connecting-rod, crank, shaft, etc.) will not need to be made so large as in a simple engine having the same mean effective pressure, and the volume of whose cylinder equals the volume of the low-pressure cylinder. This is, however, a disadvantage when the engine has a very heavy load to start, as in the case of a locomotive, for in that case the compound engine might not be able to start, although it could keep itself in motion after it had once been started.

FLY-WHEELS.

1326. The office of the *fly-wheel* is somewhat similar to that of the governor, since each is used to obtain regularity of speed. It is the duty of the governor to adjust the effort of the engine to any large or permanent variation of the

load, such as would be caused by throwing the machinery in or out of gear. It is the duty of the fly-wheel, on the other hand, to adjust the effort of the engine to sudden fluctuations of the resistance, which may occur during a single stroke of the engine. It is also the duty of the fly-wheel to equalize the varying tangential effort on the crank pin by storing energy while the piston is in the middle of the stroke, where the crank effort is greater than the resistance, and restoring it when the crank is at the dead point, where the tangential effort is zero.

1327. In Fig. 315, let *A C B* represent the tangential pressures on the crank-pin during one stroke of the piston. *A B* is the length of the semi-circumference of the crank-pin circle; *A M* is the mean ordinate, and *A B N M* represents the constant resistance to the turning of the crank. The effort and resistance must be equal; therefore, area *A C B* = area *A B N M*. It follows, then, that area *C D E* = area *A M D* + area *B N E*. At *A* the tangential effort is zero, since the crank is at a dead point. From *A* to *S* the tangential effort remains less than the resistance. The work

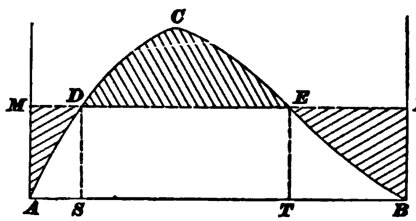


FIG. 315.

done on the crank during that period is represented by the area *A D S*, while the resistance is represented by the area *A M D S*. Hence, the resistance is greater than the effort by the area *A M D*. At *S* the effort and resistance are equal. From *S* to *T* the effort is greater than the resistance by an amount represented by the area *D C E*. At *T* the effort is again equal to the resistance, and for the remainder of the stroke it again becomes less than the resistance.

From *A* to *S* the work done by the steam was shown to be less than the resistance to be overcome; therefore, the work represented by the area *A M D* must have been

obtained from some of the moving parts of the engine. The kinetic energy of a moving body is $\frac{Wv^2}{2g}$. In order to give up energy, the v of the above expression must be diminished. That is what actually takes place. All of the moving parts of the engine, the reciprocating parts, shaft, and fly-wheel, slow down a little, and, in so doing, give up enough of their kinetic energy to overcome the resistance and carry the engine past the dead center. From S to T the effort is greater than the resistance, and, consequently, the surplus energy represented by the area $C\,D\,E$ is stored up in the moving parts—that is, their velocity is increased during the part of the stroke in question, and with it their kinetic energy. For the remainder of the stroke the moving parts again slow down and give up enough energy to overcome the excess resistance $B\,N\,E$.

1328. The **weight of the fly-wheel** may be found in the following manner:

Let V_1 = the greatest velocity of the crank-pin in feet per second;
V_2 = the least velocity of the crank-pin in feet per second;
V_0 = average velocity of crank-pin in feet per second;
W = required weight of fly-wheel in pounds;
H = the number of foot-pounds per square inch of piston represented by the area $C\,D\,E$;
A = area of piston in square inches;
n = ratio between radius of fly-wheel and length of crank;

$$E = \frac{V_1 - V_2}{V_0} = \text{coefficient of unsteadiness.}$$

The average velocity V_0 is known, since it is $\frac{\pi}{2}$ times the piston speed in feet per second. V_1 and V_2 are assumed so that the fraction $\frac{V_1 - V_2}{V_0} = E$ shall not exceed a certain value.

The following values of E agree well with ordinary practice:

Pumping engines, $\frac{1}{20}$.
Engines driving machine tools, $\frac{1}{35}$.
" " textile machinery, $\frac{1}{40}$.
" " spinning machinery, $\frac{1}{50}$ to $\frac{1}{100}$.
" " electric machinery, $\frac{1}{125}$ to $\frac{1}{150}$

H can be found directly from the diagram by multiplying the area $C\ D\ E$ in square inches by the vertical scale of pressures and the horizontal scale of distances. A is, of course, known, and n may be assumed at pleasure.

At the point D the crank-pin has the velocity V_2; since the radius of the fly-wheel rim is n times the length of crank, the velocity of the rim at D must be $n\ V_2$. Likewise, at E the crank-pin will have its greatest velocity V_1, and the fly-wheel rim the velocity $n\ V_1$.

The kinetic energy of the fly-wheel rim at D is $\frac{W(n\,V_2)^2}{2g}$; and at E it is $\frac{W(n\,V_1)^2}{2g}$. The kinetic energy stored up in passing from D to E is, therefore, $\frac{Wn^2\,V_1^2}{2g} - \frac{Wn^2\,V_2^2}{2g} = \frac{Wn^2}{g}\left(\frac{V_1^2 - V_2^2}{2}\right)$ foot-pounds. But this kinetic energy stored up is equal to the work represented by the area $C\,D\,E$. Hence,

$$\frac{Wn^2}{g}\left(\frac{V_1^2 - V_2^2}{2}\right) = A \times H. \quad (a).$$

But $$\frac{V_1 + V_2}{2} = V_0, \text{ the average velocity,}$$

and $$\frac{V_1 - V_2}{V_0} = E;$$

therefore, $$\left(\frac{V_1 - V_2}{V_0}\right)\left(\frac{V_1 + V_2}{2}\right) = \frac{V_1^2 - V_2^2}{2V_0} = E\,V_0,$$

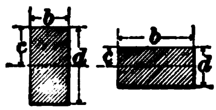
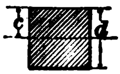
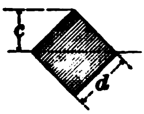
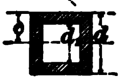
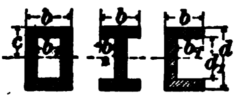
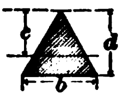
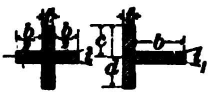
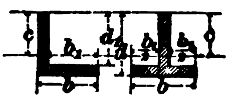
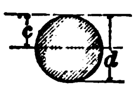
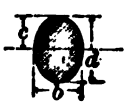
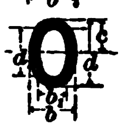
or $$\frac{V_1^2 - V_2^2}{2} = E\,V_0^2.$$

Substituting this in (a),

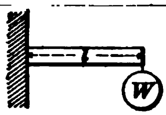
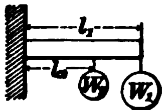
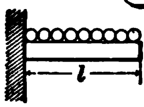
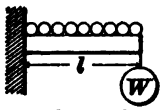
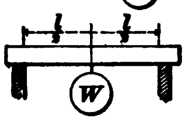
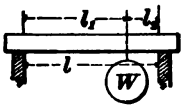
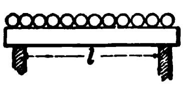
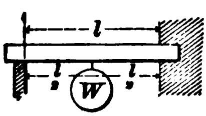
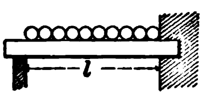
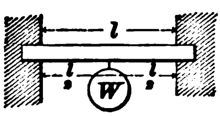
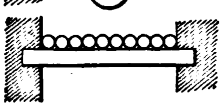
$$\frac{Wn^2}{g}\left(\frac{V_1^2 - V_2^2}{2}\right) = \frac{Wn^2\,E\,V_0^2}{g} = A\,H.$$

Whence, $$W = \frac{A\,H\,g}{n^2\,E\,V_0^2}. \qquad \textbf{(107.)}$$

MOMENTS OF INERTIA.

Form of Section.	Dotted Line Shows Position of Neutral Axis.	A	I	c
1. Rectangle...		bd	$\frac{1}{12}bd^3$	$\frac{1}{2}d$
2. Square..		d^2	$\frac{1}{12}d^4$	$\frac{1}{2}d$
3. Axis through Diagonal		d^2	$\frac{1}{12}d^4$	$.707d$
4. Hollow Square		$d^2-d_1^2$	$\frac{1}{12}(d^4-d_1^4)$	$\frac{1}{2}d$
5. Hollow Rectangle, I or Channel Iron		$bd-b_1d_1$	$\frac{1}{12}(bd^3-b_1d_1^3)$	$\frac{1}{2}d$
6. Triangle		$\frac{1}{2}bd$	$\frac{1}{36}bd^3$	$\frac{2}{3}d$
7. Cross....		$td+t_1b$	$\frac{1}{12}(td^3+bt_1^3)$	$\frac{1}{2}d$
8. Angle Iron.....		$bd-b_1d_1$	$\frac{(bd^2-b_1d_1^2)^2-4bdb_1d_1(d-d_1)^2}{12(bd-b_1d_1)}$	$\frac{d}{2}+\frac{b_1d_1}{2}\left(\frac{d-d_1}{bd-b_1d_1}\right)$
9. Circle....		$\frac{\pi}{4}d^2$	$\frac{\pi d^4}{64}$	$\frac{1}{2}d$
10. Hollow Circle...		$\frac{\pi}{4}(d^2-d_1^2)$	$\frac{\pi(d^4-d_1^4)}{64}$	$\frac{1}{2}d$
11. Ellipse...		$\frac{\pi}{4}bd$	$\frac{\pi bd^3}{64}$	$\frac{1}{2}d$
12. Hollow Ellipse..		$\frac{\pi}{4}(bd-b_1d_1)$	$\frac{\pi(bd^3-b_1d_1^3)}{64}$	$\frac{1}{2}d$

BENDING MOMENTS AND DEFLECTIONS.

	Manner of Supporting Beams.	Maximum Bending Moment, M.	Maximum Deflection, s.	Remarks.
1.		Wl	$\frac{1}{3}\frac{Wl^3}{EI}$	Cantilever, load at free end.
2.		$W_1l_1 + W_2l_2$		Cantilever, more than one load.
3		$\frac{wl^2}{2}$	$\frac{1}{8}\frac{W'l^3}{EI}$	Cantilever, uniform load w lb. per unit of length. $W' = w\,l$.
4.		$\frac{wl^2}{2} + Wl$	$\frac{1}{3}\frac{Wl^3}{EI} + \frac{1}{8}\frac{W'l^3}{EI}$	Cantilever, load partly uniform, partly concentrated.
5.		$\frac{Wl}{4}$	$\frac{1}{48}\frac{Wl^3}{EI}$	Simple beam, load at middle.
6.		$W\frac{l_1l_2}{l}$		Simple beam, load at some other point than the middle.
7.		$\frac{wl^2}{8}$	$\frac{5}{384}\frac{W'l^3}{EI}$	Simple beam, uniformly loaded.
8.		$\frac{3}{16}Wl$	$.0182\frac{Wl^3}{EI}$	One end fixed, other end supported, load in the middle.
9.		$\frac{wl^2}{8}$	$.0054\frac{W'l^3}{EI}$	One end fixed, other end supported, uniformly loaded.
10.		$\frac{Wl}{8}$	$\frac{1}{192}\frac{Wl^3}{EI}$	Both ends fixed, load in the middle.
11.		$\frac{wl^2}{12}$	$\frac{1}{384}\frac{W'l^3}{EI}$	Both ends fixed, uniformly loaded.

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